Motivation to the Crosstalk Phenomenon

- Efficient modeling of the crosstalk phenomenon is nowadays highly demanded in nanotechnology industry.
- Crosstalk phenomenon for electro-magnetic systems, in particular, for the electrical circuits, refers to the influence of the induced disturbances between electrical elements affecting the entire system.
- Our approach to model the crosstalk is via bilateral coupling of the circuit equations formulated by modified nodal analysis system (differential-algebraic equations) and non-stationary Maxwell equations (partial differential equations).
- Each set of equations is presented as an input-output system.
- Bilateral coupling relations, the so-called coupling and re-coupling relations, connect the given output of one set to the input of the other set.

Circuit Equations

The influence of the crosstalk in the entire system is caused by the induced current disturbance and the induced voltage disturbance where they can be modeled by the controlled current source $CC$ and the controlled voltage source $CV$, respectively.

Then, this effect on the electrical circuit consisting of the linear electrical elements with the associated incidence matrices $A_d$, where $d_1 \in \{C, R, L, V, CV, I, CC\}$ is described via the following modified nodal analysis system

\[
A_CCA_C^T \partial_t \eta = -A_RGA_R^T \eta - A_LUL - A_VVV - A_CVV - A_CI - A_CCC
\]

\[
L \partial_t LL = A_L^T \eta - u_V
\]

\[
0 = A_C^T \eta - u_CV.
\]

Therein,

\[
i_{CC} = [i_{CC_1} \ldots i_{CC_3}]
\]

\[
i_{CV} = [i_{CV_1} \ldots i_{CV_3}]
\]

and $\eta$ denotes vector of nodal potentials associated to the electrical circuit. In addition for the elements $d_2 \in \{L, V, CV, I, CC\}$ and $d_3 \in \{V, CV\}$, the corresponding currents and voltages are respectively denoted by $i_{d_2}$ and $u_{d_3}$.

Maxwell Equations

We consider the inducing element $i$ within the observation time interval $I$. Considering magnetic flux density $B$, electric field $E$, and magnetic field $H$ defined as vector fields $B, E, H : \Omega_i \times I \rightarrow \mathbb{R}^3$, the non-stationary Maxwell equations read as

\[
\nabla \cdot B = 0 \quad \text{in} \; \Omega_i \times I
\]

\[
\nabla \times E = \eta \quad \text{in} \; \Omega_i \times I
\]

\[
\nabla \cdot H = \partial_t \eta \quad \text{in} \; \Omega_i \times I
\]

\[
\nabla \times E = n \times E_{\text{in}} \quad \text{on} \; \Gamma i \times I
\]

\[
n \cdot B = n \cdot B_{\text{in}} \quad \text{on} \; \Gamma i \times I
\]

\[
0 = n \times E \quad \text{on} \; \Gamma i \times I
\]

\[
0 = B \quad \text{on} \; \Gamma i \times I
\]

where impressed current density, polarization density, vacuum permittivity, conductivity and electric charge density are denoted respectively by $J$, $P$, $\epsilon_0$, $\sigma$ and $\rho$. The outer normal vector is denoted by $n$. Furthermore, one has to investigate

\[
\nabla \times E_{\text{in}} = \partial_t B_{\text{in}} \quad \text{in} \; \mathbb{R}^3 \setminus \Omega_i \times I
\]

\[
\nabla \cdot (\epsilon_0 E_{\text{in}} + P) = \rho \quad \text{in} \; \mathbb{R}^3 \setminus \Omega_i \times I
\]

Index Analysis

- Spatial semi-discretization of the obtained coupled system, which is in fact partial differential-algebraic equations, leads to differential-algebraic equations (DAEs).
- Structural properties of the obtained DAEs can be presented by its index, where high index of a system of DAEs might lead to instabilities and inconsistencies. Therefore, it is necessary to determine and analyze index of the system before time integration.
- Index of the obtained DAEs system is increased by 1 while the underlying DAEs, i.e. the circuit equations, are coupled with the Maxwell equations.
- The smoothness requirements are increased due to the high index.

Future Research Plans

- Regularization
- Numerical time integration

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