Reduced Models for Uncertainty Quantification in the Cardiovascular Network via Domain Decomposition

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MOTIVATION

Reduced 1D models of the **cardiovascular system** are widely employed to study the interplay between **anomalous pressure waves** and pathologies like amputations, stenoses or devices like stents.

However, the **parameters** that characterize reduced 1D models are often **unknown**, and feature **variability** not only from patient to patient, but also within the same individual, depending on physiological conditions (e.g., rest vs. stress, and young vs. old). This motivated the design of mathematical and numerical techniques to ***quantify*** the **uncertainties** in these models.

CHALLENGES

- The employment of **full 3D models** for uncertainty quantification (UQ) analysis is **extremely costly** and requires computational resources that may not be easily accessible by users like hospitals, for financial, privacy or time constraints.
- **Reduced 1D models** may be **inaccurate** in capturing anomalies of the physiology in presence of cardiovascular pathologies like stenoses or aneurysms. This introduces additional (epistemic) uncertainty that should also be characterized.

OBJECTIVES

- Designing ad-hoc solvers for UQ in the cardiovascular system (CVS) that promote **parallelism** and **scalability**.
- Enhancing the **accuracy** and **reliability** of the 1D reduced models, approximately at the same computational cost.

UQ via DOMAIN DECOMPOSITION

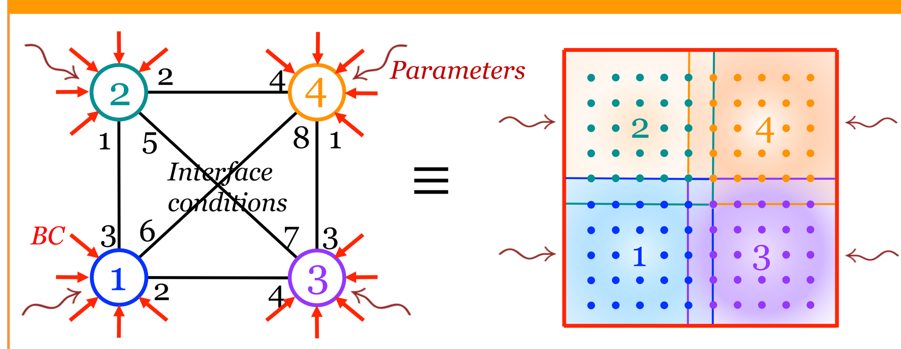


Figure: Network formulation of a DD problem.

The Domain Decomposition Uncertainty Quantification (DDUQ) approach [1] performs **UQ** at the **subsystem** level, and propagates **polynomial chaos** coefficients via DD.

NUMERICAL RESULTS: *2D steady non-linear heat equation*

Accuracy: the error is bounded and independent of the distance from the boundary (left fig.).

Scalability: computational time is reduced for different relaxation schemes (right fig.).

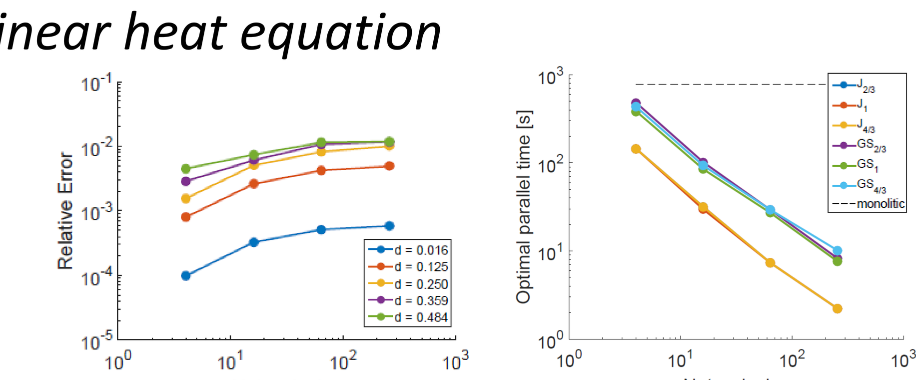


Figure: Relative error and optimal parallel time vs. network size.

"EDUCATED" REDUCED MODELS

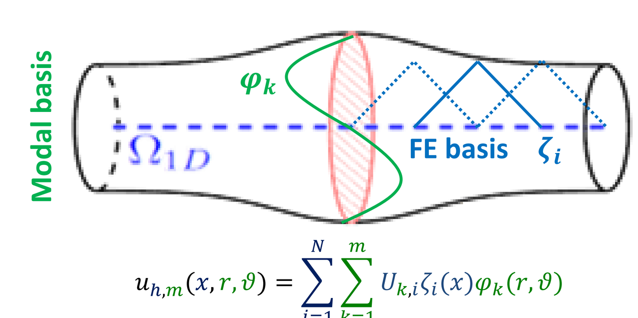


Figure: Sketch of HiMod discretization.

The **local cross-sectional dynamics** discarded by 1D models can be retrieved via ***educated* reduced models** such as the Transversally Enriched Pipe Element Method (TEPEM) [2], or Hierarchical Model (HiMod) reduction methods [3].

NUMERICAL RESULTS: 3D *unsteady Navier-Stokes equations*

Accuracy of 3D solvers can be achieved at computational cost comparable to 1D models (left).

Local **transverse dynamics** is preserved (right).

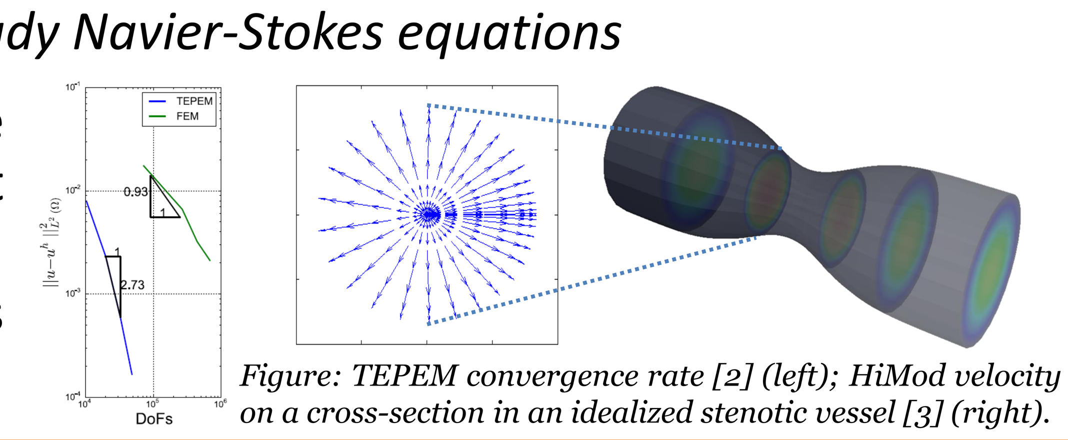


Figure: TEPEM convergence rate [2] (left); HiMod velocity on a cross-section in an idealized stenotic vessel [3] (right).

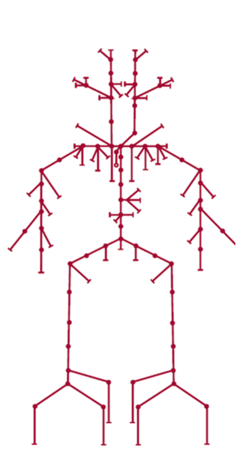
UQ in the CARDIOVASCULAR NETWORK via DD

DETERMINISTIC 1D REDUCED MODEL

Bifurcation

- Balance of fluxes (Q)
- Continuity of pressure (P_T)
- Extrapolation of Riemann invariants ($W_{1,2}$)

$$\begin{cases} \sum_{i=1}^{N_d+1} Q_i = 0 \\ P_{T,1} = P_{T,i+1}, \quad i = 1:N_d \\ W_{1,1} = W^{\star}_{1,1}, \quad W_{2,i} = W^{\star}_{2,i} \end{cases}$$



Vessel (1D Euler equations)

Conservation law of mass and momentum averaged on the cross-section:

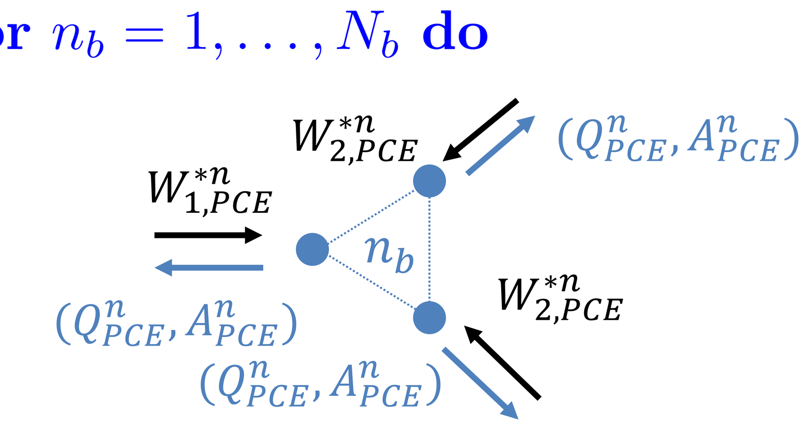
$$\begin{cases} \dfrac{\partial A}{\partial t} + \dfrac{\partial Q}{\partial x} = 0 \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{A}\right) + \dfrac{A}{\rho}\dfrac{\partial p}{\partial x} + K_R\dfrac{Q}{A} = 0 \end{cases}$$

DDUQ NUMERICAL MODEL

for $n = 1, \ldots, N_t$ **do**

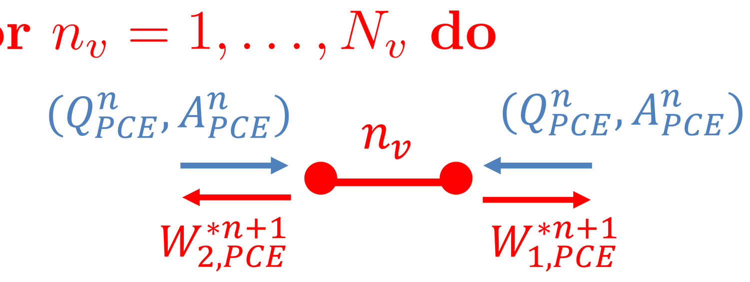
Update exogenous inputs

for $n_b = 1, \ldots, N_b$ **do**



end for

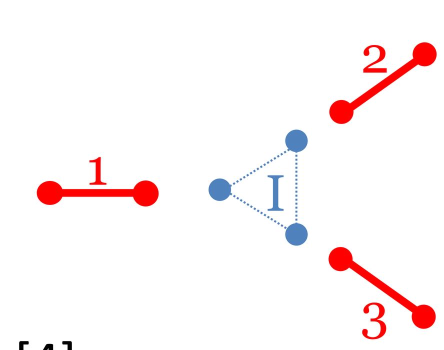
for $n_v = 1, \ldots, N_v$ **do**



end for

end for

NUMERICAL VALIDATION



Pulse wave travelling in a tapered bifurcated vessel with reflecting BC and 3 stochastic parameters.

Good **agreement** with the literature [4].

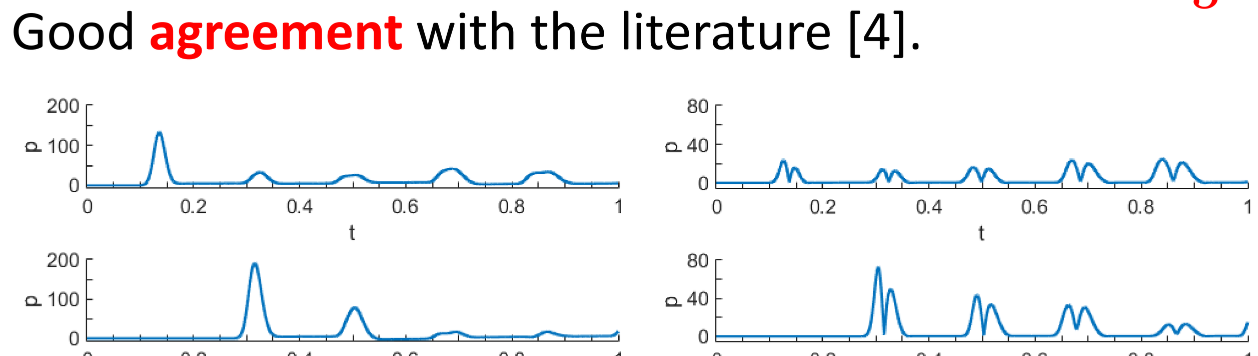


Figure: Mean (left) and STD (right) of pressure for parent (top) and daughter (bottom) vessel at the center of the pipe.

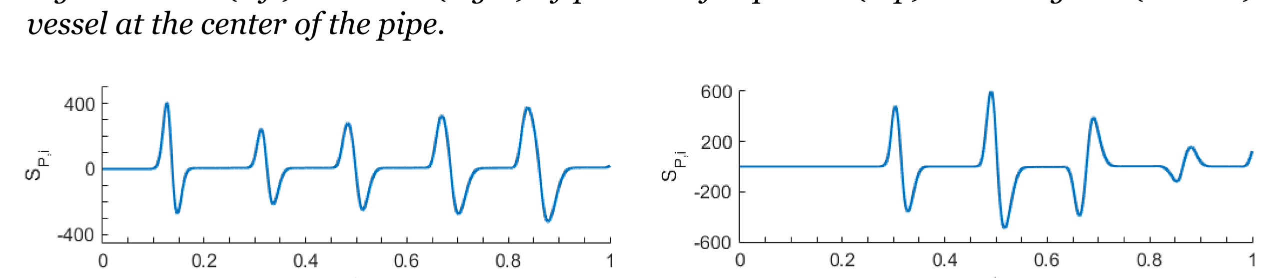


Figure: Sensitivity of pressure with respect to the stochastic parameter β in the parent (left) and daughter (right) vessel at the center of the corresponding pipe.

CONCLUSIONS and FUTURE WORK

DISCUSSION

- The **computational cost** of UQ in the cardiovascular system **can be reduced** by promoting the independence of the subsystems and avoiding full-system simulations;
- The **accuracy** of 1D solvers **can be improved** by **educated reduced models**, at roughly the same computational cost.

WORK IN PROGRESS

- DDUQ numerical simulations in real cardiovascular networks;
- Embedding of the educated reduced models in the DDUQ solver;
- Parallelization and scalability test;
- Comparison with 1D models in terms of accuracy and computational cost.

References

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