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STEADY MAGNETOHYDRODYNAMIC NATURAL CONVECTION COUETTE FLOW WITH VARIABLE ELECTRICAL CONDUCTIVITY

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The aim of the paper is to investigate the effect of Hartman number (M) and Grashof number (Gr) on steady magnetohydrodynamic natural convection couette flow of viscous in compressible and electrically conducting fluid having variable electrical conductivity between two parallel horizontal plates, when one of the plate is set into motion.

The dimensionless differential equation as well as the energy equations are solved analytically using the method of undetermined coefficient. The analytical solution are presented numerically in form of linear graphs and in tabular form; given in terms of velocity and skin friction. The result revealed that, the effect of the Hartman number and the Grashof number are to reduce and increases the velocity respectively. Similarly, the skin friction on the moving and stationary plates increase with increase in Grashof number and decreases with increase in Hartmann number, a comparative study revealed that, the behavior of the velocity and the skin friction with respect to Hartmann and Grashof numbers are same when the magnetic field is fixed relative to either the fluid or the moving plate.

KEY WORD: MHD, Natural Convection, couette flow, variable electrical conductivity.

1. Introduction:

The first recorded use of the word magnetohydro dynamics is by Hannes Alfvén, in 1942; for which he received the noble prize in physics in 1970.

The study of MHD Natural Convection Couette flow of viscous incompressible and electrically conducting fluid in the presence of transverse magnetic field is of special technical significant because of its frequent occurrence in many industrial applications such as magnetohydrodynamic power generators, pumps, cooling of nuclear reactions geothermal systems, thermal insulators, nuclear waste disposal, petroleum and polymer technology, heat exchange and others. This area has engaged the attention of several researchers. One of the earliest studies in this field was carried out by Agrawal [(1)], He investigated the couette flow of a viscous incompressible electrically conducting fluid between two infinite plate in the presence of transverse magnetic field when one of the plates starts impulsively from rest. Ahmed *et al*[(2)] investigated two dimensional steady free convection and mass transfer flow through a porous medium boundary by two stationary infinite vertical porous plates in the presence of heat source and chemical reaction. Aydin *et al*[(3)] analyzed the effect of viscous dissipation on the heat transfer in a poiseuille flow between parallel plates, Hartmann *et al*[(4)] analyzed the influence of the effects of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid exiting through parallel stationary plates that are insulated. Jha *et al*[(5)] studied the combined effects of halls and ion-slip currents on unsteady MHD couette flow in a rotating system, they observed that the resultant skin friction on the moving plate decrease with an increase in both the hall ion-slip parameters when the magnetic field is fixed relative to the fluid, while the opposite behaviour is noticed when the magnetic field is fixed relative to the moving plates. Jha *et al*[(6)]. In another article studied the combined effects of halls and ion-slip currents on unsteady MHD couette flow of viscous fluid in annulus. The effects of transverse magnetic field on the couette flow of an electrically conducting fluid between two infinite parallel plates when one of the plate have been set into ramp motion was investigated by Jha *et al*[7]. Katagiri[8] studied the couette flow of a viscous incompressible electrically conducting fluid between two infinite plates in the presence of transverse magnetic field, when one of the plate starts impulsively from rest; he presented his analysis by taking the magnetic lines of forces fixed relative to the fluid. Makind *et al*[9] investigated the generalized couette flow and heat transfer with variable viscousity and electrical conductivity. Sarris *et al*[10] investigate anumerical study of unsteady two dimensional natural convection of an electrically conducting fluid in a laterally and volumetrically heated square field. Singh *et al*[11] analyzed the problem discussed by Katagiri by assuming magnetic lines of forces fixed relative to the moving plate having impulsive or accelerated motion. Ziya *et al*[12] investigate MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in poros domain with the presence of radiation.

To the best of our knowledge, no work seems to have address the effect of Hartmann number (M) and Grashof number (Gr) on the steady MHD natural convection couette flow of viscous incompressible and electrically conducting fluid having variable electrical conductivity, raped between two horizontal parallel plates, when one of the plate is set into motion in the presences of transverse magnetic field. This will be the focus of this present work. The lower plate of the system under consideration is set into motion with a velocity U_0. While the upper plate is kept stationary. The relevant physical problem is modeled into ordinary differential equation, i.e Navier – stokes equation, the continuity equation. The dimensional momentum and energy equation together with the boundary condition are reduced to dimensionless form by using suitable parameters. The analytical solution are then presented for the velocity and the skin friction using, the method of undetermined coefficient for both constant and variable electrical conductivity. The effect of the various governing parameters of the problem. e.g. the Hartmann number & Grashof number on both the velocity and the skin friction are discussed for the two cases under consideration (i.e. when the magnetic lines of force are fixed relative to the moving fluid and when fixed relative to the moving plate).

2. Mathematical Analysis:

In this work we consider the motion of a viscous, incompressible and electrically conducting fluid filling the gap between two infinite horizontal parallel plates, the two plate are located on the Y axis at h distance apart due to the ramped motion of one of the plate (y = 0) as shown in the figure below.

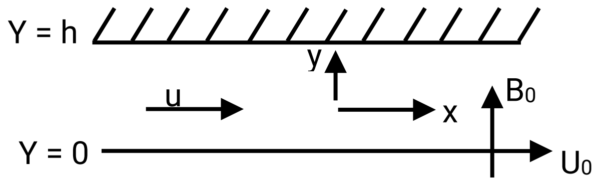


Figure 1.1 Motion of viscous incompressible and electrical conducting fluid between two horizontal parallel plates.

The x axis is assumed to be on the horizontal direction and y axis is on the radial direction. The fluid flow between the two parallel plates in the x – direction in the presences of a uniform magnetic field acting perpendicular to the main flow direction. It is assumed that the plates are electrically non-conducting and that the applied uniform magnetic field B_0 is acting perpendicular to the flow direction. By assuming a small magnetic Reynolds number, the induced magnetic field is neglected in compression to the B is a constant B = (0,0,B0) and is considered as the total magnetic field acting on the fluid. Also since no applied polarization voltage is imposed on the flow field the electric field vector E=0. This corresponds to the case where no energy is added or extracted from the fluid by the electric field.

Initially, both the fluid, the plates and the magnetic lines of force are assumed to be at rest then the lower plate starts moving in its own plane with a velocity U_0 where U is a constant and the upper plate remains stationary. The Y axis is assumed to be normal to the plates. The mathematical model governing the present physical situation is given as

$$\frac{vd^2u}{dy^2} - \frac{\sigma B_0^2 u}{\rho} + g\beta\left(T-T_0\right) = 0 \qquad (1)$$

$$\frac{d^2T}{dy^2} = 0 \qquad (2)$$

Equation (1) is valid when the magnetic field is fixed relative to the fluid. If the magnetic field is fixed relative to the moving plates equation (1) can be replaced with.

$$\frac{vd^2u}{dy^2} - \frac{\sigma B_0^2}{\rho}\left[u-U_0\right] + g\beta\left(T-T_0\right) = 0 \qquad (3)$$

Equations (1) and (3) can be combined together to obtain.

$$\frac{vd^2u}{dy^2} - \frac{\sigma B_0^2}{\rho}\left[u-KU_0\right] + g\beta\left(T-T0\right) = 0 \qquad (4)$$

Where K = { 0 when B_0 is fixed relative to the fluid; 1 when B_0 is fixed relative to the plate. }

The initial and the boundary conditions for the problem are

$u = U_0$, $T = Tw$ at $y = 0$

$u = 0$, $T = 0$ at $y = h$ (5)

Equation (4) and (5) describe the general flow, the equations are valid for both cases of when the magnetic lines of force are fixed relative to the fluid and when fixed relative to the moving plate. We introduced the following dimensionless quantities in eq. (4) as

$$Y = {}^{y}/_{h} \quad U = {}^{u}/_{U_0,} \quad \theta = (T-T_0)/(T_w-T_0)$$

$$M^2 = \frac{B_0^2 h^2 \sigma}{v\rho}, \quad Gr = \frac{g\beta\left(T_w-T_0\right)h^2}{U0v}, \quad \sigma = \frac{\sigma 0}{\left(1+{}^{y}/_{h}\right)^2}$$

Where M is the Hartmann number, Gr is the Grashof number, h is the distance between the two plate, U_0 is the velocity at which the lower plate is moving with, σ is the electrical conductivity. After rearrangement we have,

$$\frac{d^2U}{dY^2} - \frac{\sigma_0 B_0^2 h^2\left[U-K\right]}{\rho v\left(1+Y\right)^2} + \frac{g\beta\left(T_w-T_0\right)h^2}{U0v}\theta = 0 \qquad (6)$$

$$\frac{d^2U}{dY^2} - \frac{M^2\left(U-K\right)}{\left(1+Y\right)^2} = -Gr\theta \qquad (7)$$

Subject to the following dimensionless initial and boundary conditions.

$$\left.\begin{array}{ll} U = 1, \theta = 1 & \text{at } Y = 0 \\ U = 0, \theta = 0 & \text{at } Y = 1 \\ \frac{d^2\theta}{dy^2} = 0 & \end{array}\right\} \qquad (8)$$

Equation (7) can be solved using method of undetermined coefficient. Define θ = AY + B, $1+Y = e^t$

The solution of eq. (7) under the boundary condition (8) is

$$U_Y = \left[C_1\left(1+Y\right)^{\lambda} + C2\left(1+Y\right)^{-\lambda}\right]\left(1+Y\right)^{1/2} + K - Gr\left[\frac{2(1+Y)^2}{(2-M^2)} - \frac{(1+Y)^3}{(6-M^2)}\right] \qquad (9)$$

Where $\lambda = [1/4 + M^2]^{1/2}$

The velocity can be obtained in (9) using MATLAB.

2.1 Skin Friction.

The skin friction τ is defined as ${}^{dU}/_{dY}$, which is obtained by differentiating eq. (9) with respect to Y.

$$\tau = \frac{dU}{dY} = \left[C1\left(\frac{1}{2}+\lambda\right)(1+Y)^{\left(\lambda-\frac{1}{2}\right)} + C2\left(1+Y\right)^{-\left(\frac{1}{2}+\lambda\right)}\right]\left(\frac{1}{2}-\lambda\right) - Gr\left[\frac{4(1+Y)}{2-m^2} - \frac{3(1+Y)^2}{(6-m^2)}\right] \qquad (10)$$

Skin friction of the upper and the lower plates can be obtained from eq. (10) using MATLAB. On the lower plate (Y =0) we have the skin friction to be

$$\tau_0 = \frac{dU}{dY}_{Y=0} = \left[C1\left(\lambda+\frac{1}{2}\right) + C_2\left(\frac{1}{2}-\lambda\right)\right] - Gr\left[\frac{4-3}{(2-M^2)(6-M^2)}\right]$$

$$=[\lambda (c_1 - c_2) + \frac{1}{2}(c_1+c_2) - \frac{Gr\,(18-M^2)}{(2-M^2)(6-M^2)} \qquad (11)$$

On the upper plate (y = 1), we have

$$\tau_1 = \frac{dU}{dY}\Big|_{Y=1}$$

$$=\lambda\left[c_1(2)^{\left(\lambda-\frac{1}{2}\right)} - c_2(2)^{\left(\lambda+\frac{1}{2}\right)}\right] + \left[c_1(2)^{\lambda} + C2\,(2)^{-\lambda}\right]$$

$$(2)^{-(3/2)} - \frac{4Gr\,[M^2+6]}{(2-M^2)(6-M^2)} \qquad (12)$$

Constant Electrical Conductivity

The constant electrical conductivity can be obtained from equation (6) by replacing $\frac{\sigma_0 B_0^2 h^2}{\nu\rho\ (1+Y)^2}$ with M^2 where M is the Hartmann number. Hence equation (6) become

$$\frac{d^2U}{dY^2} - M^2U = -\,Gr\,\theta - M^2k \qquad (13)$$

The boundary conditions for the present problem are U = 1, θ = 1 at Y = 0

U = 0, θ = 0 at Y = 1 (14)

Equation (13) can be solved using the method of undetermined coefficient under the boundary condition (14) to obtained the velocity and skin friction at constant electrical conductivity respectively. Define $U = K_0 + K_1Y$

$$U_Y = \frac{\sin h\,\{M\,(1-Y)\}}{\sin h\,(M)}$$

$$+ K\left[1 - \frac{\sin h\,(MY)}{\sin h\,(M)} - \frac{\sinh\,(M\,(1-Y)}{\sin h\,(M)}\right]$$

$$+ \frac{Gr}{M^2}\left[1 - \frac{\sin h\,(M\,(1-Y))}{\sin h\,(M)} - Y\right] \qquad (15)$$

The corresponding skin friction τ_y at constant electrical conductivity can be obtained by differentiating eg. (15) with respect to Y

$$\tau_y = \frac{dU_Y}{dY}$$

= −

$$\frac{-\cos h\,(m\,(1-Y))m}{\sin h\,(m)} + k$$

$$\left[\frac{-\cos h\,(mY)m + \cos h\,(m\,(1-Y))m}{\sin h\,(m)}\right] +$$

$$\frac{Gr}{M^2}\left[\frac{M\cos h\,(m\,(1-Y))}{\sin h\,(m)} - 1\right]$$

Skin friction at the upper and the lower plate can be obtained from eq. (16) using MATLAB. On the lower plate (y = 0) we have the skin friction to be

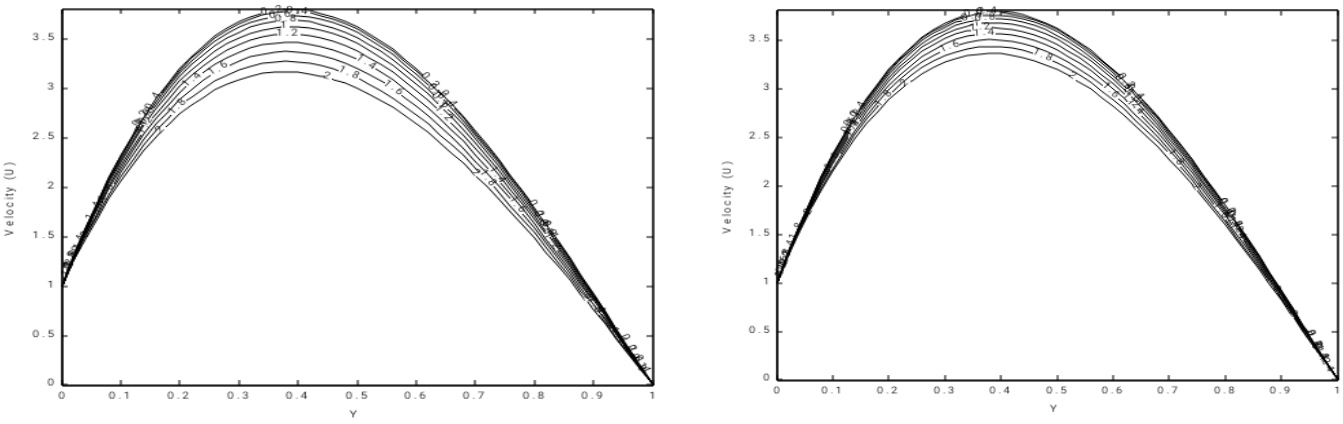
$$\tau_0 = \frac{dU}{dy}\Big|_{Y=0}$$

$$= \left[\frac{(k-1)M\cos h\,(M) - KM}{\sin h\,(M)}\right] + \frac{Gr}{M^2}\left[\frac{M\cos h\,(M)}{\sin h\,(M)} - 1\right]$$

On the upper plate (Y = 1) we have

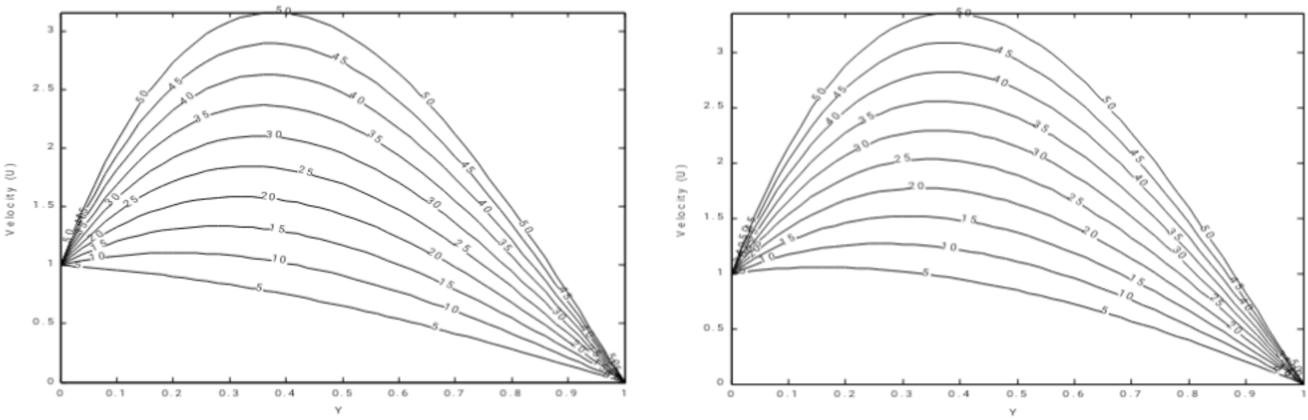
$$\tau_1 = \frac{dU}{dY}\Big|_{Y=1}$$

$$= \frac{[m\,(k-1) - KM\cos h\,(m)]}{\sin h\,(m)} + \frac{Gr}{m^2}\left[\frac{m}{\sin h\,(m)} - 1\right]$$



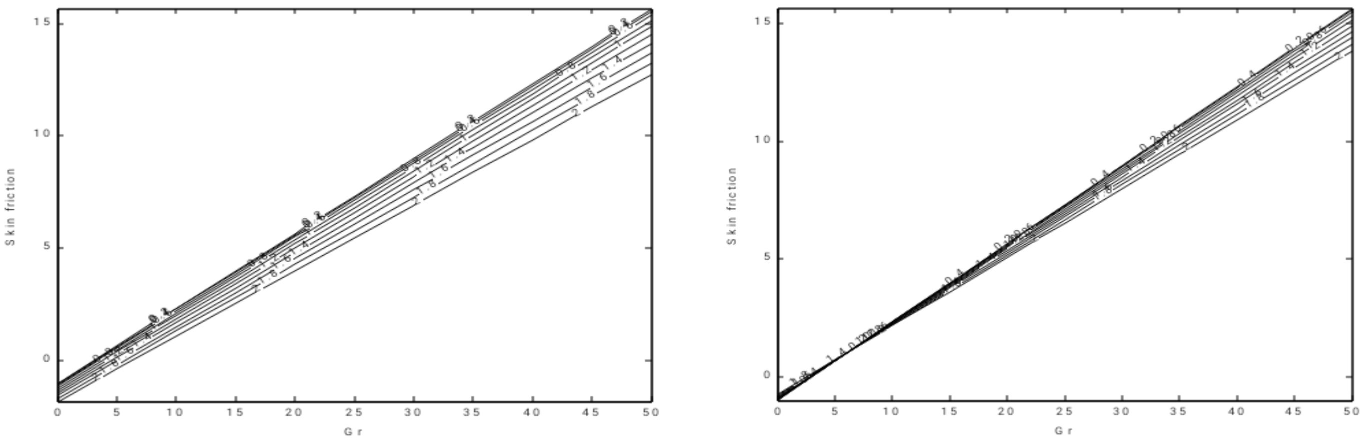
(a) (b)

Fig. 1: velocity profile showing the effect of magnetic field parameter (M) with K = 0 and 1 and Gr=50 represented by (a) and (b) respectively.



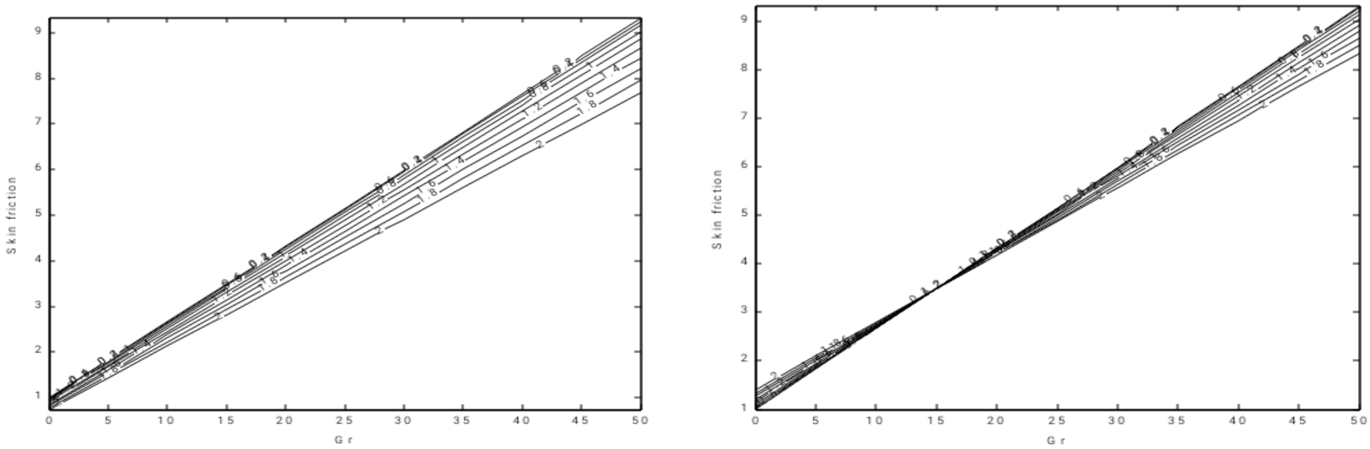
(a) (b)

Fig. 2: velocity profile showing the effect of Grashof number (Gr) with K = 0 and 1, M = 2.0 represented by (a) and (b) respectively.



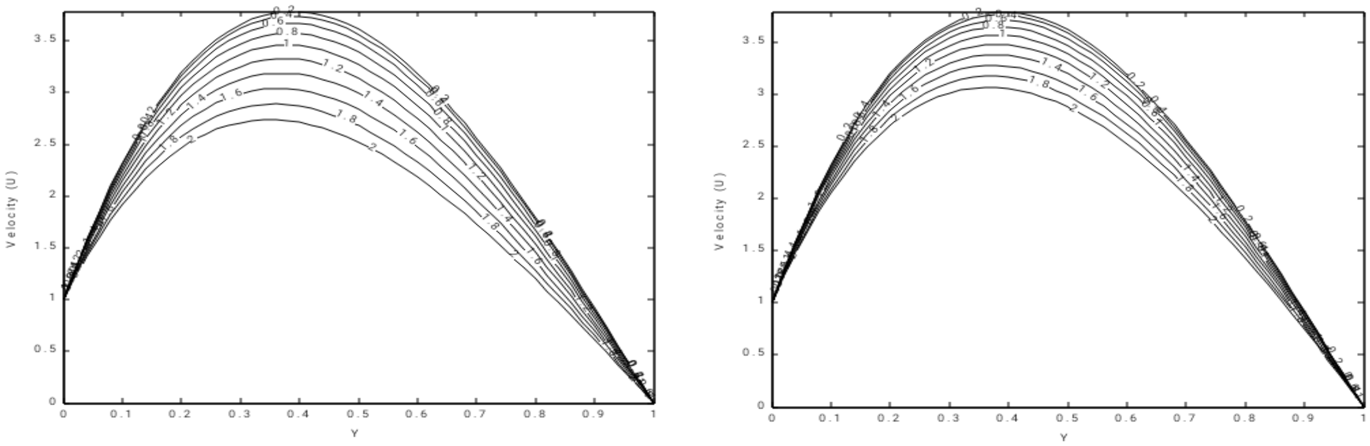
(a) (b)

Fig. 3: variation of skin friction versus Gr at y = 0 and 1 for different values of M with K = 0 and 1 represented by (a) and (b) respectively.



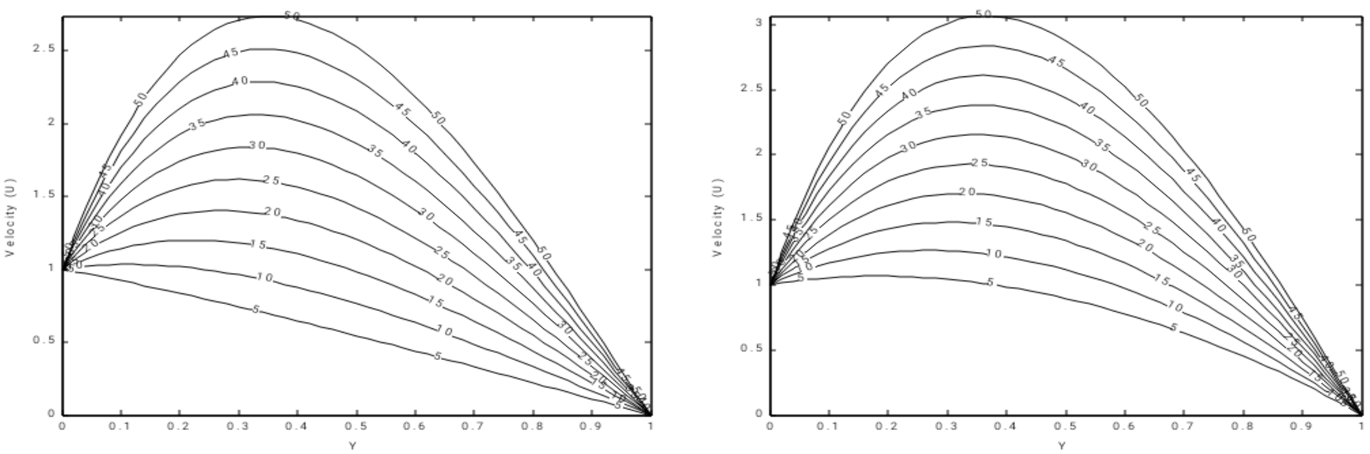
(a) (b)

Fig. 4: variation of skin friction versus Gr at y = 0 and 1 for different values of M with K = 0 and 1 represented by (a) and (b) respectively.



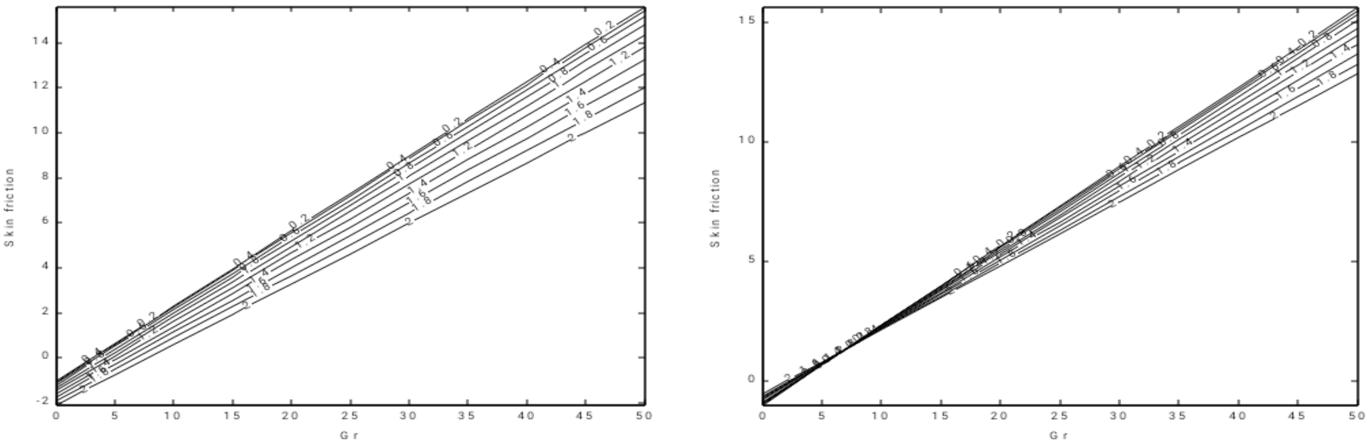
(a) (b)

Fig. 5: velocity profile showing the effect of magnetic field parameter (M) wit K = 0 and 1 and Gr=50 represented by (a) and (b) respectively.



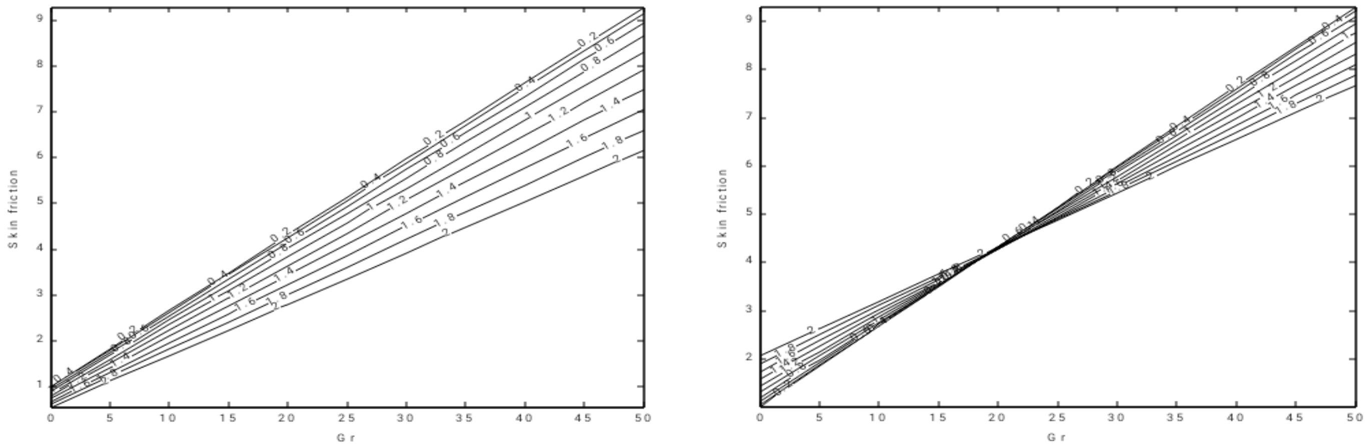
(a) (b)

Fig. 6: velocity profile showing the effect of Grashof number (Gr) with K = 0 and 1 and M = 2.0 represented by (a) and (b) respectively.



(a) (b)

Fig. 7: variation of skin friction versus Gr at y = 0 and 1 at different values of m with K = 0 and 1 represented by (a) and (b) respectively.



(a) (b)

Fig. 8: variation of skin friction versus Gr at y = 0 and 1 at different values of m with K = 0 and1 represented by (a) and (b) respectively.

Below is table one and two showing a comparative study of the skin friction at y = 0 and y = 1 with three different values of M at K = 0 and K = 1 at constant value of Gr (= 50) for both constant and variables electrical conductivity.

Table 1: Present the result for variable electrical conductivity.

M	0.5	0.5	1.0	1.0	1.5	1.5
Gr	50	50	50	50	50	50
K	0	1	0	1	0	1
τ(y=0)	15.4562	15.5322	14.8506	15.1464	13.9185	14.5560
τ(y=1)	9.2112	9.2590	8.8622	9.0477	8.3329	8.7309

Table 2: Present the result for constant electrical conductivity.

M	0.5	0.5	1.0	1.0	1.5	1.5
Gr	50	50	50	50	50	50
K	0	1	0	1	0	1
λ(y=0)	15.3134	15.4358	14.3387	14.8008	12.9470	13.8997
λ(y=1)	9.0560	9.1785	8.3050	8.7671	7.2719	8.2247

Result and Discussion:

Consider steady MHD natural convection couette flow with variable electrical conductivity. The present physical situation governing the flow is modeled into an ordinary differential equation, the analytical solution were obtained using the method of undetermined coefficient.

In order to have a good understanding of the results of the problems, a MATLAB program is coded to compute and generate the graphs for the velocity and skin friction; also a table for skin friction for different values of the Grashof number (Gr) and Hartmann number (M) parameters.

Results for variable electrical conductivity are presented in the form of line graphs in figures 1-4 to analyzed the effect of these dimensionless parameters on velocity and skin friction. Similarly, figures 5-8 present the results is the form of line graphs for constant electrical conductivity. Each of the figures is presented in two case of the magnetic field being fixed to; (a) the fluid (k = 0) and (b) the moving plate (k = 1).

Figure 1 and 2 show the velocity profile with respect to y, indicating the effect of Hartmann (M) and Grashof number (Gr) respectively. It is observed that velocity increase with decrease in Hartmann number (M) for both k = 0 and k = 1 while increasing the Grashof number (Gr) causes increase in velocity for both k = 0 and k = 1.

The variation of skin friction versus Gr for different values of M at the stationary plate (y = 0) and at the moving plate (y = 1) are represented in the figures 3 and 4 respectively. It is found that the skin friction increases with Gr and reduces with increase in M for both k = 0 and k = 1.

The effect nof the Gr and M on velocity and skin friction for the case of constant electrical conductivity is presented in figure 5 -8. Figures 5 and 6 indicate velocity profile showing the effect of M and Gr respectively. The results show that the effect of (M) is to reduces the fluid velocity, while that of Gr is to promote the fluid velocity for both k = 0 and k = 1. Also, the effect of M and Gr on skin friction at the stationary plate (y = 0) and moving plate (y = 1) are respectively show in figures 7 and 8. It is clear that skin friction reduces with increase in M and increase with Gr with both k = 0 and k = 1.

A comparative study shows that, the effect of these parameters on velocity and skin friction in the case of variable electrical conductivity is approximately same with that of constant electrical conductivity.

In like manner in each of the figures, the effect of M and Gr on velocity and skin friction for k = 0 and k =1 are approximately equal.

Conclusion:

The Steady MHD natural convection couette flow of visous incompressible and electrically conducting fluid having variable electrical conductivity when one of the plates is set into motion is studied. It is discovered that the effect of Hartmann and Grashof number are to reduces and promote the velocity respectively. Similarly the skin friction on the moving and stationary plates increase with Grashof number and decrease with increase in Hartmann number. The behaviour of the velocity and skin friction with respect to Hartmann and Grashof number are same when the magnetic field is fixed relative to either the fluid or moving plate.

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