Modeling of the Crosstalk Phenomenon for Electro-Magnetic Systems by Bilateral Coupling of DAEs and PDEs

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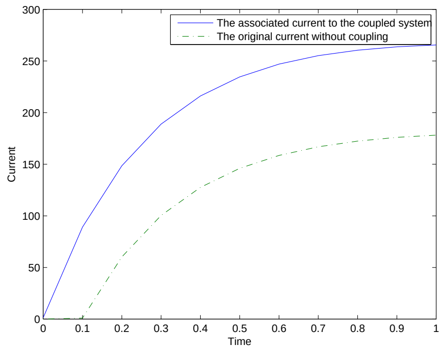
Motivation to the Crosstalk Phenomenon

- Efficient modeling of the crosstalk phenomenon is nowadays highly demanded in nanotechnology industry.
- Crosstalk phenomenon for electro-magnetic systems, in particular, for the electrical circuits, refers to the influence of the induced disturbances between electrical elements affecting the entire system.
- Our approach to model the crosstalk is via **bilateral coupling** of the circuit equations formulated by modified nodal analysis system (**differential-algebraic equations**) and non-stationary Maxwell equations (**partial differential equations**).
- Each set of equations is presented as an input-output system.
- Bilateral coupling relations, the so-called **coupling** and **re-coupling** relations, connect the given output of one set to the input of the other set.



Circuit Equations

The influence of the crosstalk in the entire system is caused by the induced current disturbance and the induced voltage disturbance where they can be modeled by the controlled current source CC and the controlled voltage source CV, respectively. Then, this effect on the electrical circuit consisting of the linear electrical elements with the associated incidence matrices A_{d_1} where $d_1 \in \{C, R, L, V, CV, I, CC\}$ is described via the following modified nodal analysis system

$$\begin{aligned} A_C C A_C^T \partial_t \eta &= -A_R G A_R^T \eta - A_L \imath_L - A_V \imath_V \\ &\quad - A_{CV} \imath_{CV} - A_I \imath_I - A_{CC} \imath_{CC} \\ L \partial_t \imath_L &= A_L^T \eta \\ 0 &= A_V^T \eta - u_V \\ 0 &= A_{CV}^T \eta - u_{CV}. \end{aligned}$$

Therein,

$$\imath_{CC}^T = [\imath_{CC_1}^T \cdots \imath_{CC_i}^T \cdots]$$
$$u_{CV}^T = [u_{CV_1}^T \cdots u_{CV_i}^T \cdots]$$

and η denotes vector of nodal potentials associated to the electrical circuit. In addition for the elements $d_2 \in \{L, V, CV, I, CC\}$ and $d_3 \in \{V, CV\}$, the corresponding currents and voltages are respectively denoted by $\imath_{d_2}$ and u_{d_3}.

Maxwell Equations

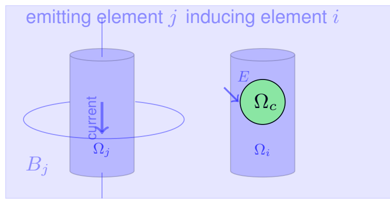
We consider the inducing element i within the observation time interval $\mathbb{I}$. Considering magnetic flux density B, electric field E, and magnetic field H defined as vector fields B, E, H: $\Omega_i \times \mathbb{I} \to \mathbb{R}^3$, the non-stationary Maxwell equations read as

$$\begin{aligned} \nabla \cdot B &= 0 && \text{in } \Omega_i \times \mathbb{I} \\ \nabla \times E &= -\partial_t B && \text{in } \Omega_i \times \mathbb{I} \\ \nabla \times H &= \\ \partial_t(\epsilon_0 E + P) &+ \sigma E + J && \text{in } \Omega_i \times \mathbb{I} \\ \nabla \cdot (\epsilon_0 E + P) &= \rho && \text{in } \Omega_i \times \mathbb{I} \\ n \times E &= n \times E^{in} && \text{on } \Gamma_i \times \mathbb{I} \\ n \cdot B &= n \cdot B^{in} && \text{on } \Gamma_i \times \mathbb{I} \\ 0 &= n \times E && \text{on } \Gamma_c \times \mathbb{I} \\ 0 &= n \cdot B && \text{on } \Gamma_c \times \mathbb{I} \end{aligned}$$

where impressed current density, polarization density, vacuum permittivity, conductivity and electric charge density are denoted respectively by J, P, ϵ_0, σ and ρ. The outer normal vector is denoted by n. Furthermore, one has to investigate

$$\begin{aligned} \nabla \times E^{in} &= -\partial_t B^{in} && \text{in } \mathbb{R}^3 \setminus \Omega_i \times \mathbb{I} \\ \nabla \cdot (\epsilon_0 E^{in} + P) &= \rho && \text{in } \mathbb{R}^3 \setminus \Omega_i \times \mathbb{I} \end{aligned}$$

Geometry of Problem



Induction/Coupling

The current and the voltage disturbances induced within the inducing element i can be prescribed by the suitable operator based model with $x = (x_1, x_2, x_3)$

$$\imath_{CC_i}(t) = \mathbf{\Psi}_i(\Omega_i, H(x,t))$$
$$u_{CV\,i}(t) = \mathbf{\Xi}_i(\Omega_i, E(x,t)).$$

Emission/Re-Coupling

The corresponding magnetic flux density to the emitting element j in $\mathbb{R}^3 \setminus (\Omega_i \cup \bar{\Omega}_j)$ is defined as

$$B_j(x,t) = \int_{\Omega_j} \mathbf{\Phi}_j(x, y, \imath_L, \eta, \partial_t \eta)\, \mathrm{d}y$$

and the total magnetic flux density is compounded by

$$B^{in}(x,t) = \sum_j B_j(x,t).$$

Index Analysis

- Spatial semi-discretization of the obtained coupled system, which is in fact partial differential-algebraic equations, leads to differential-algebraic equations (DAEs).
- Structural properties of the obtained DAEs can be presented by its index, where high index of a system of DAEs might lead to instabilities and inconsistencies. Therefore, it is necessary to determine and analyze index of the system before time integration.
- Index of the obtained DAEs system is increased by 1 while the underlying DAEs, i.e. the circuit equations, are coupled with the Maxwell equations.
- The smoothness requirements are increased due to the high index.

Future Research Plans

- Regularization
- Numerical time integration

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