A quantum theory of spacetime in spinor formalism and the physical reality of cross-ratio representation: the equation of density parameters of dark energy, matter, and ordinary matter is derived: $\Omega_M^2 = 4 \Omega_b \Omega_\Lambda$

Jackie C.H. Liu*

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA, USA
*Corresponding author’s e-mail address: jackieliu0@gmail.com

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ABSTRACT
By theorizing the physical reality through the deformation of an arbitrary cross-ratio, we leverage Galois differential theory to describe the dynamics of isomonodromic integratable system. We found a new description of curvature of spacetime by the equivalency of isomonodromic integratable system and Penrose's spinor formalism of general relativity. Using such description, we hypothetically quantize the curvature of spacetime (gravity) and apply to the problem of the evolution of the universe. The Friedmann equation is recovered and compared so that the mathematical relationship among dark energy, matter (dark matter + ordinary matter), and ordinary matter, $\Omega_M^2 = 4 \Omega_b \Omega_\Lambda$, is derived; the actual observed results are compared to this equation (calculated $\Omega_M = 0.33$ vs. observed $\Omega_M = 0.31$); the model might explain the origin of dark energy and dark matter of the evolution of the universe.

INTRODUCTION
We looked for the simplest mathematical object to identify the underlying reality of nature, and we found it to be cross-ratio. By defining cross-ratio over Riemann sphere, infinity is just another usual point; similarly, there shouldn't be any point in the universe more special than others. However, the variety of nature must be realized as a condition for such an underlying object.

In "Cross-ratio arbitrariness and the constraint to the parameter space of projective space basis" section, the article explains how potential physical varieties come from different representations of the same thing – cross-ratio deformation. So, the cross-ratio consists of both simplicity and variety. A successful example is like Einstein's masterpiece – general theory of relativity. Although Einstein's field equation is simple, many interesting solutions emerged.

In "Cassidy's work on isomonodromic system" section, we introduce Galois differential theory and related Cassidy’s work. It is a mathematical machinery to manifest the deformation of cross-ratio. Cassidy's work consists of introducing a 2 by 2 matrix differential equation and related isomonodromic integratable system, so it can describe the deformation. By such machinery, we formulate an alternative theory of the dynamics of curvature of spacetime to recover the spinor general relativity equivalent counterpart (for which a brief introduction is given in "Brief overview of spinor formulation of general relativity" section). By such connection, we hypothetically claim the origin of spacetime is from the isomonodromic integratable system, and spacetime is more fundamentally described by the curvature rather than metric or coordinated mathematical framework, that is, spinor formulation of general relativity might be more fundamental than classical general relativity; a similar argument was postulated by Penrose (1960).

In "As an application to the problem of modeling the universe evolution" section, we apply the calculation of the deformation of the isomonodromic integratable system with certain simplified conditions so a solution is found. The solution is used to recover Friedmann equation and related density parameters such that observed and calculated results are compared. This proposes an explanation of the origin of dark matter and dark energy without new kind of matter or energy, as they are new kind of gravitational field of spacetime's curvature.

BRIEF OVERVIEW OF SPINOR FORMULATION OF GENERAL RELATIVITY
Penrose's spinor approach to general relativity
Spinor formulaism of general relativity (i.e., Spinor GR) (Penrose, 1960) adopted a coordinate-free approach. The
correspondence between tensors and spinors is obtained by the use of a hybrid spinor $\sigma_{\mu}^{CD}$ (2 × 2 Hermitean matrix per CD index, denoted as $\alpha$, in short) (Einstein’s summation notation is used in this article):

$$\sigma_{\mu}^{AC} \sigma_{\nu}^{BC} + \sigma_{\nu}^{AC} \sigma_{\mu}^{BC} = g_{\mu\nu}^{AB} \text{ for } A, B, C = 0, 1, \mu, \nu = 0, 1, 2, 3,$$

(1)

where $\epsilon^{BC}$ represents 2 × 2 skew-symmetric “metric” spinor (components’ values are 0 or ± 1). To raise or lower spinor index, either one of $\epsilon^{BC}$, $\epsilon^{CB}$, $\epsilon^{BC}$, and $\epsilon^{BC}$ is used:

$$\zeta^A = \epsilon^{AB} \xi_B, \quad \xi_B = \epsilon^{AB} \zeta^A$$

(2)

where $\xi$ denotes spinor. Primed indexes refer to the complex conjugate spin space. Please note that

$$\epsilon^{BD} \epsilon^{DF} = -\delta^B_F, \quad \delta^B_F = 0 (if \ B \neq F)$$

(3)

$$\sigma_{\mu}^{AC} \rightarrow g^{\mu\nu} \sigma_{\nu}^{AC} \epsilon_{CABDF}$$

(4)

is Hermitian because $g^{\mu\nu}$ (g is the metric) is real.

Spinor GR requires the covariant derivatives of $\sigma_{\mu}^{CD}$, $g_{\mu\nu}$, and $\epsilon_{CABDF}$ to be zero. The spinor equivalent of tensor $G_{\mu\nu}$ is stated as

$$G_{ACBD} = \sigma_{AC} \sigma_{BD} G_{\mu\nu}, G_{\mu\nu} = \sigma_{AC} \sigma_{BD} G_{ACBD}.$$  

(5)

Curvature tensors, symmetry and Einstein tensors in spinor general relativity

By $R_{ACBFCD}^{EFGH}$ (spinor version of Riemann-Christoffel tensor $R_{\mu\nu\rho\sigma}$), the curvature in spinor GR is defined by curvature spinors $\chi_{ABCD}$ and $\Phi_{ABCG}$ (Penrose, 1960):

$$R_{ACBFCD}^{EFGH} = \frac{1}{2} \left( \chi_{ACBD} \epsilon^{EF} \epsilon^{GF} + \chi_{CD} \Phi_{ABFG} \epsilon^{EF} + \chi_{AB} \Phi^{*}_{EFCD} \epsilon^{FG} + \chi_{AB} \Phi^{*}_{ACFG} \epsilon^{BD} \right)$$

(6)

(* indicates complex conjugate), which satisfy the following equations (Penrose, 1960):

$$\left( \nabla_{AC} \nabla_{BD} - \nabla_{AD} \nabla_{BC} \right) \zeta^E = \chi_{ABCD} \zeta^E$$

(7)

where $\nabla_{AC}$ and $\nabla_{BD}$ are spinor covariant derivatives and raised spinor derivatives, respectively. They can be modified to

$$\epsilon^{EF} \left( \nabla_{AC} \nabla_{BD} - \nabla_{AD} \nabla_{BC} \right) \zeta^E = \epsilon^{EF} \chi_{ABCD} \zeta^E$$

(8)

$$\left( \nabla_{AC} \nabla_{BD} - \nabla_{AD} \nabla_{BC} \right) \zeta^E = \chi_{ABCD} \zeta^E$$

(9)

$$\left( \nabla_{AC} \nabla_{BD} - \nabla_{AD} \nabla_{BC} \right) \zeta^E = \chi_{ABCD} \zeta^E$$

(10)

$$\left( \nabla_{AC} \nabla_{BD} - \nabla_{AD} \nabla_{BC} \right) \zeta^E = \chi_{ABCD} \zeta^E$$

(11)

$$\left( \nabla_{AC} \nabla_{BD} - \nabla_{AD} \nabla_{BC} \right) \zeta^E = \chi_{ABCD} \zeta^E$$

(12)

$$\left( \nabla_{AC} \nabla_{BD} - \nabla_{AD} \nabla_{BC} \right) \zeta^E = \chi_{ABCD} \zeta^E$$

(13)

APPLICATION OF GALOIS DIFFERENTIAL THEORY TO SPINOR FORMULATION OF GENERAL RELATIVITY

Cassidy’s work on isomonodromic system

For a commuting set of derivations $\frac{\partial}{\partial z}$ (i=0,…,m), there exists an integratable (parameterized second-order linear differential) system [Cassidy & Singer, 2005: Prop.6.3(3)]
with integrability condition: \[ [B_{zi}, B_{zj}] = \frac{\partial}{\partial z_i} B_{zj} - \frac{\partial}{\partial z_j} B_{zi} \]

\((i, j = 0, \ldots, m)\), where \(Y\) is in \(GL_2(f(z_i))\), \(B_{zi} = \frac{\partial}{\partial z_i} \left( f(z) \right)\), \(B_{zi} = g_{zi} \left( z_0_1 \right)\) \((k = 1, \ldots, m)\), \(f(z_0)\) is a set of functions which \(\frac{\partial}{\partial z_i}\) acts on. The variable set \(\{z_0\}\), for \(k = 1, \ldots, m\), is called parameters-set. This system is called isomonodromic system. Furthermore, the related Gal-group is a proper Zariski dense subgroup of \(SL_2(c^{n\theta})\), where Gal-group is the parameterized Picard-Vessiot group (PPV-group) and \(c^{n\theta}\) is the differential constant of \(\frac{\partial}{\partial z_i}\) (but function of parameters-set) or called \(\frac{\partial}{\partial z_i}\) invariant. \(Y\) is called the fundamental solution matrix. The PPV-group is a \(\delta\)-autormorphism group such that \(Gal \in \) PPV-group if Gal: \(Y \rightarrow Y\) and Gal \(\delta = \delta\) Gal \(\forall\) \(\delta\). Therefore, Gal(\(Y\)) is another fundamental solution matrix to the system. Let us define the Lie algebra corresponding to one-parameter subgroup of Gal by \(G\) through exponential map:

\[ Gal = \text{Exp}(\kappa G), \text{here } \kappa \text{ is a real parameter} \]

By expanding the exponential map with parameter \(\kappa \rightarrow 0\) and commuting property of Gal \(\delta\) (i.e., Gal \(\delta = \delta\) Gal), the commuting property of Lie algebra \(G\) and \(\delta\) exists:

\[ G\delta = \delta G, \forall \delta \]

It is obvious that \(G \in \text{sl}_2(c^{n\theta})\).

There is a property of integrability condition that change of variables from parameter-set \(z_i\) to \(z_i'\) preserves this property (by change of variables) if there exists smooth maps \(\frac{\partial}{\partial z_i}\) for \(z_i(z_i')\). The transformed new integratable system (together with \(\frac{\partial}{\partial z_i} Y = B_{zi} Y\)) is

\[ \frac{\partial}{\partial z_i} Y = B'_{zi} Y, \text{ where } B'_{zi} = \sum_i \frac{\partial z_i}{\partial z_i'} B_{zi} \text{ for } i = 1, \ldots, m \]

\[ 1, \ldots, m \]

(16)

note that \(n\) is not necessarily equal to \(m\).

**Cross-ratio arbitrariness and the constraint to the parameter space of projective space basis**

A cross-ratio (https://en.wikipedia.org/wiki/Cross-ratio) can be defined by a function of four points on Riemann sphere:

\[ \text{Cross Ratio} = \frac{(z_0 - z_2)(z_1 - z_3)}{(z_1 - z_2)(z_0 - z_3)} \]

If we allow any arbitrary cross-ratio fixed value to be assigned, those four points are then constrained (only three points are free to move). If we further assume any arbitrary reference fixed point to be assigned, say for \(z_0\), then those other points are also constrained by four real parameters \(\{x_0, x_1, x_2, x_3\}\), that is,

\[ \text{Cross Ratio} = \frac{(z_0 - z_2)(z_1 - z_3)}{(z_1 - z_2)(z_0 - z_3)} \]

So, the arbitrariness of description of a cross-ratio allows four real parameters. Physically, we mean the principle of cross-ratio arbitrariness is natural because no particular point over Riemann sphere is more special and no reference point is more special than others.

By Yoshida (Chapters I and IV), the realization of cross-ratio has direct relationship with hypergeometric function (three regular singular points):

\[ z(1-z)\frac{d^2 y}{dz^2} + (c - (a + b + 1)) \frac{dy}{dz} - aby = 0, \]

where \(a, b, c\) are complex numbers.

Since hypergeometric function is a solution of related hypergeometric differential equation, so if we deform the hypergeometric differential equation, we also deform the related hypergeometric function and its underlying cross-ratio realization.

Referring to Hypergeometric function (https://en.wikipedia.org/wiki/Hypergeometric_function), any second-order differential equation with three regular singular points can be converted to hypergeometric differential equation by change of variables, so we can describe the hypergeometric differential equation by second-order differential equation. Leveraging Cassidy’s work in previous section, we can deform the hypergeometric differential equation by the deformation of second-order differential equation. Physically, we mean we can deform the realization of cross-ratio by deformation of related isomonodromy integratable system by a deformation space of four real parameters:

\[ \frac{\partial Y}{\partial X_\mu} Y = \frac{\partial}{\partial X_\mu} (\partial Y) Y = B_\mu Y \text{ or } \partial Y \delta_\phi Y^\phi = \delta_\phi Y^\phi \overline{\delta_\phi} F Y^F = B_\mu Y^F \]

(17)

the later is in spinor form, where \(B_\mu(z_0, x_v) = \sum_i \frac{\partial z_i(x_v)}{\partial X_\mu}\) \(B_\mu(x_0, z_j)\) for \(\mu, v = 0, \ldots, 3; i, j = 1, \ldots, 3; x_v \in \text{Reals}\).

Id is \(2 \times 2\) identity matrix; \(\overline{\delta_\phi} F = \overline{\partial Y} \delta_\phi ; \text{ note that } B_\mu(z_0, x_v) \)

\(\text{and } B_\mu(x_0, z_j)\) are matrices \(gl_2\), and \(\overline{\delta_\phi} F, B_\mu Y^F\) are hybrid spinors like \(\sigma^{AB}\) in spinor GR (Robert, Chapter 13).

**Operators of integrability condition and quantized Bianchi equation for curvature spinors**

Based on the isomonodromy integratable system formulated in previous section, we look for a set of the Lie algebras associated with Gal that can produce the integrability. In order to do so, some fundamental operators, such as derivation operator and integratability operator, are introduced. They are related to mathematical objects in spinor GR so that
Bianchi identity is recovered by an operator equation. Finally, the equivalency of spinor GR and this theory is summarized. Let us define spinor (derivation) operator:

$$\widehat{D}_{AB}E_F = \sigma^\mu_{AB} \widehat{D}_\mu E_F$$

(18)

and $\widehat{D}_\mu E_F$ is the differential hybrid spinor operator from previous section (so it is associated with an integratable system), not covariant derivative.

We demand $\sigma^\mu_{AB}$ (denoted as $\sigma^\mu$ in short) is the differential constant of $\widehat{D}_\mu$ in the sense that $\widehat{D}_\mu E_F = \text{invariant}$ $|\widehat{D}_\mu E_F| = |\sigma^\mu_{AB} (\widehat{D}_\mu E_F)|$ for any spinor $X$, so it is consistent with spinor GR requirement (covariant derivative of $\sigma^\mu_{AB}$ is zero) except $\nabla \rightarrow \widehat{D}$. $\sigma^\mu_{AB}$ defined here satisfies the same relationship in Equation (4).

Note that commutative $\partial_\mu$ implies commutative $\widehat{D}_{AB}E_F: \widehat{D}_{AB}E_F \widehat{D}_{CD}^E = \widehat{D}_{CD}^E \widehat{D}_{AB}E_F^G$.

By Equations (16) and (18) as well as $\sigma^\mu_{AB}$ being $\widehat{D}_\mu E_F$ - invariant, there is a commuting set of spinor operator for an integratable system:

$$\widehat{D}_{AB}E_F Y^{FG} = \widehat{D}_{CD}^E \widehat{D}_{AB}E_F Y^{FG} \leftrightarrow \widehat{D}_{CD}^E \widehat{D}_{AB}E_F Y^{FG}$$

where

$$\widehat{D}_{AB}E_F Y^{FG} \equiv \sigma^\mu_{AB} \partial_\mu Y^{FG}$$

(19)

($\leftrightarrow$ indicates complex conjugate, so prime and unprime indices are interchanged, and note that the ordering among prime and unprime indices are irrelevant (Robert, Chapter 13).

The integratability condition is used to facilitate a definition on a set of spinor operator:

$$\widehat{J}_{ABCD}E_F$$

is called integratability operator (or, in short, int operator), acting on $Y$ |

$$\widehat{J}_{ABCD}E_F Y^{FG} = (\widehat{H}_{AB} E_F H_{CD} E_F - \widehat{H}_{CD} E_F H_{AB} E_F) Y^{FG}$$

$$= \widehat{H}_{[AB} E_F H_{CD]} E_F Y^{FG},$$

and

$$\widehat{J}_{ABCD}E_F Y^{FG} = (\sigma^\mu_{AB} \partial_\mu H_{CD} E_F) Y^{FG}$$

$$\widehat{J}_{ABCD}E_F Y^{FG} = (\sigma^\mu_{AB} \partial_\mu H_{CD} E_F) Y^{FG},$$

[AB' and CD'] are anti-commuting indices notation as usual in spinor GR. So, by integratability condition (14), this implies

$$\widehat{J}_{ABCD}E_F Y^{FG} = (\widehat{H}_{AB} E_F H_{CD} E_F) Y^{FG},$$

$$\widehat{J}_{ABCD}E_F Y^{FG} = (\widehat{H}_{AB} E_F H_{CD} E_F) Y^{FG},$$

(20)

There are a total of four sets of spinor operators from the contraction of $\widehat{J}$ and $\widehat{J}^*$, which have a close relationship with int operators and curvature spinors:

$$\widehat{J}_{AC}E_F = \widehat{J}_{AC}E_F = \epsilon^{BCD} \widehat{J}_{ABCD}E_F,$$

$$\widehat{J}_{AC}E_F = \widehat{J}_{AC}E_F = \epsilon^{BCD} \widehat{J}_{ABCD}E_F,$$

$$\widehat{J}_{AC}E_F = \widehat{J}_{AC}E_F = \epsilon^{BCD} \widehat{J}_{ABCD}E_F,$$

$$\widehat{J}_{AC}E_F = \widehat{J}_{AC}E_F = \epsilon^{BCD} \widehat{J}_{ABCD}E_F.$$
As a short summary, we look for an integrable system, where \( G \) is generated by the int operator \( f \), and \( G \) is a solution to the fundamental (equation (21)). This system leads to quantized Bianchi identity.

Let us define a set of spinor operators \( \hat{\delta}_{ABCD}^E_F \) such that

\[
(\hat{\delta}_{ABCD}^E_F - \hat{\delta}_{CDAB}^E_F)Y^F_H = \left( \hat{\mathbb{D}}_{[AB} \gamma^C (H_{CD}]^E_F) \right) Y^F_H
\]

By contracting over \( B' \) and \( D' \), we get

\[
\epsilon^{B'D'}(\hat{\delta}_{ABCD}^E_F - \hat{\delta}_{CDAB}^E_F)Y^F_H = \epsilon^{B'D'}(\hat{\mathbb{D}}_{[AB} \gamma^C (H_{CD}]^E_F)) Y^F_H
\]

Using the integratability condition property (Equation 20) for \( J \) operator on the right-hand side, we get

\[
(\hat{\delta}_{AB'C}^B_F + \hat{\delta}_{CD'A}^D_F)Y^F_H = \hat{\mathcal{J}}_{AC}^E \gamma^F_H
\]

The right-hand side is just \( \mathcal{J}_{AC}^E \) (Equation 20b), and the equation becomes

\[
(\hat{\delta}_{AB'C}^B_F + \hat{\delta}_{CD'A}^D_F)Y^F_H = \hat{\mathcal{M}}_{BD}^F \gamma^F_H
\]

Let us do the same for (Equation 22), but contracting over \( A \) and \( C \):

\[
\epsilon^A_C(\hat{\delta}_{ABCD}^F - \hat{\delta}_{CDAB}^F)Y^F_H = \epsilon^A_C(\hat{\mathbb{D}}_{[AB} \gamma^C (H_{CD}]^F) Y^F_H
\]

After the same operations, we get

\[
(\hat{\delta}_{AB'C}^B_F + \hat{\delta}_{CD'A}^D_F)Y^F_H = \mathcal{J}_{AC}^E \gamma^F_H
\]

By decomposing \( Y^F_H \) as matrix into \( \xi^F \) and \( \eta^F \) (both are column vectors of dimension 2) into Equations (23) and (24); \( Y \rightarrow \xi \oplus \eta \), we get: \( \xi^F = \mathcal{J}_{AC}^E \gamma^F \). (Equation 25) and \( \mathcal{J} \) are equivalent to the equations of curvature spinors in Equation (7) if

\[
\hat{\mathcal{J}}_{AC}^E = \mathcal{J}_{AC}^E, \mathcal{M}_{BD}^F = \Phi^B_D \gamma^F \text{ (not the operators) (26)}
\]

In conclusion, spinor GR requires a well-defined spinor covariant derivative \( \nabla \). \( \nabla \) gives raise to the relationship as Equation (7), so that curvature spinors \( \chi \) and \( \Phi \) are well established, which obey Bianchi identity (Equation 9). These conditions are said to be equivalent to "generate" spacetime manifold of \( \{+,\ldots\} \) (Penrose, 1960) (except certain torsion-free symmetric properties). In this article, the author claims that the spinor covariant derivative \( \nabla \) is corresponding to the operator \( \hat{\delta} \); curvature spinors \( \Phi \) and \( \chi \) are generated by \( \hat{\mathcal{J}} \) and \( \mathcal{M} \) operators on \( Y \); Bianchi identity is satisfied by the operators \( \hat{\mathcal{J}} \), \( \mathcal{M} \) in such quantized form.

To illustrate the correspondences between spinor GR and this part of the theory (i.e., isomonodromic system):

- Spinor covariant derivatives: \( \nabla \nabla \rightarrow \hat{\delta}_{ABCD}^E_F \)
- Curvatures: \( \Phi, \chi \rightarrow \hat{\mathcal{J}}, \mathcal{M} \)
- Bianchi identity (dynamics of curvatures):

\[
\nabla^B \mathcal{Z}_{ABCD}^E = \nabla^A \Phi^B_{DCE} \rightarrow (\hat{\mathbb{D}}_D \Phi_{E} \mathcal{J}_{AB}^F) Y^G_H
\]

Note that \( \hat{\mathcal{J}} \) and \( \mathcal{M} \) are just linear combinations of int operators, which must exist in an integrable system, while \( Y \) is a special solution to quantized Bianchi equation, which is a result of commuting relationship of \( \hat{\mathcal{J}}, \mathcal{M} \) and \( \hat{\mathbb{D}} \). The consequence is that, we can only claim that the equivalence of spinor GR to this theory is up to particular set of solution \( Y \) and its associated values of \( \hat{\mathcal{J}} \) and \( \mathcal{M} \), that is, spinor GR is equivalent to this part of the theory in such quantized sense.

### The symmetry of curvature spinor

Certain symmetries of curvature spinors are described by Penrose in previous sections, and those symmetries are consequence of classical general relativity. We show that certain symmetries are naturally arisen by the operators introduced in previous section, while some others are torsion related and needed to be met additionally. However, those additional conditions are not considered as mandatory because the key objective is to describe that spacetime’s curvature can be manifested by the deformation of isomonodromy integrable system, not to explain the naturalness of torsion.

From Equation (26) and the traceless property of matrices generated from int operators (Equation 20), it is concluded that \( \mathcal{Z}_{AB}^C \) and \( \Phi^B_{DCE} \) are traceless matrices (if considering \( G \) and \( E \) as the matrix indices), and

\[
\int \mathcal{J} \rightarrow \text{symmetry of } \chi, \Phi : \mathcal{Z}_{AB}^C \oplus \Phi^B_{DCE} \text{ (30)}
\]

Because of the symmetry properties of \( \mathcal{J}_{AC}^F \) and \( \mathcal{M}_{BD}^F \), it implies trivially

\[
\mathcal{Z}_{AB}^C \oplus \Phi^B_{DCE} \text{ (31)}
\]

Besides these two symmetries, spinor GR also requires symmetry:

\[
\mathcal{Z}_{AB}^C \oplus \Phi^B_{DCE} \text{ (32)}
\]

and the \( \lambda \) reality (Equation 10):

\[
\lambda = \frac{\mathcal{Z}_{AB}^C}{2} = \frac{1}{2} \mathcal{Z}_{ABCD}^E \Phi^B_{DCE} \rightarrow \frac{1}{2} \mathcal{Z}_{AB}^C \Phi^B_{DCE} \text{ (32)}
\]

By Gomez-Lobo and Martin-Garcia (2012), GR symmetry (Equation 31) and \( \lambda \) reality are consequences of the torsion-free property of original GR theory. However, torsion free is known that it is not a mandatory requirement; for example, Einstein–Carton theory removes such constraints.
In conclusion, to recover GR symmetry \( x_{ABFE} \approx x_{FAEB} \) and \( \Phi_{D'CFE} = \Phi_{E'DCF} \) and \( \lambda \) reality, there are conditions for spinors generated by operators \( \tilde{J} \) and \( \tilde{M} \) to meet. However, the discussion of sufficient and necessary conditions for \( \tilde{J} \) and \( \tilde{M} \) to meet these symmetries is not covered yet in this paper.

**Translation to ordinary Ricci, Einstein tensors, and Bianchi equation**

The purpose of this section is to show the explicit links between related mathematical objects spinor GR (spinors of Ricci and Einstein tensors) and the isomonodromy integrable counterparts. Because those are key components of classical general relativity, showing explicit relationships helps visualize the equivalency and claim the origin of related tensors. Recall the spinor equivalences of Ricci tensor (Equation 11), Ricci scalar (Equation 12), and Einstein tensor (Equation 13): \( R_{ACBD} = \Phi_{ACBD} - \Phi_{ABC} \cdot R = 4 \lambda \), and \( G_{ACBD} = -\Phi_{ACBD} - \Phi_{ABC} \cdot R \). The following are \( \tilde{J} \) and \( \tilde{M} \) equivalences:

\[
R_{ACBD} = \left( \frac{1}{2} \epsilon^{FGH} \mathcal{J}_{GH} \right)_{ABCD} - (M_{AB} \epsilon^{CD} + \epsilon^{E} \epsilon^{E})^* \\
R = 2 \epsilon^{BD} \tilde{J}_{ABCD}
\]

\[
G_{ACBD} = -\left( \frac{1}{2} \epsilon^{FGH} \mathcal{J}_{GH} \right)_{ABCD} - (M_{AB} \epsilon^{CD} + \epsilon^{E} \epsilon^{E})^*
\]

Follow the ordinary way of spinor GR to translate from spinor to tensor (Equation 5), Einstein tensor is equivalent to

\[
G_{\mu \nu} = \sigma_{\mu}^{AC} \sigma_{\nu}^{BD} \left( -\frac{1}{2} \epsilon^{FGH} \mathcal{J}_{GH} \right)_{ABCD} - (M_{AB} \epsilon^{CD} + \epsilon^{E} \epsilon^{E})^* \]  

(36)

If we use the property of \( M_{AB} \epsilon^{CD} + \epsilon^{E} \epsilon^{E} = M_{AB} \epsilon^{CD} + \epsilon^{E} \epsilon^{E} \), for which the symmetries are proved in the previous section, then

\[
G_{\mu \nu} = \sigma_{\mu}^{AC} \sigma_{\nu}^{BD} \left( -\frac{1}{2} \epsilon^{FGH} \mathcal{J}_{GH} \right)_{ABCD} - (M_{AB} \epsilon^{CD} + \epsilon^{E} \epsilon^{E})^* = \sigma_{\nu}^{BD} \sigma_{\mu}^{AC} \left( -\frac{1}{2} \epsilon^{FGH} \mathcal{J}_{GH} \right)_{ABCD} - (M_{AB} \epsilon^{CD} + \epsilon^{E} \epsilon^{E})^* = G_{\mu \nu}
\]

(where, in the last right-hand side, we rename the dummy indices). So, it is a clear symmetric for \( G_{(\mu \nu)} \) as expected.

Bianchi identity (Equation 9) in spinor GR describes the dynamics of curvature. Unlike Einstein field equation, which the dynamic of curvature is described by solving (up to) second-order differential equation of metric tensor; spinor GR handles the same problem by first-order differential equation - Bianchi identity (Penrose, 1960). However, spinor GR is coordinate-free yet a classical theory; there is no metric differential analog. Nevertheless, quantized Bianchi equation do provide quantized analog to spinor GR as

\[
\nabla_{D} x_{AB} \epsilon^{E} = \nabla_{A} \epsilon^{C} \Phi_{D'CFE} \epsilon^{E} \rightarrow (\hat{D}_{D} \epsilon^{E} \mathcal{J}_{AB} \epsilon^{E})^{YGH} = (\hat{D}_{A} \epsilon^{E} \mathcal{M}_{D} \epsilon^{E})^{YGH}.
\]

Once we solve the operators \( \tilde{\delta}, \hat{D}, \tilde{J}, \tilde{M}, \) and fundamental solution matrix \( Y \), the dynamics of curvature spinors \( \chi \) and \( \Phi \) can be found by equations Equation (26). The ordinary curvature components such as Ricci tensors and Ricci scalar as well as Einstein tensors can be found.

From last three chapters of this section, the overall approach of spacetime dynamics is done by first identifying related classical curvature spinors and the derivatives equations (i.e., Bianchi identity), then manifesting the spacetime by a deeper structure from isomonodromy system; finally, illustrating related tensors such as Ricci and Einstein tensors equivalence by associated spinors by associated objects from isomonodromy system. Physically, the approach claims a more fundamental reality from isomonodromy system over classical spacetime just like Penrose mentioned that spinor GR may be more deep-rooted than tensors (Penrose, 1960).

**Einstein condition and the connection to quantum operators**

It is well known that Einstein’s theory (theory of general relativity) does not assign a definite stress-energy tensor to the gravitational field.

When solving Einstein field equation, we guess the form of metric and energy-momentum-stress tensor \( T_{\alpha \beta} \) because the physical interpretation to define \( T_{\alpha \beta} \) is not yet known without knowing the metric first. In this article, the author uses the same approach to define \( T_{\alpha \beta} \) but claims that the curvature is caused by \( \delta \) on \( Y \), which constitutes energy-momentum-stress distribution. As required, the energy–momentum conservation is guaranteed in "quantized sense" by quantized Bianchi equation (just as in classical GR, Bianchi identity guarantees conservation). However, it is expected that there must exist a condition that local energy/mass and momentum density do contribute proportional to \( G_{\alpha \beta} \) as Einstein field equation: \( G_{\alpha \beta} = -8\pi \mathcal{T}_{\alpha \beta} \) (\( \mathcal{G} \) is gravitational constant).

As \( G_{\alpha \beta} \) is related to \( \sigma_{\alpha}^{BD} \sigma_{\beta}^{EF} \) multiplying linear combination of \( \delta \) (by construction of \( \tilde{J} \) and entries of \( \mathcal{M}^* \) (Equation 36), let’s examine how to find \( \tilde{J} \) and \( \tilde{M} \). To find \( \tilde{J} \) and \( \tilde{M} \), which are contractions of int operators \( \tilde{J} \) on \( Y \) (Equation 20), Equation (22) explains that we can get them by \( \tilde{J}_{ABCD} \epsilon^{E} \). Considering the form of the operator \( \delta \):

\[
\tilde{J}_{ABCD} = \tilde{D}_{AB} \epsilon^{E} \tilde{G}_{CD} \epsilon^{E} + \sigma_{\alpha}^{AB} \sigma_{CD}^{\alpha \beta} (Y_{\alpha \beta} \epsilon^{E} \tilde{G}_{E} \epsilon^{E}),
\]

(37)

where \( Y_{\alpha \beta} \epsilon^{E} \) is the unknown hybrid-spinor. This implies that int operator \( \tilde{J} \) can be found by

\[
\tilde{J}_{ABCD} \epsilon^{E} \epsilon^{F} = \tilde{D}_{AB} \epsilon^{E} \tilde{G}_{CD} \epsilon^{E} + \sigma_{\alpha}^{AB} \sigma_{CD}^{\alpha \beta} (Y_{\alpha \beta} \epsilon^{E} \tilde{G}_{E} \epsilon^{E})
\]

(38)

for which we use the fact that \( \mu \) and \( \nu \) are dummy indexes, and \( \tilde{D}_{\mu} \epsilon^{E} \) and \( \tilde{D}_{\nu} \epsilon^{E} \) commute; this implies \( \tilde{J} \) and \( \tilde{M} \) are related to terms of \( \sigma_{\alpha}^{AB} \sigma_{CD}^{\alpha \beta} (Y_{\alpha \beta} \epsilon^{E} \tilde{G}_{E} \epsilon^{E}) \).
where \((Y^{-1})_{ik}\) is the inverse of \(Y\). The introduction of \(\tilde{\sigma}\) (Equation 37) is to change from dealing with \(\mathbb{H}_{AB}^{\mu}G\) to \(\Gamma_{\mu\nu}^{EG}\), while this form \(\tilde{\sigma}\) seems not unique. Let \(B_{\mu}^{F}\) is invertible (if considering \(E\) and \(F\) are considered as the matrix indices), that is, in GL, we may naturally define the hybrid-spinor \(\Gamma_{\mu\nu}^{EG}\) by

\[
\Gamma_{\mu\nu}^{EG} = \left(\tilde{\sigma}_{\mu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right).
\]

So it implies

\[
\begin{align*}
\sigma_{\mu}^{E\mu\nu} & = \left(\tilde{\sigma}_{\mu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right) = \sigma^{E\mu\nu}_{\mu}^{\gamma} = \left(\tilde{\sigma}_{\mu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right)F_{\mu}\,
\sigma_{\mu\nu}^{E\mu\nu} & = \left(\tilde{\sigma}_{\mu\nu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right) = \sigma_{\mu\nu}^{E\mu\nu}_{\mu}^{\gamma} = \left(\tilde{\sigma}_{\mu\nu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right)F_{\mu}\end{align*}
\]

where we use core equation (Equation 17) and condition, that is, \(\sigma_{\mu\nu}^{E\mu\nu}\) is \(D\)-invariant. So, we get the int operator working as desired.

For scalar case of \(\tilde{\mathbb{D}}_{\mu}^{E\mu\nu}\), we currently consider \((\tilde{\mathbb{D}}_{\mu}^{E\mu\nu} \equiv \partial_{\mu}\delta_{\mu}^{E\mu\nu}\) as previously stated):

\[
\mathbb{D}_{\mu}^{E\mu\nu}F_{\mu} = \mathbb{D}_{\mu}F_{E} = \frac{\mathbb{D}_{\mu}F_{E}}{\mathbb{h}} (39)
\]

where \(\mathbb{D}_{\mu}\) is the usual quantum energy, momentum operator, while, for simplicity, \(p_{i} (i=1,2,3)\) is negative to usual momentum; we can plug it into the following expression and get the relationship:

\[
\begin{align*}
\sigma_{\mu}^{E\mu\nu} & = \left(\mathbb{D}_{\mu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right) = \sigma_{\mu}^{E\mu\nu} = \left(\mathbb{D}_{\mu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right)F_{\mu},
\sigma_{\mu\nu}^{E\mu\nu} & = \left(\mathbb{D}_{\mu\nu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right) = \sigma_{\mu\nu}^{E\mu\nu} = \left(\mathbb{D}_{\mu\nu}^{E\mu\nu}(B_{\mu}^{F})(B_{\mu}^{-1})^{G}\right)F_{\mu}\end{align*}
\]

note that we choose to ignore the complex conjugate sign \(\star\) of \(\mathcal{M}\) and detail of the indices per entity as we just want to illustrate the possible condition to match \(T_{\mu\nu}\) (as operator-sense) of Einstein field equation. It motivates the definition of a hybrid-spinor density to match the unit of Einstein field equation:

\[
\Gamma_{\mu\nu}^{EG} \equiv i\hbar\,\mathcal{G}[\text{Density}]
\]

such that \(\tilde{\sigma}\) with \(\Gamma_{\mu\nu}^{EG}\) density generates the curvature by the source of local energy–momentum from related ordinary quantum mechanics operators \((i\hbar\partial_{\mu})\). The detail of \(\sigma_{\mu}^{E\mu\nu}\), \(\sigma_{\mu\nu}^{E\mu\nu}\), and \(\Gamma_{\mu\nu}^{EG}\) results in the freedom that one can assume the structure of \(T_{\mu\nu}\) as we normally solve problems in classical general relativity (demonstrated in later sections). Nevertheless, one must solve the operators and required equations in section 3.3, in order to find \(\Gamma_{\mu\nu}^{EG}\) density for a complete solution.

The generalization to gauge operators for spacetime

The gauge theory adopted in this theory is, as usual, to allow certain transformation of mathematical objects including the solution (\(Y\) here) such that same set of equations are invariant. So, if we found those gauge transformation, a new dynamic description of the curvature of spacetime exists, which we call it generalization to the gauge operators (or called fields). The core theory of this article naturally suggests the spinor operation over \(2 \times 2\) matrix (the base mathematical field) instead of scalar because of core equation (Equation 17). So in this section (ONLY), the author illustrates the equations in spinor over \(2 \times 2\) matrix, that is, core equation reads from \(\mathbb{D}_{\mu}^{E\mu\nu}F_{\mu} = B_{\mu}^{E\mu\nu}F_{\mu} / \mathbb{h}\) to \(\mathbb{D}_{\mu}Y = B_{\mu}Y\), so it is easy to see the natural simplicity.

Gauge operators (or fields) \(\mathcal{G}\) operators used to define the transformation of following objects as well as \(Y\) simultaneously while keeping following equations invariant:

Objects to transform by gauge fields: \(\mathbb{D}_{\mu}^{\mu\nu}, \mathcal{G}_{\mu\nu}^{\mu\nu}, B_{\mu}^{\mu\nu}J_{\mu\nu}\)

Equations to be invariant:

\[
\begin{align*}
\text{commutativity of } \mathbb{D}_{\mu}^{\mu\nu}, \mathbb{D}_{\mu}^{\mu\nu} & : \mathbb{D}_{\mu}^{\mu\nu}Y = 0, \\
\text{invariant: } \mathbb{D}_{\mu} \Sigma_{\mu\nu}^{\mu\nu}X & = \Sigma_{\mu\nu}^{\mu\nu} \mathbb{D}_{\mu}X \quad (\Sigma_{\mu\nu}^{\mu\nu}\text{ denotes as generalized }\sigma_{\mu\nu}^{\mu\nu}, \text{ so it is } 2 \times 2\text{ matrix in general }\forall\text{ indices of }\mu,\nu) \}
\end{align*}
\]

Equations of system (Equation 19):

\[
\mathbb{D}_{\mu}^{\mu\nu}Y = B_{\mu}^{\mu\nu}Y, \quad \mathbb{D}_{\mu}Y = B_{\mu}^{\mu\nu}Y
\]

Integratability: \([B_{\mu}^{\mu\nu}] = \mathbb{D}_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu} \text{ and } \mathbb{D}_{\mu}B_{\mu}^{\mu\nu} = \mathbb{D}_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu} = 0\)

\(\Gamma\) operator (if any): \(\Gamma_{\mu\nu} = (\mathbb{D}_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu})^{-1}\)

\(\text{comummutativity of } \mathbb{D}_{\mu}^{\mu\nu}, \mathbb{D}_{\mu}^{\mu\nu} \text{ and } \mathcal{M}: \mathbb{D}_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu} / \mathcal{M}[\mathbb{D}_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu}] = 0, \text{ and } \mathcal{M} / \mathcal{M} = 0\)

Fundamental equation (Equation 21): \(\mathcal{K}^{\mathcal{K}}[\mathcal{A}^{\mathcal{K}}B_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu}]Y = \mathcal{K}^{\mathcal{K}}[\mathcal{A}^{\mathcal{K}}B_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu}]Y(\\text{original form})\)

One can easily observe that only certain transformation is allowed because the equations above must be simultaneously satisfied. Physically, it means only certain gauge field related to transformation of curvature of spacetime is allowed. In the next section, we will exercise this generalization in certain way to apply to evolution model of the universe.

A key symmetric structure of fundamental equation in second form \((\mathbb{D}_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu}X)Y = (\mathbb{D}_{\mu}^{\mu\nu}B_{\mu}^{\mu\nu}X)Y \text{ is obvious that the author defines this in the original version. As it has Hermitian conjugate structure if considering the indices of }\mathcal{A}\) and some \(\mathcal{J}\) and \(\tilde{\mathbb{D}}\).

**AS AN APPLICATION TO THE PROBLEM OF MODELING THE UNIVERSE EVOLUTION**

In this section, some forms of operators and solution \(Y\) are deployed such that a simple solution of equation (21) is obtained. An equation explaining the evolution model of the universe is derived; usual Friedmann equation is compared as well as density parameters of dark matter, ordinary matter, and dark energy is proposedly originated; furthermore, the prediction of density parameter of ordinary matter is compared to the observation of Planck data (Carroll & Ostlie, 2007; Density data: http://hyperphysics.phy-astr.gsu.edu/hbase/astro/denpar.html). At last, certain equations of unsolved fields are discussed for the consistency of the theory.
Basic cosmological model

There are some assumptions on the form of $Y^{GH}$, $\hat{D}_F^E$, and $\Gamma_{\mu E F}$ as well as the new gauge field - hybrid spinor $\Lambda_{\mu E F}$ to the application of a cosmological model we interest:

\[ Y^{GH} \rightarrow Y^{GH}(t) = \begin{pmatrix} g^{\nu(t)} & \frac{\partial}{\partial t} \\ 0 & 1 \end{pmatrix}, \] (\(\nu(t)\) is a scalar function of \(t\))

\[ \hat{D}_F^E \rightarrow 0 + \alpha \Lambda_{\mu E F}, \quad \Lambda_{\mu E F} \text{ is } t \text{- independent}, \]

parameter space are \( t = \chi_0, x = \{x_1, x_2, x_3\}; \lambda_{\nu E F}(t, x) \) are 2 \( \times \) 2 commuting matrices (if considering \(E\) and \(F\) as the matrix indices) such that \([\Lambda_{\mu}, \Lambda_{\lambda}] = 0\), and \(a(t)\) is a real scalar field for t-dependent;

\[ \Gamma_{\mu E F} \rightarrow a(t) \alpha X(x) \chi_{\mu E F}, \quad (41) \]

where \(a(x)\) is the real scalar field for \(x\)-dependent only, \(\chi_{\mu E F}\) is a constant spinor such that

\[ \chi_{\mu E F} = 0 \quad \text{for } \nu \neq 0, \chi_{\mu E F} = \chi \delta_{EF} \text{ is a real constant if } \nu = 0. \]

Finally, we assume $B_{\mu E F}$ is an invertible matrix (if considering \(E\) and \(F\) as the matrix indices).

Let's examine $\hat{f}_{EF}(Y^{KL})$ from Equation (38):

\[ \hat{f}_{EF}(Y^{KL}) = \chi_{EF} E \left( \Gamma_{\mu E F} \frac{\partial}{\partial \mu} \right) Y^{KL} \]

where we use the commutativity of the form of $\hat{D}_F$, form of $\hat{D}_E$, form of $\Gamma_{\mu E F}$ (Equation 41), and $\sigma_{EF}$ is $\hat{D}_F$-invariant. We further apply the form of $\hat{f}_{EF}$ (Equation 41) that $\hat{f}_{EF} = 0$ for $\nu \neq 0$ and $Y^{KL}$ is $t$-dependent only:

\[ \hat{f}_{EF}(Y^{KL}) = \chi_{EF} E \left( \Gamma_{\mu E F} \frac{\partial}{\partial \mu} \right) \chi_{KL} \]

Let us study the overall $t$-scaling (i.e., $t$-dependent) effect of the equation by introducing length scaling factor $l$ from a:

\[ a = l^{-3} \rightarrow [\text{length scaling factor}], \quad \hat{f}_{EF}(Y^{KL}) = \hat{f}_{EF}(Y^{KL}) \]

where a new spinor from combination of $\sigma_{EF} : \Sigma_{EF}^{GH} \equiv (\Sigma_{i=1}^{\nu} \sigma_{EF} \sigma_{0}^{GH} - \sigma_{EF}^{GH} (\Sigma_{i=1}^{\nu} \sigma_{0}^{GH})).$

Then we get $\mathcal{J}$ and $\mathcal{M}$ by $\mathcal{J}, \mathcal{M}$ and the inverse of $Y^{GH}$:

\[ \mathcal{J}_{AC}^F = (\mathcal{J}_{AC}^F) (Y^{GH}(Y^{-1})_{HF} = (\mathcal{J}_{AC}^F E Y^{GH}(Y^{-1})_{HF} = \left( \begin{array}{cc} \epsilon^{BD} & \Sigma_{ABCD} \end{array} \right) Y^{GH} \left( Y^{-1} \right)_{HF} \]

\[ = \left( \begin{array}{cc} \theta^{C} a(X) \Lambda_{\mu E F}(x) \chi_{\nu E F} \end{array} \right) \frac{\partial}{\partial \mu} \left( Y^{GH} \right) \left( Y^{-1} \right)_{HF} \]

\[ \mathcal{M}_{BD}^E = (\mathcal{M}_{BD}^E) \left( Y^{GH}(Y^{-1})_{HF} = (\mathcal{M}_{BD}^E) \left( Y^{GH}(Y^{-1})_{HF} \right) \]

\[ = \left( \begin{array}{cc} \epsilon^{AC} \Sigma_{ABCD} \end{array} \right) \frac{\partial}{\partial \mu} \left( Y^{GH} \right) \left( Y^{-1} \right)_{HF} \]

\[ = \left( \begin{array}{cc} \epsilon^{AC} \Sigma_{ABCD} \end{array} \right) \frac{\partial}{\partial \mu} \left( Y^{GH} \right) \left( Y^{-1} \right)_{HF} \]

where $\Sigma_{AC} \equiv \epsilon^{BD} \Sigma_{ABCD}$, $\Sigma_{BD} \equiv \epsilon^{AC} \Sigma_{ABCD}$.

Let's examine the Einstein spinor $G_{\mu \nu}$ (Equation 35) based on these assumptions and the calculation above. $G_{\mu \nu}$ is contributed by $\lambda$ and $\mathcal{M}_{\mu \nu}^E$; $\lambda$ is contributed by $\mathcal{J}_{\mu \nu}^E$ Equation 32, and Einstein tensor is derived by $\Gamma_{\mu \nu}^E \equiv \sigma_{EF}^0 \sigma_{EF}$. $G_{\mu \nu}$ is $\hat{D}_F$-invariant, $\sigma_{EF}$ is $\hat{D}_F$-invariant. We can also choose $\sigma_{EF}$ and $\sigma_{EF}$ to be near constants to make computation easy; however, one must check the consistency of the choice with the core equations (refer to section: the generalization to gauge operators for spacetime).

In conclusion, $G_{\mu \nu}$, is contributed by the term

\[ \chi_{\nu E F} \left( \frac{\partial}{\partial \mu} \right) \]

and by the following definition (to be consistent with Einstein equation):

\[ \text{Ordinary matter density tensor } \equiv \frac{8 \pi G}{\chi_{\nu E F}} \]

the $t$-dependency of ordinary matter density in this case is proportional to $l^{-3}$. Note that, the ordinary matter density defined here is to match Einstein equation for this model only.

Calculation of scaling factor and comparison to Friedmann equation

Solving Equation (21, the fundamental equation) is the core task to get the dynamics of the cosmological model, or more generally the physical structure of spacetime. The first step is to calculate the term, $\mathcal{J}_{\mu \nu}^E \hat{D}_F^E$, which is left side of the
fundamental equation. After using the form of $\mathcal{D}$, $\Gamma^{\mu\nu}_{\rho}$ (41) and $\mathcal{D}$-invariant, it implies:

$$\mathcal{J}_{CD}^{EF} \mathcal{D}_{AB}^{EF} \phi^{Y_{GH}} = \epsilon^{DEF} (\sigma^{\rho}_{AB} \sigma^{\nu}_{CD} \sigma^{\mu}_{DF} + \cdots)$$

where $\sigma^\mu$. After substituting these results to the equation above, all terms are matrices operating on $Y^{GH}$ only (if considering $G, H$ as the matrix indices), so we can take it out as $Y^{GH}$ is in GL. The equation becomes

$$\epsilon^{D\alpha} a \mathcal{X} \Gamma^{EF} (\Sigma_{CFDF}) (a^{\rho}_{AB} (a^{2}\Lambda_{F}^{E} \Lambda_{G}^{E} + \Lambda_{E}^{E}(a' + 2a\phi))$$

and note that $\Lambda_{E}^{E}$ is $t$-independent. Then, there exists a solution:

$$0 = a^{2}\Lambda_{E}^{E} \Lambda_{G}^{E} + (a' + 2a\phi) \Lambda_{E}^{E} + (\phi^2 + \phi') \delta^{E}_{E}.$$

Let’s pick an abelian structure for $\Lambda_{E}^{E}$ (if considering $F$ and $G$ as the matrix indices): below:

$$\Lambda_{E}^{E} = \left( \begin{array}{c} s(x) \\ y(x) \end{array} \right),$$

and we have the set of differential equations for the solution:

$$0 = 2ay(a + s + \phi) + ya'$$

$$0 = a^2 - s^2 + a^2 y^2 + 2a s \phi + \phi^2 + sa' + \phi'.$$

and we pick a solution that $a \neq 0$:

$$a' = -2a(s + \phi) + \phi' = a^2(s^2 - y^2) - \phi^2.$$

To get an equation to compare with usual Friedmann equation, let’s converting the solution to length scaling factor $l$ and getting a scalar $\mathcal{H} \equiv \frac{l}{R}$

$$l' = 2(l^3 \phi + s) \frac{s}{3l}, \mathcal{H} = \frac{2(l^3 \phi + s)}{3l}.$$

The equation to be compared with Friedmann equation is

$$\left( \mathcal{H}(l, \phi) \right)^2 = \frac{4}{9l^2} \frac{s^2}{l^2} + 8s \phi \frac{4\phi^2}{9l^2}, \quad (43)$$

while Friedmann equation is (spatial curvature $k$ is assumed to be zero, $\Lambda_{cos}$ is usual cosmological constant, $\Omega_n$ are the usual density parameters for different types of matter):

$$\left( \mathcal{H}(l) \right)^2 = \sum_n \Omega_n l^{-n} = \frac{8\pi G}{3H_0^2} \left( \sum_n \rho_n (l^{-n}) \right) + \frac{\Lambda_{cos}}{3H_0^2}.$$
Densities of dark energy, matter, and ordinary matter ($\Omega_\Lambda, \Omega_M, \Omega_b$)

As usual in $\Lambda$ CDM model, we identify the zeroth and negative power terms of scaling factor ($P^1, P^{-3}$ and $\Omega^{-6}$), in equation $(\Omega^B)^2$ (Equation 43), so there are three types of matter to be responsible for the evolution of the scaling factor, and their density parameters are namely $\Omega_\Lambda$, $\Omega_M$, and $\Omega_b$, respectively. The $\Omega_b$ is supposedly to be responsible for ordinary matter density as the t-dependency of ordinary matter density is proportional to $t^{-6}$ in preceding discussion:

$$\Omega_b = \frac{4\pi^2}{9} \Omega_\Lambda = \frac{4\pi^2}{9} \Omega_M = \frac{8\pi^2}{9} \Omega_b.$$  

$\Omega_M^2 \simeq 4\Omega_\Lambda \Omega_b$ (approximation = equal sign because of the approximation of the model)

To compare observation results from observation data (Carroll & Ostlie, 2007; Density data: http://hyperphysics.phy-astr.gsu.edu/hbase/astro/denpar.html), $\Omega_\Lambda$ and $\Omega_b$ are used:

$$\Omega_\Lambda \to 0.69, \quad \Omega_b \to 0.04;$$

the calculated and observed $\Omega_M$ are

Calculated $\Omega_M = 0.33$ vs

Observed $\Omega_M = 0.31$ (upper bound for best fit)

The observation results of $\Omega_b$, $\Omega_\Lambda$, $\Omega_M$ motivate the speculation for this model to claim the relationship of densities; it is similar that Maxwell speculated that electromagnetic wave is the light itself because the speed of electromagnetic wave from his equations was found accidentally the same as light speed.

The author claims that, more accurate relationship among those sources of evolution of the universe should be manageable after certain assumption is removed such as time-independent of $\Lambda_\Omega$. Moreover, complete proof of calculation of $\Omega_b$ requires particle theory, which is also not covered in this article.

Spinor gauge field-$\Lambda_{\mu}^{E_F}$ dynamics

In preceding discussion of calculation of the solution for fundamental equation (Equation 21), a set of PDE related to $\Lambda_{\mu}^{E_F}$ is used. In fact, they are the results of core constraints of the theory.

Let us look at the commutativity of $\hat{D}$ after applying gauge field $\Lambda_{\mu}^{E_F}$:

$$\hat{D}_{AB}^{E_F}\hat{D}_{CD}^{E_F}Y^{GH} = \hat{D}_{CD}^{E_F}\hat{D}_{AB}^{E_F}Y^{GH},$$

there exist a solution

$$\partial_{i}(\alpha \Lambda_{\mu}^{E_F})Y^{GH} = 0;$$

by $\Lambda_{\mu}^{E_F}$ is abelian (if considering E and F as the matrix indices), it implies

$$\partial_{i}(\alpha \Lambda_{\mu}^{E_F})Y^{GH} = 0.$$  \hspace{1cm} (44)

Then, let us look at integrability of the system: $B_{\mu}^{E_F} = \hat{D}_{\mu}^{E_F}B^{E_F}$ as $\Gamma_{\mu}^{E_F} = 0$ for $\nu \neq 0$, and it provides $\hat{D}_{\mu}^{E_F}B^{E_F}$ by Equation (38) $\Gamma_{\mu}^{E_F} = \left( \hat{D}_{\mu}^{E_F}(B_{\mu}^{E_F}(H))\right)(B_{\nu}^{-1})^{GH}$, so we get

$$B_{\mu}^{E_F}B^{E_F} = 0, B_{\mu}^{E_F}B^{E_F} = 0, B_{\mu}^{E_F}B^{E_F} = -a\alpha \Gamma B_{\mu}^{E_F}(i,j) = 1, 2, 3; \mu = 0, 1, 2, 3).$$

By the core equation $\hat{D}_{\mu}^{E_F}Y^{FG} = B_{\mu}^{E_F}Y^{FG}$, $B_{\mu}^{E_F}$ is solved by $(\partial_{\mu}\delta_{\mu}^{E_F} + a\Lambda_{\mu}^{E_F})Y(t)^{FG} = B_{\mu}^{E_F}Y(t)^{FG}$; $\Rightarrow$ exist a solution $B_{\mu}^{E_F} = a\Lambda_{\mu}^{E_F}$, it is consistent with $B_{\mu}^{E_F}B^{E_F} = 0$ and $\Lambda_{\mu}^{E_F}$ abelian property (if considering E and F as the matrix indices). $\hat{D}_{\mu}^{E_F}B^{E_F} = 0$, equation leads to:

$$a\Lambda_{\mu}^{E_F}\Lambda_{\nu}^{E_F} + \partial_{i}(\Lambda_{\mu}^{E_F}) = 0$$

$$a^2\Lambda_{\mu}^{E_F}\Lambda_{\nu}^{E_F} + a\partial_{i}(\Lambda_{\mu}^{E_F}) = 0.$$  

Combining the above equation and Equation (44), the dynamics of gauge field $\Lambda_{\mu}^{E_F}$ is got:

$$a\Lambda_{\mu}^{E_F} + a\Lambda_{\mu}^{E_F} = 0$$

$$\partial_{i}(\Lambda_{\mu}^{E_F}) = -a\Lambda_{\mu}^{E_F} + a\Lambda_{\mu}^{E_F} = 0$$

$$\partial_{i}(\Lambda_{\mu}^{E_F}) = -a\Lambda_{\mu}^{E_F} + a\Lambda_{\mu}^{E_F} = 0$$

for $i,j=1,2,3(a$ is t-derivative of $a$).

Please note that the calculation of cosmological model in this section does not include solving gauge field-$\Lambda_{\mu}^{E_F}$; however, the theory requires that the dynamics of $\Lambda_{\mu}^{E_F}$ must exist in order to be consistent.

DISCUSSION

This article provides another approach of describing the physical nature by simplicity of cross-ratio. However, the complete correspondence between cross-ratio deformation and isomonodromic integrable system used in this article is not yet covered. Potentially, additional property beyond our spacetime might emerge.

The description of quantum mechanical wave function is also not discussed, and this is what this theory should extend to. With such description and spacetime description in this article, a more complete picture of matter and spacetime should be reviewed.

Finally, the author proposes that the physical reality is the general deformation of same cross-ratio.

References


**Competing Interests**
The author declares no competing interests.

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