Model Order Reduction of Nonlinear Magnetodynamics with Manifold Interpolation

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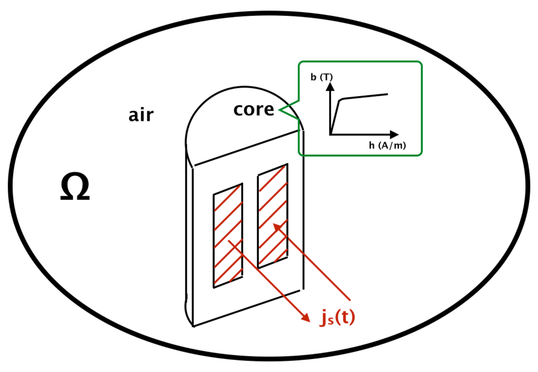
Introduction & Goal

The aim of this research is to determine efficiently the solution of a magnetodynamic problem for an unknown input parameter set based on pre-computed information. This information comes from pre-solved problems for different input parameter sets. Therefore, we have to:

- Construct reduced order magnetodynamic models (ROM).
- Reuse information from different pre-computed solutions.

This method is demonstrated on a nonlinear inductor-core system where the input frequency is a parameter.



Formulations

The magnetodynamic problem is ruled by (1) and can be split into linear and nonlinear parts as in (2).

$$\mathbf{curl}\,(\nu\,\mathbf{curl}\,a(t)) + \sigma\partial_t a(t) = \mathbf{j_s}(t) \quad (1)$$

$$\Leftrightarrow \underbrace{\mathbf{curl}\,(\nu_0\,\mathbf{curl}\,a(t)) + \sigma\partial_t a(t) - \mathbf{j_s}(t)}_{\text{Linear part}} = \underbrace{-\mathbf{curl}\,(\tilde{\nu}\,\mathbf{curl}\,a(t))}_{\text{Nonlinear part}} \quad (2)$$

POD [1, 2]

Proper Orthogonal Decomposition (POD) is very efficient to obtain reduced order magnetodynamic models in the linear case, or in the nonlinear case when the input parameters do not vary. The reduced and full vector of unknowns, resp. a_r and a, are linked with the reduction basis Ψ by

$$a = \Psi a_r$$

where $a \in \mathbb{R}^{n\times 1}$, $a_r \in \mathbb{R}^{r\times 1}$ and $\Psi \in \mathbb{R}^{n\times r}$. We expect $r << n$ to reduce the number of unknowns.

The basis is obtained by a applying a thin SVD on a snapshot matrix S. This matrix S gathers the solutions for all time steps:

$$S = \left[a_1, a_2, \cdots, a_T\right] \in \mathbb{R}^{n\times T}$$

where T is the number of time steps. The reduced basis Ψ is obtained by

$$[U, S, V] = \mathrm{svd}(S),$$
$$\to \Psi = U.$$

Since the number of time steps is smaller than the size of the initial unknown vector, the matrix U doesn't have to be truncated as it is usually done. Here $r = T$.

In our case, $n = 553$ and $r = 20$.

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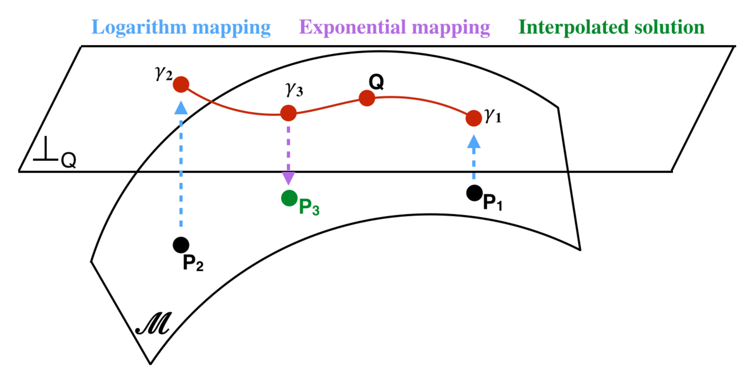
Interpolation on Manifolds [3]

The reduction bases Ψ lie on the Grassmann manifold where interpolation can be performed. The reduction bases are mapped to the tangent space at point Q using a logarithm mapping

$$(I - \Psi_Q\Psi_Q^T)\Psi_i(\Psi_Q^T\Psi_i)^{-1} = U\Sigma V^T \text{ (thin SVD)}, \quad (3)$$
$$\gamma_i = U\tan^{-1}(\Sigma)V^T. \quad (4)$$

These projections lie on a plane space and can be interpolated (e.g. via Lagrange intepolation) to obtain the projection of Ψ^*, γ^*. This interpolated projection is mapped back onto the Grassmann manifold using the corresponding exponential mapping

$$\gamma^* = U^*\Sigma^*{V^*}^T \text{ (thin SVD)}, \quad (5)$$
$$\Psi^* = \Psi_Q V^*\cos(\Sigma^*) + U^*\sin(\Sigma^*). \quad (6)$$

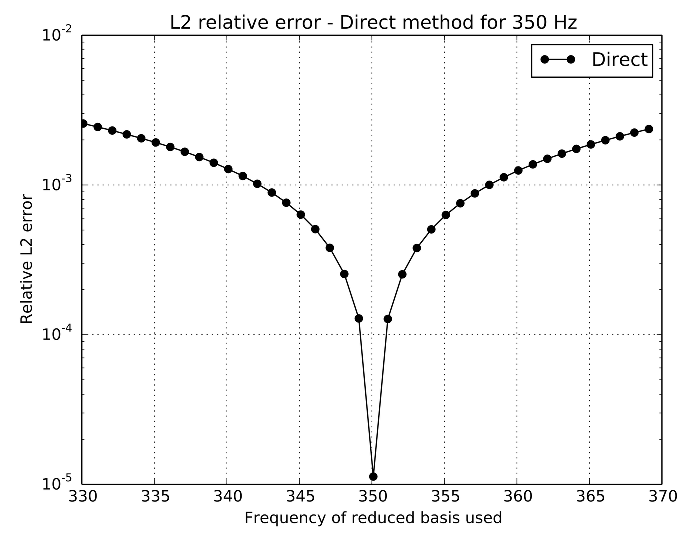


In our case, P_3 is the unknown reduced basis Ψ^*.

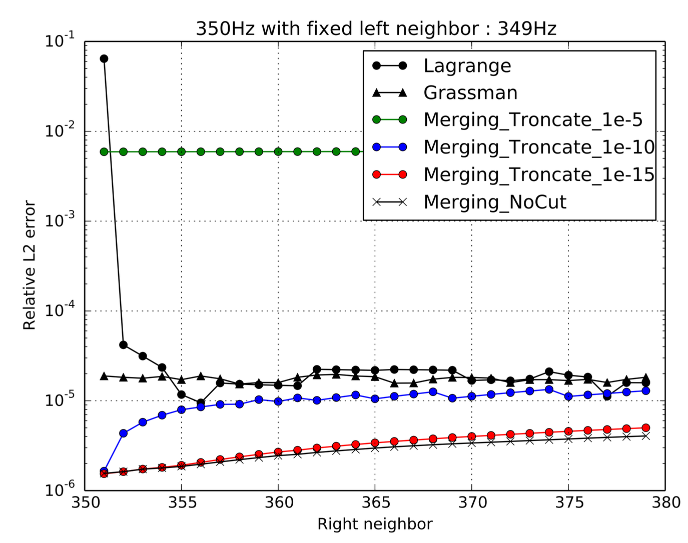
Results

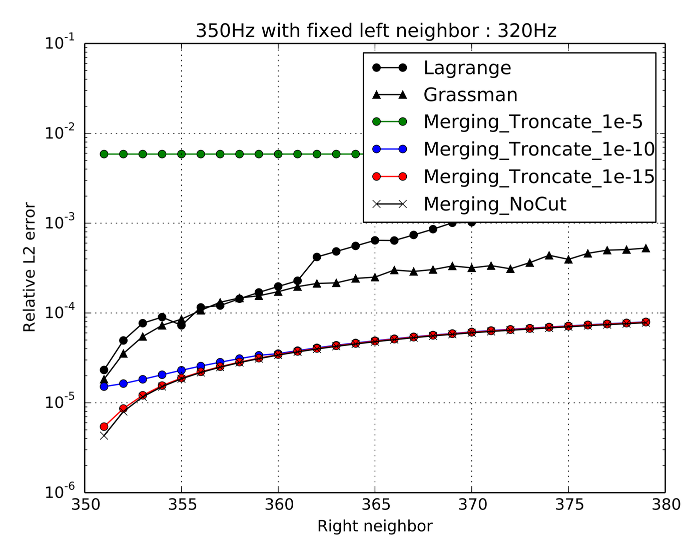
The input excitation of the nonlinear inductor-core system is given by $\mathbf{j_s}(t) = J\sin(\omega t)$. Due to dynamical effects, the solution spatially changes with the input frequency. Here frequency $f = 350$ Hz.

The ROM is directly constructed with a reduced basis Ψ without interpolation. The L2 error between the ROM and the full model is plotted below where the reduction basis varies from 330 Hz to 370 Hz.



The ROM is constructed with an interpolated reduced basis or a reduced basis based on merged snapshots of different solutions. The interpolation uses two basis/snapshots around 350Hz. The first one is fixed from solution at 320Hz. The second one is varying from 351Hz to 380Hz.





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