Article title: A Machine-Level Interpretation of Deductive Rules on Enact I and Enact II Model Theorems.

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Preprint statement: This article is a preprint and has not been peer-reviewed, under consideration and submitted to ScienceOpen Preprints for open peer review.

DOI: 10.14293/S2199-1006.1.SOR-.PPVOHXY.v2

Preprint first posted online: 04 November 2020

Keywords: model, composites, enactment, logic, proof, syntactic, theorem, enactprover, machine, Program

SYNTACTIC COMPUTATION OF MODEL COMPOSITES OF ENACTMENT LOGIC.

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Extended Abstract[†]. This research is a continuation on provable forms based on syntactic theorem using Kleene Axiom schema on Enact model VII, VIII and IX of propositional formulas from enactment logic. Enactment logic is proven in terms of theorems based on deductive rules. This work proves by deduction rules that Enact Model VII, VIII and IX are deduction theorems.

Keywords. model III, model IV, model V, composites, enactment, logic, proof, syntactic, theorem.

Year of Study: 2016

Year of Publication: 2020

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1 INTRODUCTION

This research continues on axiomatic schemes[3] of Kleene that gives theorems based on proofs of enactment models[1]. Syntactic computing[3] in

the propositional calculus is based on using axioms and deduction rules[3] to produce theorems. Kleene[3] give the following axiom schemas:

- (1a) $\alpha \rightarrow (\beta \rightarrow \alpha)$.
- (1b) $(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \gamma))$.
- (2a) $\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta))$,
- (2b) $(\alpha \rightarrow \beta) \rightarrow \alpha$.
- (2c) $(\alpha \wedge \beta) \rightarrow \beta$.
- (3a) $\alpha \rightarrow (\alpha \vee \beta)$,
- (3b) $\beta \rightarrow (\alpha \vee \beta)$,
- (3c) $(\alpha \rightarrow \gamma) \rightarrow ((\beta \rightarrow \gamma) \rightarrow ((\alpha \vee \beta) \rightarrow \gamma))$.
- (4a) $(\alpha \rightarrow \beta) \rightarrow ((\alpha \rightarrow \neg \beta) \rightarrow \neg \alpha)$,
- (4b) $(\neg \neg \alpha) \rightarrow \alpha$.

Results of Work[1]:The models of enactment logic are:

- (1) $enact_E \rightarrow enact_L$
- (2) $a_i \rightarrow a$
- (3) $a_j \rightarrow l$
- (4) $rank_i \rightarrow t$
- (5) $enact_E \rightarrow (enact_E \rightarrow a_i)$
- (6) $enact_L \rightarrow (a_j \rightarrow l)$
- (7) $a_i \rightarrow (a_j \rightarrow l)$
- (8) $a_j \rightarrow (l \rightarrow rank_i)$
- (9) $l \rightarrow (rank_i \rightarrow t)$

Axiom is produced by replacing the Greek variable of an axiom schema by formula. Models (7), (8) and (9) will be proved using Kleene expression axioms (1a) to (4b).

2 THEOREM PROVING

Let $Enact_E$ to be represented by E_E .

Let $Enact_L$ to be represented by E_L .

Let $rank_i$ to be represented by r_i .

(7) $a_i \rightarrow (a_j \rightarrow l)$: **Enact Model VII**

(7) is proven by theorem 7 as shown below:

Theorem 7:

An a_i implies consequence of $\rightarrow$ -composites of $a_j \rightarrow l$, expressed as:

$a_i \rightarrow (a_j \rightarrow l)$, then it is the consequence of
 $((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l))$.

Proof. Theorem 5 is expressed as $((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l))$.

Axiom.

- (1) $((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l))$. given can be proved from Kleene's Axiom.
- (2) $((a_i \rightarrow (a_j \rightarrow l)) \rightarrow (a_i \rightarrow a_j) \rightarrow (a_j \rightarrow (a_i \rightarrow l)))$, axiom of schema (1a).
- (3) $((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow (a_j \rightarrow l)))$ modus ponens of lines 1 and 2.

Deduction Rule. $\frac{\alpha, (\alpha \rightarrow \beta)}{\beta}$, where β is a consequence of α

and $(\alpha \rightarrow \beta)$.

$$\frac{(a_i \rightarrow (a_j \rightarrow l)) = \alpha, (\alpha \rightarrow (a_i \rightarrow a_j) \rightarrow (a_j \rightarrow (a_i \rightarrow l))) = \beta}{\beta}$$

$(a_i \rightarrow a_j) \rightarrow (a_j \rightarrow (a_i \rightarrow l)) = \beta$ is the consequence of
 $(a_i \rightarrow (a_j \rightarrow l)) = \alpha$ and $(a_i \rightarrow a_j) \rightarrow (a_j \rightarrow (a_i \rightarrow l)) = \beta$.

- (4) $((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow (a_j \rightarrow l)))$
 $\rightarrow (((a_i \rightarrow a_j) \rightarrow ((a_i \rightarrow (a_j \rightarrow l)) \rightarrow (a_i \rightarrow l))) \rightarrow ((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l)))$

axiom of schema (1b).

- (5) $((a_i \rightarrow a_j) \rightarrow ((a_i \rightarrow (a_j \rightarrow l)) \rightarrow (a_i \rightarrow l))) \rightarrow ((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l))$
 modus ponens of lines 3 and 4.

Deduction Rule. $\frac{\alpha, (\alpha \rightarrow \beta)}{\beta}$, modus ponens 3 and 4 gives,

$$\frac{\alpha_4 = \alpha_3, (\alpha_4 = \alpha_3 \rightarrow \beta_4)}{\beta_4} .$$

β_4 is a consequence of $\alpha_3 = \alpha_4$ and $(\alpha_3 = \alpha_4 \rightarrow \beta_4)$.

$$\frac{(a_i \rightarrow a_j) \rightarrow (a_i \rightarrow (a_j \rightarrow l)) = \alpha_3 = \alpha_4, (\alpha_3 = \alpha_4 \rightarrow \beta_4)}{((a_i \rightarrow a_j) \rightarrow ((a_i \rightarrow (a_j \rightarrow l)) \rightarrow (a_i \rightarrow l))) \rightarrow ((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l)) = \beta_4}$$

(6) An axiom of schema (1b) gives ,

$$(a_i \rightarrow a_j) \rightarrow ((a_i \rightarrow (a_j \rightarrow l)) \rightarrow (a_i \rightarrow l)) .$$

(7) Modus ponens on line 5 and 6 gives,

$$\frac{\alpha_5 = \alpha_6, (\alpha_5 = \alpha_6 \rightarrow \beta_5)}{\beta_5}$$

$$\frac{(a_i \rightarrow a_j) \rightarrow ((a_i \rightarrow (a_j \rightarrow l)) \rightarrow (a_i \rightarrow l)) = \alpha_5 = \alpha_6, (\alpha_5 = \alpha_6 \rightarrow \beta_5)}{((a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l)) = \beta_5}$$

β_5 is the consequence of $\alpha_5 = \alpha_6$ and $(\alpha_5 = \alpha_6 \rightarrow \beta_5)$. Thus

$$(a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l) . \text{ Hence } a_i \rightarrow (a_i \rightarrow a_j) \vdash (a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l) .$$

$$a_i, a_j, l \vdash (a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l)$$

$(a_i \rightarrow a_j) \rightarrow (a_i \rightarrow l)$ is a theorem.

(8) $a_j \rightarrow (l \rightarrow \text{rank}_i)$: **Enact Model VIII**

(8) is proven by theorem 8 as shown below:

Theorem 8:

An a_j implies consequence of $\rightarrow$ -composites of $a_j \rightarrow l$, expressed as:

$$a_j \rightarrow (l \rightarrow r_i) , \text{ then it is the consequence of}$$

$$((a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i)).$$

Proof. Theorem 8 is expressed as $((a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i))$.

Axiom. (1) $((a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i))$. given can be proved from Kleene's Axiom.

(2) $((a_j \rightarrow (l \rightarrow r_i)) \rightarrow (a_j \rightarrow l) \rightarrow (l \rightarrow (a_j \rightarrow r_i)))$, axiom of schema (1a).

(3) $((a_j \rightarrow l) \rightarrow (l \rightarrow (a_j \rightarrow r_i)))$ modus ponens of lines 1 and 2.

Deduction Rule. $\frac{\alpha, (\alpha \rightarrow \beta)}{\beta}$, where β is a consequence of α

and $(\alpha \rightarrow \beta)$.

$$\frac{(a_j \rightarrow (l \rightarrow r_i)) = \alpha, (\alpha \rightarrow (a_j \rightarrow l) \rightarrow (l \rightarrow (a_j \rightarrow r_i))) = \beta}{\beta}$$

$(a_j \rightarrow l) \rightarrow (l \rightarrow (a_j \rightarrow r_i)) = \beta$ is the consequence of $(a_j \rightarrow (l \rightarrow r_i)) = \alpha$ and $(a_j \rightarrow l) \rightarrow (l \rightarrow (a_j \rightarrow r_i)) = \beta$.

(4) $((a_j \rightarrow l) \rightarrow (l \rightarrow (a_j \rightarrow r_i)))$
 $\rightarrow (((a_j \rightarrow l) \rightarrow ((l \rightarrow (a_j \rightarrow r_i)) \rightarrow (a_j \rightarrow r_i))) \rightarrow ((l \rightarrow a_j) \rightarrow (l \rightarrow r_i)))$

axiom of schema (1b).

(5) $((a_j \rightarrow l) \rightarrow ((l \rightarrow (a_j \rightarrow r_i)) \rightarrow (a_j \rightarrow r_i))) \rightarrow ((l \rightarrow a_j) \rightarrow (l \rightarrow r_i))$

modus ponens of lines 3 and 4.

Deduction Rule. $\frac{\alpha, (\alpha \rightarrow \beta)}{\beta}$, modus ponens 3 and 4 gives,

$$\frac{\alpha_4 = \alpha_3, (\alpha_4 = \alpha_3 \rightarrow \beta_4)}{\beta_4}$$

β_4 is a consequence of $\alpha_3 = \alpha_4$ and $(\alpha_3 = \alpha_4 \rightarrow \beta_4)$.

$$\frac{(a_j \rightarrow l) \rightarrow (l \rightarrow (a_j \rightarrow r_i)) = \alpha_3 = \alpha_4, (\alpha_3 = \alpha_4 \rightarrow \beta_4)}{((a_j \rightarrow l) \rightarrow ((l \rightarrow (a_j \rightarrow r_i)) \rightarrow (a_j \rightarrow r_i))) \rightarrow ((l \rightarrow a_j) \rightarrow (l \rightarrow r_i)) = \beta_4}$$

(6) An axiom of schema (1b) gives,

$$(a_j \rightarrow l) \rightarrow ((a_j \rightarrow (l \rightarrow r_i)) \rightarrow (a_j \rightarrow r_i))$$

(7) Modus ponens on line 5 and 6 gives,

$$\frac{\alpha_5 = \alpha_6, (\alpha_5 = \alpha_6 \rightarrow \beta_5)}{\beta_5}$$

$$\frac{(a_j \rightarrow l) \rightarrow ((a_j \rightarrow (l \rightarrow r_i)) \rightarrow (a_j \rightarrow r_i)) = \alpha_5 = \alpha_6, (\alpha_5 = \alpha_6 \rightarrow \beta_5)}{((a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i)) = \beta_5}$$

β_5 is the consequence of $\alpha_5 = \alpha_6$ and $(\alpha_5 = \alpha_6 \rightarrow \beta_5)$. Thus

$$(a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i)$$

Hence $a_j \rightarrow (l \rightarrow r_i) \vdash (a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i)$.

$$a_j, l, r_i \vdash (a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i)$$

$(a_j \rightarrow l) \rightarrow (a_j \rightarrow r_i)$ is a theorem.

(9) $l \rightarrow (rank_i \rightarrow t)$: **Enact Model IX**

(9) is proven by theorem 9 as shown below:

Theorem 9:

1 implies consequence of $\rightarrow$ -composites of $r_i \rightarrow t$, expressed as:

$l \rightarrow (r_i \rightarrow t)$, then it is the consequence of

$((l \rightarrow r_i) \rightarrow (l \rightarrow t))$.

Proof. Theorem 9 is expressed as $((l \rightarrow r_i) \rightarrow (l \rightarrow t))$.

Axiom. (1) $((l \rightarrow r_i) \rightarrow (l \rightarrow t))$. given can be proved from Kleene's

Axiom.

(2) $((l \rightarrow (r_i \rightarrow t)) \rightarrow (l \rightarrow r_i) \rightarrow (r_i \rightarrow (l \rightarrow t)))$, axiom of schema (1a).

(3) $((l \rightarrow r_i) \rightarrow (r_i \rightarrow (l \rightarrow t)))$ modus ponens of lines 1 and 2.

Deduction Theorem.

$\frac{\alpha, (\alpha \rightarrow \beta)}{\beta}$, where β is a consequence of α and $(\alpha \rightarrow \beta)$.

$\frac{(l \rightarrow (r_i \rightarrow t)) = \alpha, (\alpha \rightarrow (l \rightarrow r_i) \rightarrow (r_i \rightarrow (l \rightarrow t))) = \beta}{\beta}$

β
 $(l \rightarrow r_i) \rightarrow (r_i \rightarrow (l \rightarrow t)) = \beta$ is the consequence of $(l \rightarrow (r_i \rightarrow t)) = \alpha$ and
 $(l \rightarrow r_i) \rightarrow (r_i \rightarrow (l \rightarrow t)) = \beta$.

(4) $((l \rightarrow r_i) \rightarrow (l \rightarrow (r_i \rightarrow t)))$
 $\rightarrow (((l \rightarrow r_i) \rightarrow ((l \rightarrow (r_i \rightarrow t)) \rightarrow (l \rightarrow r_i))) \rightarrow ((r_i \rightarrow l) \rightarrow (r_i \rightarrow t)))$

axiom of schema (1b).

(5) $((l \rightarrow r_i) \rightarrow ((l \rightarrow (r_i \rightarrow t)) \rightarrow (l \rightarrow t))) \rightarrow ((r_i \rightarrow l) \rightarrow (r_i \rightarrow t))$

modus ponens of lines 3 and 4.

Deduction Rule. $\frac{\alpha, (\alpha \rightarrow \beta)}{\beta}$, modus ponens 3 and 4 gives,

$\frac{\alpha_7 = \alpha_8, (\alpha_7 = \alpha_8 \rightarrow \beta_8)}{\beta_8}$.

β_8 is a consequence of $\alpha_7 = \alpha_8$ and $(\alpha_7 = \alpha_8 \rightarrow \beta_8)$.

$$\frac{(l \rightarrow r_i) \rightarrow (l \rightarrow (r_i \rightarrow t)) = \alpha_3 = \alpha_4, (\alpha_3 = \alpha_4 \rightarrow \beta_4)}{((l \rightarrow r_i) \rightarrow ((l \rightarrow (r_i \rightarrow t)) \rightarrow (l \rightarrow t))) \rightarrow ((r_i \rightarrow l) \rightarrow (r_i \rightarrow t)) = \beta_4}$$

(6) An axiom of schema (1b) gives ,

$$(l \rightarrow r_i) \rightarrow ((l \rightarrow (r_i \rightarrow t)) \rightarrow (r_i \rightarrow t)) .$$

(7) Modus ponens on line 5 and 6 gives,

$$\alpha_5 = \alpha_6, (\alpha_5 = \alpha_6 \rightarrow \beta_5)$$

β_5

$$\frac{(l \rightarrow r_i) \rightarrow ((l \rightarrow (r_i \rightarrow t)) \rightarrow (r_i \rightarrow t)) = \alpha_5 = \alpha_6, (\alpha_5 = \alpha_6 \rightarrow \beta_5)}{((l \rightarrow r_i) \rightarrow (l \rightarrow t)) = \beta_5}$$

β_5 is the consequence of $\alpha_5 = \alpha_6$ and $(\alpha_5 = \alpha_6 \rightarrow \beta_5)$. Thus

$$(l \rightarrow r_i) \rightarrow (l \rightarrow t) .$$

Hence $l \rightarrow (r_i \rightarrow t) \vdash (l \rightarrow r_i) \rightarrow (l \rightarrow t)$.

$$l, r_i, t \vdash (l \rightarrow r_i) \rightarrow (l \rightarrow t)$$

$(l \rightarrow r_i) \rightarrow (l \rightarrow t)$ is a theorem.

3 CONCLUSION

This concludes work on a deduction theorem proving on Enact model VII, VIII, and IX of enactment logic in [1,2]. Thw work discusses the three remaining enact models (VII), (VIII) and (IX) of enactment logic. It is little difficult to follow but bear with me and follow the Kleene Axioms.

Compliance with Ethical Standards

(In case of funding) Funding: This is research is funded by King's Alumni Group, Association of Engineering with ISAreference grant number: 204424 20821845.

Conflict of Interest:

Author, Dr. Frank Appiah declares that he has no conflict of interest .

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