Abstract: The study on how Marx created the theory of social capital reproduction shows that this theory ranks among his original contributions to economics. In this theory, Marx's analysis of Department II and the exchange relation reveals the conditions that capitalist reproduction must follow if it is to proceed smoothly. The fully indirect exchange process indicates that the capitalists' individual consumption preferences, and changes in these preferences, are factors not only in determining capital accumulation but also in destroying the proportional relations of reproduction. Although Marx proposed the decomposition of Department II and made a theoretical analysis of the law of the simple reproduction of social capital after decomposition, he did not construct a complete model or make the same decomposition of the two major departments in the analysis of the reproduction of social capital on an extended scale. As a result, it is necessary to develop a technical supplement on the basis of Marx's department decomposition principle, so as to cast light on the correctness of his theory of social capital reproduction and on the inevitability of capitalist reproduction crises.

Keywords: department decomposition; technical supplement; reproduction crisis; social capital reproduction

The Creation of Marx’s Theory of Social Capital Reproduction

Marx’s theory of social capital reproduction is an important part of Marxist economics. It is generally believed that this theory, like Marx’s economic theory as a whole, was created on the basis of a critical inheritance of the theory of classical
economic on the reproduction of social capital. Despite its prevalence, this view is not entirely consistent with reality. From the process that led to the establishment of Marx’s theory of social capital reproduction, it can be argued that this theory should be considered to be Marx’s original creation.

The development of Marx’s theory of social capital reproduction was combined with his observation and analysis of the economic crisis of capitalism. In the late 1840s, Marx had explored the contradiction between capitalist production and the market in works such as The Poverty of Philosophy (Marx 1847a) and Wage Labor and Capital (Marx 1847b), which demonstrated for the first time that the capitalist competition to increase labor productivity in order to extract more profit would deepen and expand the division of labor and the scale of production while impoverishing the workers, thus creating a contradiction between the continuous expansion of production and the narrowing of the market. As a result, periodic economic crises would arise in response to the times and conditions.

In 1851, in a manuscript entitled “Reflections on Money,” Marx ([1851] 1977) observed that Adam Smith had divided the exchange of social products into the trade between industrialists and the trade between industrialists and consumers. The former is the transfer of capital, and the latter is the exchange of income and capital. It is important to distinguish between the departments involved here, but Smith failed to account for the connection between the two kinds of trade and the two kinds of currency. Smith was unable to show this connection because he attributed all products to income and concluded that the trade volume between industrialists and consumers was equal to that between industrialists. It is on the basis of Smith’s theory that some economists believe that the two kinds of trade must be balanced; thus denying the existence of crises of overproduction. Marx pointed out that all crises in fact show that trade between industrialists always exceeds the limits set by the trade between industrialists and consumers. For this reason, there is an imbalance between the two kinds of trade. All the assertions used by economists to prove the impossibility of a general crisis of overproduction concern only the trade between industrialists, failing to see that this trade is restricted by the trade between industrialists and consumers, which in turn is restricted by the income of the working class whose members constitute the largest part of the consumers. At the same time, trade between industrialists will create trade between industrialists and consumers. As a result, trade between industrialists and consumers mostly runs up against trade between industrialists, and the crisis always occurs first in trade between industrialists and consumers. Therefore, overproduction is not only attributable to disproportionate production, but also to the relationship between the capitalist class and the working class (Marx and Engels 1982, 154–156). Marx also analyzed the problem of currency circulation in the process of capital realization, which actually involves the realization of the
value of capital (Marx and Engels 1982, 156–159). These facts show that Marx’s analysis reflects his thoughts on the realization of aggregate social products even though it is developed under the premise of Smith’s division of two kinds of trade.

In his economic manuscripts of 1857 and 1858, Marx initially constructed a logical system for reproducing the capitalist mode of production in theory, and explicitly included the reproduction and accumulation of capital in its circulation process as his topic of “The Circulation Process of Capital,” which is the second section of “The Chapter of Capital.” In the manuscript, Marx studied the implementation of aggregate social products from the perspective of the unity of the production and circulation processes of capital. He pointed out that capital represents the unity of the value preservation process of the capital used, the value proliferation process of capital and the value implementation process of products. However, the three processes are separate in time and space. Although these three processes have intrinsic unity, they are independent of each other, and each process is a prerequisite for another. For this reason, capital must be in circulation after the end of the production process. As a commodity, capital must be of use-value and must be exchanged for money. In order to be renewed, all the products, i.e., the aggregate social product, must be converted into money and must be fully realized. Marx proposed two implementation conditions for the commodity. The first condition is the consumption itself, i.e., the demand for the commodity, and the second is that there must be an equivalent for the commodity (Marx and Engels 1979, 383–390). To illustrate the implementation of the aggregate social product, Marx divided the aggregate social product into constant capital, variable capital and surplus-value in terms of value, as well as raw materials (A, B), the machinery (C), workers’ necessaries (E) and surplus products (D) in terms of the use of products. On the basis of the use of products, Marx divided social reproduction into five major sectors, and established a capitalist economy composed of five major production sectors. Taking the implementation process of surplus-value of sector E as an example, Marx analyzed in detail the process of value compensation and in-kind replacement of each component of the aggregate social product, and described the implementation conditions of the aggregate social product (Marx and Engels 1979, 432–437). If we combine and sum up sectors A, B and C, which belong to the same sector as the means of production, and combine and sum up sectors D and E, which belong to the sector of the means of consumption, respectively, and the five implementation conditions are summed up by the nature of sector consolidation, then the first and second departments and their corresponding outputs defined by Marx in Volume II of Capital as well as the three main points of exchange and the three basic implementation conditions of social capital reproduction can be obtained. This shows that Marx solved the basic problem of social capital reproduction.
In his manuscripts from 1861 to 1863, Marx provided an in-depth critique of Smith’s theory of commodity-value formation, i.e., “Smith’s dogma.” On this basis, Marx divided social production into consumer goods production (A) and means of production (B), using a recursive method to construct a capitalist economy composed of 729 different production spheres \( I = 1 + 2 \left( 1 + \sum_{i=1}^{5} 3^{i} \right) \), \( I \) stands for production spheres) to analyze the implementation of the aggregate social capital with simple reproduction. Through this analysis, Marx solved the problems of exchange between the two major departments (the exchange of capital and income) and of the implementation of invariant capital in Department B, and then put forward three exchange points of implementation conditions for the simple reproduction of capitalism (Marx and Engels 1972, 236–258). In the manuscript, Marx provided a scientific interpretation of Quesnay’s economic table (Marx and Engels 1972, Chapter 6, 323–366). In the meantime, Marx transformed Quesnay’s economic table schema on the basis of his own theory of reproduction, and constructed a schema of reproduction with the production of means of consumption as Department I and the production of means of production as Department II (Marx and Engels 1985, 166–168).

In the manuscript of the second volume of *Capital*, Marx abandoned the economic table with the traces of Quesnay and adopted his own original formula of social capital reproduction and the diagram of two major departments.

The above brief review shows that Marx’s theory of social capital reproduction should be regarded as his original creation, from scientific terms to theoretical content. The relation between this theory and classical economics is mainly critical, based on Marx’s profound understanding of the internal contradictions of capitalist reproduction and on his theory of labor value.

Lawrence Klein believes that Marx’s reproduction and accumulation schema is the forerunner of theoretical modeling (Klein 1964). One of the major influences exerted on contemporary economics by Marx’s theory of social capital reproduction has proceeded through the modern economic growth theory established by Feldman. The task of this thesis is to decompose Department I by following Marx’s approach to the decomposition of Department II, on the basis of his theory of social capital reproduction, and then to devise a technical supplement to his social capital reproduction model from the viewpoint of department decomposition. The significance of this supplement is that Marx did not provide the corresponding formula and model, even though he had given a theoretical explanation of the implementation process of the products after the decomposition of the two major departments. This thesis holds that it is necessary to furnish this technical supplement to Marx’s social capital reproduction model, on account of the complexity of the implementation conditions for the aggregate social product after the decomposition of the two major departments, and due to the importance of this complexity for understanding the capitalist crisis. The reproduction schema in the Marxist economic context is referred to hereafter as the reproduction model.
The Decomposition of Department I under the Condition of Decomposition of Department II, and the Exchange Relation between the Two Departments

Marx employs two assumptions in his analysis of the simple reproduction of social capital. First, the organic composition of capital of the two departments (the ratio of constant capital to variable capital) is 4:1 in each case. Second, the rate of surplus-value of the two departments is 100%. The formulas are expressed as:

\[
\begin{align*}
\text{I} & : 4000c + 1000v + 1000m = 6000 \\
\text{II} & : 2000c + 500v + 500m = 3000
\end{align*}
\]

After giving three exchange points of two departments and analyzing the exchange relation and implementation condition between them, Marx thought it essential to devote himself to the exchange problem within Department II. He stated that \( v + m \) had also to be studied in the value of products of Department II, which involves the question of how variable capital flows back to its starting point (Marx 2004, 447). He further divided the products of Department II into two major sub-departments according to their end use (Marx 2004, 448).

The Sub-department a is IIa, which produces necessities consumed by both workers and capitalists. Its products make possible the consumption of the working class and also provide part of the consumption of the capitalist class, since they consist of necessities, even if in the case of the capitalists these means are often different in terms of their quality and value from the necessities of the workers. For the purpose of his study, Marx classified these products as items of the necessities.

The Sub-department b is IIb, which consists of luxury means of consumption. These items are accessible only to the capitalist class, and so they can only be exchanged through the expenditure of surplus-value, which is absolutely not in the hands of the workers.

The variable capital of IIa can flow back directly, because the value products \( v + m \) in Sub-department IIa exist in the form of necessities. Marx pointed out:

As far as the first category is concerned it is obvious that the variable capital advanced in the production of the commodities belonging in it must flow back in money-form directly to that portion of the capitalist class II (i.e., the capitalists IIa) who have produced these necessities of life . . . This reflux is direct so far as this entire sub-division a of capitalist class I is concerned, no matter how numerous the transactions may be between the capitalists of the various pertinent branches of industry, by means of which the returning variable capital is distributed pro rata.
These are processes of circulation, whose means of circulation are supplied directly by the money expended by the labourers. (Marx 1956, 246–247, emphasis original)

However, the physical forms of the value products of Sub-department IIb are luxuries, and the variable capital of the sub-department cannot flow back directly:

The entire portion of the value produced in this sub-division, IIb(v + m), exists in the bodily form of articles of luxury, i.e., articles which the labouring class can buy no more than it can buy commodity-value I existing in the form of means of production, notwithstanding the fact that both the articles of luxury and the means of production are the products of these labourers. (Marx 1956, 247)

There is, in fact, a similar problem with the implementation of surplus-value.

To illustrate the exchange process within Department II, Marx assumed the value product of the entire Department II to be 1000, i.e., II(v + m) = 500v + 500m, of which the value of essential consumption goods is v = 400, m = 400 and the value of luxuries is 100v + 100m = 200. Assuming that 60% of the surplus-value of the capitalist is used for the consumption of necessities and 40% for the consumption of luxuries, then II(v + m) can be decomposed as:

IIa(400v + 240m + 160m)
IIb(100v + 60m + 40m)

The implementation is that 640 physical forms of IIa(400v + 240m), i.e., the supplies, are essential consumption goods, and the value forms, i.e., the demands, are the wages used by the workers for personal consumption and the surplus-value used by the capitalists for the consumption of necessaries. Supplies and demands are balanced in their total amount and the structure, which can be realized within the sub-department, and v prepaid by the capitalists can flow back directly. The physical forms of IIb(40m) are luxuries, and the value-form is the surplus-value used by capitalists to consume luxuries. Supplies and demands are also balanced in the total amount and in the structure, which can still be realized within the sub-department. The physical forms of IIa(160m) are essential consumer goods, and the value-form is the surplus-value used by capitalists to consume luxuries, while the demands are luxuries, not necessities. Even though the total amount is balanced, the structures do not match and are out of balance. Similarly, the physical forms of IIb(100v + 60m) are luxuries, but the value forms are the wages used by the workers for personal consumption and the surplus-value used by the capitalist for the consumption of necessaries. What is needed is necessities, not luxuries. Although the total value of luxuries is equal to the total value of the capitalists’
demand for necessities, the structures are still not matched. It follows that the supplies and demands are complementary, and can be replaced in kind and compensated in value by exchange between the sub-departments. Therefore, IIb(ν) can only flow back indirectly. The above implementation process is formulated by Marx as follows (Marx 2004, 451):

\[
\begin{align*}
(a) & \quad \frac{\nu}{400\nu(a)} + \frac{m}{240m(a) + 100\nu(b) + 60m(b)} = 800 \\
(b) & \quad \frac{\nu}{100m(a)} + \frac{m}{60m(a) + 40m(b)} = 200
\end{align*}
\]

The symbols in brackets indicate the exchange relationship. This will inevitably affect the exchange relation between the two departments, since Department II is divided into two sub-departments. For simplicity, Marx assumed that the capital composition (the ratio of constant capital to variable capital) of the two sub-departments of Department II is also 4:1, so the composition of the two sub-departments is as follows:

- (IIa) 1600c + 400ν + 400m = 2400
- (IIb) 400c + 100ν + 100m = 600

According to the primary implementation conditional expression I(ν + m) = IIc of the two departments, Marx pointed out:

Thereinto, 1,600 of the 2000IIc in articles of consumption, which are exchanged for 2000I(ν + m) as consumer information, are exchanged for means of production of necessities of life and 400 for means of production of luxuries. (Marx 1956, 249)

Therefore, 2000I(ν + m) should also be divided into the means of production I(800ν + 800m) = 1600 for the production of essential means of livelihood and the means of production I(200ν + 200m) = 400 for the production of luxuries. I(800ν) + I(200ν) = 1000 is realized because wages are spent on the means of consumption, and the money capital prepaid for wages will be distributed equally among the various capitalist producers of Department I while flowing back. Their respective prepaid variable capital will then be compensated in money. I(800m) + I(200m) = 1000 will receive 600IIa and 400IIb with the form of means of consumption from the remaining II(1000c) equally. The capitalists who compensate constant capital for IIa will receive 480 from 600c(IIa) and 320 from 400c(IIb) for a total of 800, and the capitalists who compensate constant capital for IIb will receive 120 from 600c(IIa) and 80 from 400c(IIb) for a total of 200. The two compensations add up to 1000, and the whole product is realized (Marx 2004, 452–453).
Nevertheless, it is obvious that the formula given by Marx for the exchange relation within the second department is not as easy to understand as that for the two major departments. At the same time, the decomposition of \( I(v + m) \) and the exchange relation of \( I(v + m) = IIc \) when Department II is decomposed into two parts have been given by Marx, but the corresponding model has not been provided. In the following, we will construct a technical model supplement based on Marx’s theory of social capital reproduction and his analysis of the implementation conditions of the two departments under the decomposition of Department II.

The exchange process within the second department given by Marx can be represented by the exchange model as:

\[
\begin{align*}
IIa & : 400v + 240m + 160m \\
IIb & : 100v + 60m + 40m
\end{align*}
\]

If the ratio of the surplus-value used by capitalists for the consumption of daily necessities is \( x_a \), and the ratio of the surplus-value used for the consumption of luxuries is \( x_b \), then the primary implementation condition of the internal exchange of the second department can be expressed as \( IIa.mx_b = IIb.(v + mx_a) \), thereinto \( x_a + x_b = 1 \). Since the second department is decomposed into two sub-departments, the first department should also be decomposed into two sub-departments. One is the production Sub-department Ia of the means of production for producing necessities, and the other is the production Sub-department Ib of the means of production for producing luxuries. According to Marx’s hypothesis, Department I after decomposition into two sub-departments can be expressed by the formulas:

\[
\begin{align*}
Ia & : 3200c + 800v + 800m = 4800 \\
Ib & : 800c + 200v + 200m = 1200
\end{align*}
\]

Two simple reproduction models of the two departments can be expressed as:

\[
\begin{align*}
Ia & : 3200c + 800v + 800m = 4800 \\
Ib & : 800c + 200v + 200m = 1200 \\
IIa & : 1600c + 400v + 400m = 2400 \\
IIb & : 400c + 100v + 100m = 600
\end{align*}
\]

Again, according to Marx’s hypothesis, the capitalists in the different departments spend 60% of their surplus-value on the consumption of necessities, i.e., \( x_a = 0.6 \); hence, we have:
Marx analyzed in detail the exchange relation between the two sub-departments within department II under the decomposition conditions, showing the realization of prepaid variable capital, but did not analyze the exchange relation between the two departments under the decomposition conditions. In fact, the exchange relation between the two departments under the conditions of department decomposition will be much more complicated.

This first concerns Ia(800c + 480v + 320m). These products are the means of production of essential consumer goods in physical form, and the demands of workers and capitalists for essential consumer goods in value-form. IIa(c) is essential consumer goods in physical form, but it is the constant capital value consumed in the production of the sub-departments in value-form, and what is needed is the means of production. Therefore, IIa(c) and Ia(800c + 480v + 320m) can be exchanged so that they are replaced in kind and compensated in value.

Second, it concerns Ib(80v + 120m). These products are the means of production of luxury consumer goods in physical form, and they are the demands of capitalists for luxuries in value-form. IIb(c) satisfies the capitalists’ need for luxuries in physical form, and it is the constant capital value consumed in the production in value-form, representing IIb’s demand for the means of production for the production of luxuries. Therefore, IIb(c) and Ib(80v + 120m) can be exchanged so that they can be compensated in value.

Third, it concerns Ia(320m). These products are the means of production of essential consumer goods in physical form, and they are the demands of capitalists for luxuries in value-form. They can only be purchased from IIb(c), but IIb(c) does not need product Ia(320m); what is needed is the product Ib.

Finally, it concerns Ib(200v + 120m). These products are the means of production of luxury consumer goods in physical form, and they are the demands of workers and capitalists for essential consumer goods in value-form. They can only be purchased from IIa(c), but IIa(c) does not need product Ib(200v + 120m); what is needed is the product Ia.

The cases of products Ia(320m) and Ib(200v + 120m) show that the supply (the physical form of products) and demand (the value-form of products) are non-synchronous and non-complementary in the process of circulation or implementation. The value compensation and physical replacement cannot be implemented at the same time, showing the structural contradiction, so they can only be implemented through the indirect exchange process. The specific process is as follows. First, Ia(320m) sells products to IIa(c) = 320, but does not buy IIa(c). Instead, it buys luxuries IIb(c) = 320 with the currency obtained. As a result, Ia(320m) realizes
physical replacement and value compensation. Second, IIb(c) uses the currency obtained to purchase products with the value of Ib(200v + 120m) = 320 from the Sub-department Ib to replace the means of production consumed in production. Thus, IIb(c) implements physical replacement and value compensation. Finally, Ib(200v + 120m) uses the currency obtained, 320, to purchase necessities IIa(c) = 320 to meet the consumption needs of workers and capitalists. Ib(200v + 120m) and IIa(c) = 320 are both implemented and the dual compensation is completed. The currency returns to the starting point and satisfies the principle that the currency must flow back.

The above process of exchange relations can be represented by the exchange Model 1 as:

\[
\begin{align*}
\text{Ia.} & \quad 3200c + 800v + 480m + 520m \\
\text{Ib.} & \quad 800c + 200v + 120m + 80m \\
\text{IIa.} & \quad 800c + 480c + 320c \\
\text{IIb.} & \quad 320c + 80c
\end{align*}
\]

Model 1. Indirect Implementation of the Value Compensation for the Production of Essential Consumer Goods and Luxuries

Here, the direction of the arrow is money flow, and the direction of the root of the arrow is material flow. It is obvious that the basic implementation condition \( I(v + m) = IIc \) of the simple reproduction of social capital under the two departments has become much more complicated. If we set IIa.800c + 480c = 1280c to be IIa.c, IIa.320c to be IIa.c1, IIb.320c to be IIb.c1, and IIb.80c to be IIb.c2, while using the superscripts \( s \) and \( d \) to denote supply and demand, then the basic equilibrium condition \( I(v + m) = IIc \) of simple reproduction under the conditions of two departments will be further developed into the basic implementation conditions of simple reproduction of social capital under the conditions of four sub-departments.

\[
\begin{align*}
\text{Ia.} & \quad v + mx_{a} = IIa. c_{1} \\
\text{Ib.} & \quad mx_{b} = IIb. c_{2} \\
\text{Ia.} & \quad mx_{a}^{d} = IIa. c_{2}^{d} \\
\text{Ia.} & \quad mx_{b}^{d} = IIb. c_{1}^{d}
\end{align*}
\]
IIb. \( c_1^d = I b. (v + mx_a) \)

IIb. \( (v + mx_a)^d = I I a. c_2^i \)

Among them, the four conditions \( I a. mx_a^d = I I a. c_2^d \), \( I a. mx_a^d = I b. c_1^i \), \( I I b. c_1^d = I b. (v + mx_a)^i \), and \( I b. (v + mx_a)^d = I I a. c_2^i \) are the equilibrium conditions that must be followed by a fully indirect exchange process determined by non-synchronous and non-complementary supply and demand relations. The basic implementation conditions of simple reproduction under the conditions of sub-departments do not change the basic relations between the two departments, but they will complicate the implementation process of the total social product. Adding the exchange relations between the sub-departments in Department II to Model 1 above, and noting that \( I a. c \) and \( I b. c \) are realized within this Sub-department, the complete exchange model of simple reproduction of social capital in the four sub-departments is as follows (Model 2):

Model 2. The Complete Exchange Model for the Simple Reproduction of Social Capital in Four Sub-departments

On the basis of the above basic implementation conditions, the following derived conditions can be deduced:

\[
I a. 3200c + 800v + 480mx_a + 320mx_b = I a. 3200c + I I a. (800c + 480c + 320c)
\]

\[
I b. 800c + 200v + 120mx_a + 80mx_b = I b. 800c + I I b. (320c + 80c)
\]

\[
I I a. 1600c + 400v + 240mx_a + 160mx_b = I a. (800v + 480mx_a)
+ I b. (200v + 120mx_a)
+ I I a. (400v + 240mx_a)
+ I I b. (100v + 60mx_a)
\]

\[
I I b. 400c + 100v + 60mx_a + 40mx_b = I a. 320mx_b + I b. 80mx_b + I I a. 160mx_b
+ I I b. 40mx_b
\]
These four derived conditions show that there are mutual conditions and constraints among the sub-departments. All the products of Sub-department Ia and Sub-department Ib must exactly meet the compensated demand of the whole society for the means of production consumed in the production of essential consumer goods and the means of production consumed in the production of luxury consumer goods under the conditions of simple reproduction, as well as the compensated demand (Ia.c) of the means of production consumed in the production of essential consumer goods and the compensated demand (Ib.c) of the means of production consumed in the production of luxuries. In addition, all the products of Sub-department IIa and Sub-department IIb must just meet the needs of workers and capitalists for essential consumer goods, as well as the needs of capitalists for luxury consumer goods.

The four derived conditions mentioned above will also give rise to some illusions, as in the case of the simple reproduction of social capital in two departments. For Sub-department Ia and Sub-department Ib, it seems that all products are composed of transferred old value, with no new value created by labor in that particular year. For Sub-department IIa and Sub-department IIb, by contrast, it seems that the entire product value is composed of new value created during that year. This kind of illusion is caused by the process of value compensation and physical replacement. For this reason, “Smith’s dogma” is also wrong under the condition of sub-departments. It is necessary to point out that the equilibrium condition IIb.(100v + 60mx_a) = IIa.160mx_b, which is implicit in the derived condition IIb.400c + 100v + 60mx_a + 40mx_b = Ia. 320mx_a + Ib. 80mx_b + IIa.160mx_b + IIb. 40mx_b, is the basic condition for the implementation of internal exchange within the second department under the conditions of sub-departments.

Although Marx in his study of the expanded reproduction of social capital did not decompose the two departments under the condition of expanded reproduction in terms of final consumer goods, his principle of department decomposition of simple reproduction is also applicable to expanded reproduction. Here, the initial formulas of expanded reproduction given by Marx will be taken as an example to illustrate the decomposition of the two departments under the conditions of expanded reproduction. Marx’s initial formulas of expanded reproduction are as follows:

I. 4000c + 1000v + 1000m = 6000
II. 1500c + 750v + 750m = 3000

It emerges that the organic compositions of capital in the two departments are 4:1 and 2:1, respectively. According to Marx’s assumptions, accumulation starts from
Department I, with an accumulation rate of 50%; that is, 500$m$ is used for accumulation. It is further assumed that the ratio of Sub-department a and Sub-department b of Department I is 4:1, and that the ratio of surplus-value used by capitalists in the sub-departments for the consumption of necessities, i.e., consumption preference, is $x_a = 0.3$; the ratio of surplus-value used for the consumption of luxuries is then $x_b = 0.7$. For the purpose of accumulation, the adjustment process of social reproduction under the conditions of the four sub-departments can be expressed as follows:

\[
\begin{align*}
\text{Ia.} & \quad 3200c + 320\Delta c + 800\nu + 80\Delta \nu + 120m x_a + 280m x_b \\
\text{Ib.} & \quad 800c + 80\Delta c + 200\nu + 20\Delta \nu + 60m x_a + 40m x_b \\
\text{IIa.} & \quad 1200c + 80\Delta c + 600\nu + 40\Delta \nu + 288m x_a + 192m x_b \\
\text{IIb.} & \quad 300c + 20\Delta c + 150\nu + 10\Delta \nu + 32m x_a + 88m x_b
\end{align*}
\]

The implementation process of expanded reproduction between the two departments under the conditions of sub-departments is shown in Model 3:

Model 3. The Implementation Process of the Expanded Reproduction Between the Two Departments under the Conditions of Sub-Departments

The implementation process within Department II under the conditions of sub-departments is shown in Model 4:

Model 4. The Process of Value Implementation within the Second Department under the Conditions of Sub-Departments
In this way, the implementation process of the total social product for the purpose of accumulation under the conditions of sub-departments can be expressed by Model 5:

Model 5. The Implementation Process of Expanded Reproduction of Two Departments for Accumulation under the Conditions of Sub-Departments

Therefore, the basic equilibrium conditions of expanded reproduction under the conditions of two departments, \( I_1(v + \Delta v + \frac{m}{x}) = I_2(c + \Delta c) \), are further developed into the basic equilibrium conditions of expanded reproduction under the conditions of four sub-departments.

\[
\begin{align*}
I_a. (v + \Delta v + mx_a) &= I_2 a. c_1 \\
I_b. mx_b &= I_2 b. c_2 \\
I_a. mx_b^i &= I_2 a. c_2^i \\
I_a. mx_b^d &= I_2 b. c_1^s \\
I_2 b. c_1^d &= I_1 b. (v + \Delta v + mx_a)^s \\
I_b. (v + \Delta v + mx_a)^d &= I_2 a. c_2^s
\end{align*}
\]

Among these six conditions, \( I_a. mx_b^s = I_2 a. c_2^d \), \( I_a. mx_b^d = I_2 b. c_1^r \), \( I_2 b. c_1^d = I_1 b. (v + \Delta v + mx_a)^s \) and \( I_b. (v + \Delta v + mx_a)^d = I_2 a. c_2^s \) are the equilibrium conditions that must be followed by a fully indirect exchange process determined by non-synchronous and non-complementary supply and demand relations. As with simple reproduction, four derived conditions can also be deduced from the basic implementation conditions of expanded reproduction of sub-departments.

\[
\begin{align*}
I_a. 3520c + 800v + 80\Delta v + 120mx_a + 280mx_b \\
= I_2 a. 3520c + I_2 a. (1000c_1 + 280c_2)
\end{align*}
\]
These constitute the conditions that must be followed in order to achieve equilibrium through resource reallocation among the sub-departments of the expanded reproduction of social capital under the conditions of sub-departments, which indicate the interdependent and interactive relations among sub-departments. On the one hand, all the products of Sub-department Ia and Sub-department Ib must be able to compensate all kinds of means of production consumed by the whole society in production, and to provide additional means of production for the expanded reproduction of each sub-department. On the other hand, all the products of Sub-department IIa and Sub-department IIb must be able to satisfy the demands of workers and capitalists for essential consumer goods and of capitalists for luxury consumer goods, as well as the demands of workers who are supplemented for expanded reproduction for additional essential consumer goods. Accordingly, the condition \( \text{IIb. } 150v + 10\Delta v + 32mx_a + 88mx_b = \text{Ia. } 280mx_b + \text{Ib. } 40mx_b 
\) + \( \text{IIa. } 192mx_b + \text{IIb. } 88mx_b \) is the basic implementation condition of internal exchange of Department II under the sub-departments, which is implied in derived condition: \( \text{IIb. } 320c + 150v + 10\Delta v + 32mx_a + 88mx_b = \text{Ia. } 280mx_b + \text{Ib. } 40mx_b + \text{IIa. } 192mx_b + \text{IIb. } 88mx_b \).

**The Crisis of Capitalist Reproduction**

Marx’s theory of department decomposition is an important part of his theory of social capital reproduction. Nevertheless, this theory often receives insufficient attention. Marx explains further the complexity of the implementation conditions of capitalist reproduction through department decomposition, and from a specific perspective, further demonstrates the inevitability of capitalist reproduction crisis.

It can be seen from the model of the simple reproduction and expanded reproduction of sub-departments that the complexity of the implementation conditions for reproduction increases when the capitalist reproduction system is decomposed from two departments into four sub-departments. There is only one basic implementation condition for both simple reproduction and expanded reproduction under the conditions of two departments, whereas there will be six under the conditions of four sub-departments. Under the conditions of two departments, what needs to flow back indirectly is the prepaid variable capital \( Iv \) in Department I. Under the
conditions of sub-departments, except for the prepaid variable capital in IIa.\(v\) (simple reproduction) and IIa.\((v + \Delta v)\) (expanded reproduction), the rest of the prepaid variable capital, whether it is in simple reproduction or expanded reproduction, has to flow back indirectly. In addition, there is a structural contradiction between non-complementary supply and demand under the conditions of department decomposition; this will lead to a fully indirect exchange process that does not exist under the conditions of two departments. Moreover, the number of exchange links in the fully indirect exchange process is equal to that of sub-departments in the department decomposition. According to this thesis, there are four sub-departments in the department decomposition and four exchange links in the fully indirect exchange process. It is apparent that the implementation conditions of social capital reproduction become much more complicated under the conditions of department decomposition, and it will be more difficult for capitalism to maintain normal reproduction. The inevitable result will be a crisis of capitalist reproduction.

Under the condition of simple reproduction of a sub-department, Marx’s theory of department decomposition indicates, the whole social reproduction will be constrained by capitalists’ consumption preferences or consumption structures, that is, by the decomposition of surplus-value into \(m_{x_a}\) and \(m_{x_b}\), especially by the capitalists’ luxury consumption quantity \(m_{x_b}\). According to the simple reproduction model of sub-departments, the quantity of Ia.\(m_{x_b}\) and Ib.\(m_{x_b}\) determines the size of IIb.\(c\). Under the condition that the organic composition of capital remains unchanged, the prepaid variable capital of each department is a constant, i.e., \(\frac{dv}{dt} = 0\); hence the size of IIa.\(c\) depends on the quantity determined by capitalists’ personal consumption preferences, i.e., there is a functional relation IIa.\(c = f(Ia. m_{x_a})\). Thus, the normal proportional relation bounded by capitalists’ personal consumption preferences or consumption structures in terms of a merely quantitative relation under the simple reproduction condition of sub-departments should be as follows:

\[
\frac{Ia. (v + m_{x_a} + m_{x_b})}{IIa. (c_1 + c_2)} = \frac{Ib. (v + m_{x_a} + m_{x_b})}{IIb. (c_1 + c_2)} = 1
\]

However, unlike neoclassical economics and Keynesian economics, Marx’s economics requires not only quantitative or aggregate equilibrium but also structural equilibrium according to the internal laws of economic operation. In the above normal proportional relation based on quantitative relation, \(Ia. (v + m_{x_a})\) and \(Ib. m_{x_b}\) are balanced in quantity and structure, whereas \(Ia. m_{x_b}\) and \(Ib. (v + m_{x_a})\) are balanced in quantity but unbalanced in structure. Ia.\(m_{x_b}\), IIa.\(c_2\), Ib.\(.(v + m_{x_a})\) and IIb.\(c_1\) belong to
non-complementary supply and demand relations, and structural equilibrium can only
be solved through a fully indirect exchange process, i.e., through the exchange relation
formed by one-way buying or one-way selling of products (as shown in Figure 1).

Ia. \( mx_a \rightarrow IIb. c_1 \rightarrow Ib. (v + mx_a) \rightarrow IIa. c_2 \)

Figure 1. The Fully Indirect Exchange Process Under the Condition of Simple Reproduction in Sub-Departments

This fully indirect exchange relation runs through each sub-department, so it can
be considered that the fully indirect exchange relation is the core implementation
condition of simple reproduction under the sub-department condition, and the
whole reproduction cannot be implemented as long as this condition is destroyed.
The complementary exchange relations of supply and demand are
Ia. \( (v + mx_a) = IIa. c_1 \), Ib. \( mx_b = IIb. c_2 \) and IIa. \( mx_a = IIb. (v + mx_a) \), and they are
the basic equilibrium conditions for the Sub-departments Ia and IIa, Ib and IIb, as
well as IIa and IIb. Moreover, in terms of IIa. \( mx_a \), the condition \( IIa. mx_a \leq IIb. (v + m) \)
must be established, that is, \( IIb. (v + m) \) is the maximum quantity of luxury con-
sumption in sub-department IIa. Among these conditions, Marx analyzed in par-
ticular the equilibrium condition \( IIa. mx_a = IIb. (v + mx_a) \), and pointed out that this
condition was of great significance in determining the structure of production
(Marx, 2004, 453–454, 457). \( Ia. mx_a + Ib. mx_b \) must be equal in quantity and struc-
ture, and the internal decomposition proportion is determined by \( Ia. mx_a : Ib. mx_b \).
Also, it must be ensured quantitatively that the decomposition within
\( Ib. (v + mx_a)^d = IIa. c_2^d \) and \( IIa. c \) is determined by \( Ia. (v + m_a) : Ia. mx_b \).

This all adds up to a clear case that the structure of the reproduction of social capi-
tal under the conditions of sub-departments is regulated by capitalists’ consumption
preferences and consumption structures. As Marx pointed out:

We saw a while ago that the proportion between the production of consumer
necessities and that of luxuries requires the division of II(v + m) between IIa and
IIb, and thus of IIC between (IIa)c and (IIb)c. Hence this division affects the character
and the quantitative relations of production to their very roots, and is an essential
determining factor of its general structure. (Marx 1956, 251)

At the same time, the proportional relation between the production of necessities
and luxuries obviously depends on capitalists’ consumption preferences and con-
sumption choices. Moreover, the change of capitalists’ consumption preferences
for luxuries in any sub-department will cause systematic adjustment within the whole reproduction system. As a result, to ensure normal reproduction, capitalists’ individual consumption choices must follow the various conditions of simple reproduction mentioned above.

Nevertheless, there is no necessary association between capitalists’ individual consumption preferences and consumption choices, and the capitalists’ preference choices based on utility maximization will not take into account the reproduction consequences of the choices. The contradiction between the proportional requirements of social reproduction and the individual choice behaviors of capitalists will inevitably undermine the proportional relations of social reproduction. Meanwhile, a fully indirect exchange relation must exclude any conflict between capitalists’ demand choices as consumers and their commodity supplies as producers. However, the standard decision-making system of capitalism, which is determined by the basic contradictions of capitalism, is formed by individual capitalists. As suppliers, capitalists can only judge the changes of supply and demand according to the market price, and thus infer capitalists’ consumption preferences and their changes as consumers. For this reason, capitalists can only make decisions using what Marx called “ostensible estimates” (Marx and Engels 1982, 156). The decision-making process thus becomes an uncertain process of capitalists’ mutual expectations (e), and the proportional relations of reproduction (B*), spontaneously determined by this process, can only be a certain expectation (p) of the objective proportional relations (B) required by normal reproduction, which is formed according to a certain probability (E(B|p)), that is B* = E(B|p). Therefore, the basic condition for the realization of simple reproduction in sub-departments is expressed in an expectation form subject to the capitalist’s individual decision, which cannot fundamentally exclude the conflict between capitalists’ demand choices as consumers and their commodity supplies as producers. It is inevitable that the reproduction ratio will be undermined.

Marx clearly states that under the condition of sub-departments, in terms of quantity, the exchange between the various parts of annual products can be carried out in proportion only if production scale and value remain stationary, and the relation will not change as a result of foreign trade (Marx 2004, 454). Nevertheless, the choice preferences of capitalists are dynamic, and the proportion of surplus-value used for consumer goods a and b varies over time, i.e., \( x_a(t) + x_b(t) = 1 \) but \( \frac{dx_a}{dt} \neq 0 \). At the same time, the consumption of capitalists will be constrained not only by the amount of surplus-value and its split proportion, but also by the quantity and structure of the corresponding real product. However, capitalists will not adjust their preferences. As a result, the condition of proportional reproduction, in which production scale and value relations remain stationary, cannot be
maintained. Therefore, capitalists’ proportional consumption choices are the conditions for normal reproduction, but it is impossible for capitalists to follow the proportional requirement consciously. Meanwhile, the systematic adjustment caused by the change of capitalists’ consumption preference choices is beyond their control. For this reason, crises of reproduction are inevitable.

The scale of reproduction under the condition of expanded reproduction in sub-departments depends not only on the capital accumulation rate \((k)\), but also on the capitalists’ consumption preferences. According to Marx’s theory of social capital reproduction, the prerequisite of expanded reproduction under the condition of sub-departments should be \(Ia.(v + m) > IIa.c\) and \(Ib.(v + m) > IIb.c\), so as to provide additional means of production for the whole social reproduction under the condition of sub-departments, which is the expansion of the condition \(I(v + m) > IIc\). In order to achieve expanded reproduction, sub-department \(Ia\) or \(Ib\) must accumulate so as to reallocate resources in accordance with the requirements; that is, accumulation must follow the following equilibrium conditions:

\[
Ia.(v + \Delta v + mx_a + mx_b) = IIa.(c + \Delta c)
\]

\[
Ib.(v + \Delta v + mx_a + mx_b) = IIb.(c + \Delta c).
\]

After expansion, the following eight specific equilibrium conditions can be obtained:

\[
Ia. (v + \Delta v + mx_a) = IIa.c_1
\]

\[
Ib. mx_b = IIb. c_2
\]

\[
Ia. mx^s_b = IIa. c_2^d
\]

\[
Ia. mx^d_b = IIb. c_1^s
\]

\[
IIb. c_1^d = Ib. (v + \Delta v + mx_a)^s
\]

\[
Ib. (v + \Delta v + mx_a)^d = IIa. c_2^s
\]

\[
IIa. (c_1 + c_2) = IIa. (c + \Delta c)
\]

\[
IIb. (c_1 + c_2) = IIb. (c + \Delta c)
\]

Meanwhile, the exchange relations \(Ia.mx^s_b = IIa.c_2^d\), \(Ia.mx^d_b = IIb.c_1^s\), \(IIb.c_1^d = Ib.(v + \Delta v + mx_a)^s\), \(Ib.(v + \Delta v + mx_a)^d = IIa.c_2^s\) are the fully indirect exchange processes (Figure 2).
It can be seen that capitalists’ consumption decisions in Sub-department Ia and Sub-department Ib, based on maximizing personal utility, directly determine the size of accumulation in Sub-department IIa and Sub-department IIb. At the same time, changes in the accumulation rates and consumption preferences or consumption structures of capitalists in Sub-department Ia and Sub-department Ib will result in a systematic adjustment in the whole of social reproduction through a process of fully indirect exchange. This requires the establishment of interconnections between individual capitals in terms of capital accumulation and consumption choices that are consistent with the conditions for the implementation of reproduction. The capitalist nature of production, however, determines that it is impossible for capitalists to establish such a relationship, and the spontaneous connection formed by the mutual expectation of capitalists can only be the expectation $B^c = E\left(B | p\right)$ of the objective connection required by normal reproduction. As a result, the basic implementation conditions for expanded reproduction can only take the form of expectation, and there must be conflicts between the spontaneous connection and the objective connection. Meanwhile, the accumulation preference ($k$) and the consumption preferences ($x_a, x_b$) of independent decision-making capitalists are all changed with time, i.e., $\frac{d}{dt} \left( \frac{x_a}{x_b} \right) \neq 0$. The accumulation dynamics can likewise only be of the expected form: $K^c_{a,b} = f[t, K(t), x_{a,b}(t), u(t)]$ The factor $u(t)$ is the capitalist’s decision variable regarding the change rate of the capital stock. Nevertheless, independent decision-making capitalists neither care about the reproduction consequences of accumulation and consumption behaviors based on maximization considerations, nor have the ability to control these consequences, which makes the destruction of the reproduction ratio inevitable.

Whether simple reproduction or expanded reproduction is involved, the decision-making behavior of capitalists as they pursue maximization determines that the capitalist reproduction process is not a system characterized by gradual stabilization. Its stability is of a saddle-point nature, in which the system is stable only if the proportional relations (stable arms, stable paths) required for reproduction are maintained. However, the individual decisions of capitalists cannot guarantee that reproduction will be carried out in proportional fashion, and a crisis of
reproduction will occur sooner or later once reproduction deviates from the required proportional relation (deviates from the stable arm, deviates from the stable path).

The above analysis shows that the capitalist’s individual consumption preference and consumption choice behavior is an internal factor of the capitalist reproduction crisis under the conditions of sub-departments. This factor further reveals, from a particular perspective, a certain internal mechanism by which the capitalist reproduction ratio is bound to be undermined. Even in the literature studying Marxist economics, it is rare to find such a mechanism.

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Note

It may be noted that this article starts with Ia(320m) to analyze the fully indirect exchange process. This is not a required analysis procedure. To reveal the complexity of the sinuous exchange process, the analysis can in fact start from any of its parts.

References