Loop Quantum Gravity meets Topological Phases of Matter at the Black Hole horizon

Andreas G. A. Pithis[1] and Hans-Christian Ruiz[2]

[1]Department of Physics, King's College London, Strand, London, WC2R 2LS, U.K.

[2]Donders Institute, Radboud University, 6525 EZ Nijmegen, The Netherlands

Abstract

In this work we investigate the role played by large diffeomorphisms of quantum isolated horizons for the statistics of Loop Quantum Gravity black holes by means of their relation to the braid group. The mutual exchange of quantum entities in two dimensions is achieved by the braid group, rendering the statistics anyonic. With this we argue that the quantum isolated horizon model of LQG based on $SU(2)_k$-Chern-Simons theory explicitly exhibits non-abelian anyonic statistics, since the quantum gravitational degrees of freedom of the horizon can be seen as flux-charge composites. In this way a connection to the theory behind the fractional quantum Hall effect and that of topological quantum computation is established, where non-abelian anyons play a significant role.

Introduction

The horizon of black holes as an inner boundary of space can be described in equilibrium locally by the isolated horizon (IH) boundary condition [1]. Physically, its introduction amounts to having no fluxes of energy across it. Technically, the boundary condition leads in the overall action of the gravitational field to a surface term for the horizon which in terms of Ashtekar-Barbero connection variables A is proportional to the action of Chern-Simons-theory on $\mathbb{R} \times S^2$. Hence, the gravitational field of the horizon resides in a topological phase. This description is fully compatible with the laws of black hole mechanics.

The quantum geometric handling of spacetimes with such an IH by means of LQG techniques describes the quantum geometry of the bulk by a spin network, whose graph pierces the horizon surface yielding a set of punctures $\mathcal{P} = \{p_i\}_{i=1}^N$ as in (1).

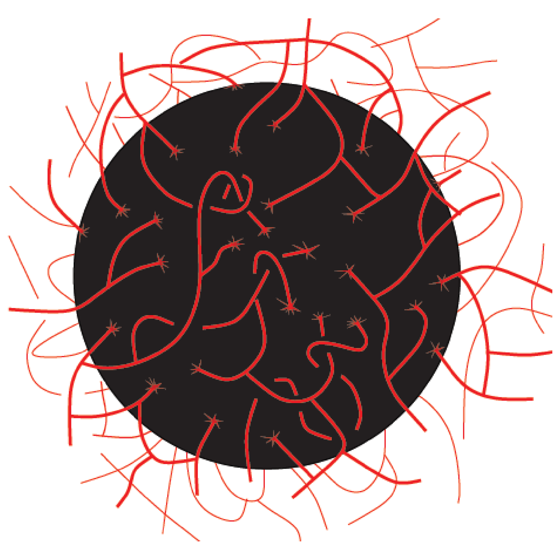

Figure 1: Bulk spin network impinging on the horizon.

The totality of the punctures forms a gas of topological defects which represent the quantum excitations of the gravitational field of the horizon. These d.o.f. are then described by $SU(2)$-CS-theory at level k given on a punctured S^2. $SU(2)$ arises from the local gauge group of gravity and $k \propto a_H/\ell_p^2$, with a_H the classical horizon area and Planck length ℓ_p. Thus

$$S_{\text{horizon}} = \frac{k}{4\pi}\int_{\mathbb{R}}\int_{S^2} \mathrm{Tr}(-A\partial_0 A + 2A_0 F)$$

with

$$F^i + \frac{4\pi}{k}\sum_{p\in\mathcal{P}} J^i_{\rho_p}\delta^2(x, x_p) = 0$$

and $i \in \{1,2,3\}$ denotes an $su(2)$-index [2].

Equipped with this, one sets out to count the microstates of the corresponding CS-Hilbert space. Together with proper notions of quasi-local energy and local temperature of the IH its statistical mechanical analysis is facilitated and an expression for the IH entropy is obtained, which is compatible with the semiclassical Bekenstein-Hawking area law [3].

Despite these successes in matching the semiclassical results, in this work we wanted to investigate whether the statistics of the IH quantum d.o.f. could actually be different from the one assumed, motivated by the well-known fact from solid state physics that quantum objects in $2d$ obey anyonic statistics. Indeed, we found that the LQG black hole model explicitly exhibits non-abelian anyonic/braiding statistics [4].

From statistical physics one knows that particle exchange is equivalent to an (adiabatic) parallel transport along paths that fall into the first homotopy group π_1 of the respective N-particle configuration space Q_N. This leads to a kinematical ambiguity in the corresponding quantum theory: Since state vectors ψ are sections of a complex line bundle over Q_N, these bundles are classifed by unitary irreducible representations of π_1,

$$\rho : \pi_1(Q_N) \to \mathrm{End}(\mathcal{H}_N), \quad \psi \to \rho\psi.$$

In $3d$ one has $\pi_1(Q_N) = S_N$, the symmetric group, giving rise to bosonic/fermionic statistics whereas in $2d$ it is the braid group B_N, giving rise to anyonic statistics. Hence, the statistical properties of a N-particle system are closely related to the global topological structure of the configuration space.

For scalar quantum theories these irreducible representations are $1d$, however, if wavefunctions are multiplets - as in case of the LQG black hole - one has higher dimensional irreps and in general one has

$$[\rho_1, \rho_2] \neq 0.$$

LQG Black hole and anyons

For LQG black holes, the kinematical ambiguity which arises in the quantization on the configuration space of N (distinguishable) punctures, yields a non-abelian phase for the multicomponent state vectors: $\rho \in \mathrm{End}(\pi_1(Q_N), \mathcal{H}_N)$, where $\pi_1(Q_N) = B_N(S^2)$ is the spherical braid group. Interestingly, it is tightly linked to the group of large diffeomorphisms via

$$M_N(S^2) \cong B_N(S^2)/\mathbb{Z}_2.$$

Hence, the quantum statistics of the horizon d.o.f. is related to the large diffeomorphism symmetry group of the problem.

To calculate such phases, assume the perspective of a local stationary observer at proper distance to horizon and that for large k the horizon looks locally like a punctured plane $\mathbb{R}^2_N$. The double exchange of two punctures p and p' is then achieved by the parallel transport of the Knizhnik-Zamolodchikov connection along a loop C, yielding

$$\psi \to \rho\psi = P\, e^{i\oint_C A_K}, \quad A_K = \frac{4\pi}{k+2}\sum_{1\le p<p'\le N} J^i_p \otimes J^i_{p'}\omega_{pp'},$$

with $\omega_{pp'} = \frac{1}{2\pi i} d\log(z_p - z_{p'}) \in H^1(\mathbb{R}^2_N;\mathbb{Z})$ and $[C] \in B_N(\mathbb{R}^2)$.

For two punctures 1 and 2 a double exchange gives the monodromy

$$M_{12} = q^{2\, J^i_1 \otimes J^i_2}, \quad q = e^{i\frac{2\pi}{k+2}},$$

whereas a half-exchange/simple braid gives

$$B_{12} = q^{J^i_1\otimes J^i_2} P_{12},$$

with the permutation P_{12} and $M = B^2$ as in (2).

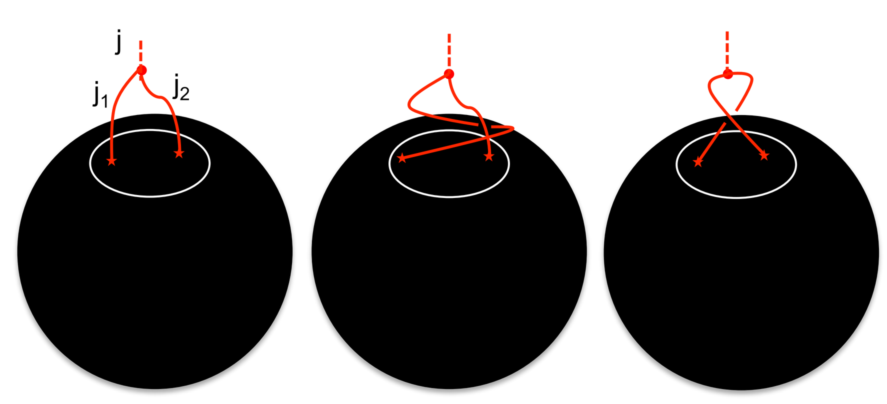

Figure 2: Two incident bulk edges piercing the horizon: unbraided (left), upon the application of M (centre) and B (right).

In the semiclassical limit, where $k \to \infty$, $M \to \mathbb{I}$ and $B \to P_{12}$, hence braiding is understood as a quantum effect, relevant for small black holes.

A local stationary observer could in principle measure the effects of the braiding, since for finite k

$$[F^i, \rho] \neq 0.$$

In analogy to the Aharonov-Bohm effect in electromagnetism, consider two punctures encircling each other as in (3). Let puncture 1 have non-abelian flux ϕ_i and puncture 2 carry non-abelian charge J^i, where the gravitomagnetic flux ϕ^i is obtained via

$$B^i = \frac{1}{2}\epsilon^{ab}F^i_{ab} = \frac{4\pi}{k}\delta^2(x, x_1)\, J^i, \quad \phi^i = \int B^i\epsilon_{ab}dx^a dx^b.$$

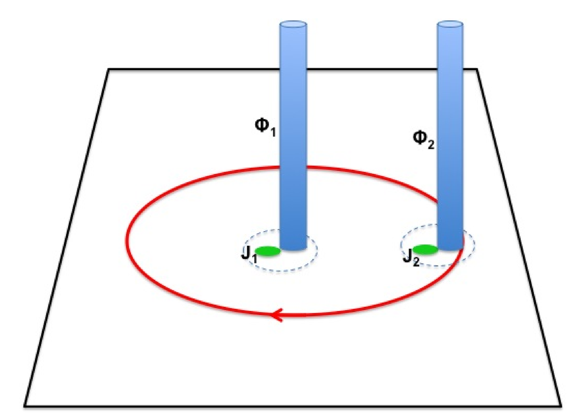

Figure 3: Incident bulk spin network edges seen as non-abelian flux-charge composites.

Comparing unbraided ψ with braided $\tilde{\psi}$ one has

$$||\tilde{\psi}||^2 = 1 + \Re(\langle\psi|M|\psi\rangle),$$

with $M = q^{2J^i_1\otimes J^i_2} = e^{i\phi^i\otimes J^i_2}$. Hence, the interference pattern changes with the non-abelian gravitomagnetic flux ϕ_i.

Conclusions and outlook

A full comparison of the mathematical structures used to describe the LQG black hole model with the structures used to describe non-abelian $su(2)_k$-anyons in condensed matter theory [5] shows that both are given by analogous unitary braided fusion categories. In this light, the Hilbert space of the horizon d.o.f. is reinterpreted as the fusion Hilbert space of non-abelian anyons [4].

The braiding amounts to non-local observable correlations among the horizon d.o.f. and hence increases order. It could be investigated to what extent this quantitatively reduces the entropy.

Apart from the qualitative arguments given in [4], that the black hole radiance spectrum should display traces of the non-trivial braiding statistics, it could also be interesting to exactly quantify how much the emission spectra proposed by LQG are then actually modified.

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