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Authors: Douglas Scott[1], Zani Ludick[2]

Affiliations: Manfred and Penny Conrad Institute for Music Therapy Research - Wilfrid Laurier University[1],
Department of Mathematical Statistics & Actuarial Science - University of the Free State[2]

Orcid ids: 0000-0002-3422-8754[1]

Contact e-mail: jdmusictuition@gmail.com

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Exploratory interpretations of power law relationships in Debussy's *Syrinx*

Douglas Scott

*Manfred and Penny Conrad Institute for Music Therapy Research
Wilfrid Laurier University
(music@dwscott.ca)*

Zani Ludick

*Department of Mathematical Statistics & Actuarial Science
University of the Free State*

Background in music analysis. Music analysis often relies on pre-formed concepts such as “binary form”, key centers, or rhythmic tropes to provide templates for comparison between works. To the extent that such tools are based primarily on semantic level considerations, they can present obstacles to translation between different 4E domains of embodied, embedded, extended, and enactive cognition. Power law analysis, representing a context neutral information theoretic approach which is amenable to task specific weighting, can help bridge the gap.

Background in applied mathematics. In this paper we investigate how deviations from the near universal phenomenon of logarithmic perception correspond to psychologically tractable interpretations of a musical score. We show that variations in the slope of a straight-line log-log regression and its residuals, together with variation in structural features produced by varying counting methods and levels of analysis correspond to academic and expert assessments in various ways. We then show how wavelet analysis can be used to smooth over level of analysis problems.

Aims. We seek to explore how violations of simple power law descriptions (Zipf's law) relate to structural features of Claude Debussy's *Syrinx* for solo flute as in the form of a variety of analyses and performances.

Main contribution. Power laws are an important implicit feature of musical notation through such characteristics such as octave equivalence and beat hierarchies, but are rarely explicitly employed as higher-level analytical tools directly, despite the fact that the scale-invariance property and empirical observation suggests that they can be useful at that level too. The simple historical reason is that generating such descriptions, while conceptually straightforward, is quite computationally intensive. Now that computation is readily available it has become possible to extend power law descriptions into these higher levels of analysis with relative ease. Our contribution is to explore some general techniques that can be expanded upon and refined as a broader research program with substantial interdisciplinary implications for the study of perception and cognition generally, since these phenomena obey similar laws.

Implications. The present study is correlational in nature, so the direct implications are limited for either music analysis, which provides the interpretive data, or power law representations, which follows well-established practices. However, having established that a set of relatively approachable mathematical operations can readily account for key music-theoretical structural features, the door is opened to a systematic application of the principles involved. This provides an opportunity to state music theories in ways that are precise, falsifiable and directly relatable to modes of description employed in other sciences. Furthermore, because these procedures have broad relevance to many domains outside of music theory, it allows for insights gained from studying musical structure to be applied directly to other, ostensibly unrelated problems.

Keywords: Music, Inverse problems, Power laws, Zipf, Benford, Debussy, *Syrinx*

1. INTRODUCTION

Music analysis represents an inverse problem, which has been described as consisting of “using the actual result of some measurements to infer the values of the parameters that characterize the system” (Tarantola, 2005). Inverse problems are generally considered problematic because models are necessarily dimensional reductions, which means that complex information is reduced to relatively simple representations, a process that can be as misleading in some contexts as it is helpful in others. This is because information loss involved in the dimensional reduction makes it impossible to work backwards from a model of events to the events themselves. Because of our finite nature we can only measure a limited flattened out snapshot of objects and events which are complex and nuanced by nature.

Nevertheless, composers of classical music clearly have a reasonable expectation that the abstracted models of music they disseminate in the form of music notation. Ethnomusicologists hope to rely on performances in the here and now to meaningfully inform about the deep-time and deep-culture of a society. Similarly, practitioners in a 4E framework of embodied, embedded, extended and enactive minds (Newen, De Bruin, & Gallagher, 2018) must rely on fragments of information from larger corpuses. If the inverse problem of working backwards from the data to source was systematically intractable, none of these approaches could hope to be fruitful.

While many of the general issues of how the concept of coherence, or harmony, between different representational domains can overcome this difficulty in practice has been dealt with elsewhere by the first author of this paper (Scott, *Music as semiotic eigenbehavior*, 2017), (Scott, 2020), here we examine specifically how differences between expected and actual power law representations can be used semiotically as markers. In this way, rather than treating the inverse reasoning as problem for cognitive theory, we consider errors introduced by failures of coherence in models as sources of denotative meaning.

To this end, we describe how the frequencies of pitch classes in the Debussy's *Syrinx* for solo flute deviate from a straight-line log-log regression with a slope of -1 predicted by Zipf's law. Karl Popper suggested that his ideas of falsifiability and induction in science originated from a conjecture about the evolution of Western polyphony: That composers use themselves as emotional test bodies for ideas to be realized in the score (Popper, 1963, p. 50), (Popper, 1976, p. 73). The basic principle of this article is that a general information theoretic law can serve as a proxy for such a test body in the same way that physical gestures can function as stand-ins for psychological tension, insofar as they indicate a divergence from a putative neutral posture of the gesticulator (Kang & Tversky, 2016).

2. POWER, ZIPF'S AND BENFORD'S LAWS

2.1 Power Laws and expectation

Zipf's law, Benford's law, the Riemann zeta distribution, pink noise, 1/f frequency distributions and fractal noise are all part of a close-knit family of empirical power-law relationships that have been observed in a wide variety of contexts, including music and language (Powers, 1998). These laws are connected to principles of information entropy (Hernando, Hernando, Plastino, & Plastino, 2013) and it can be argued that they are a property of coherent explanations in general. However, a decisive way to account for their occurrence remains elusive (Berger & Hill, 2011). While Kingsley Zipf (1949) postulated a "principle of least effort" to explain the empirical observation of the law, Benoit Mandelbrot argued that a monkey hitting "typewriter keys at random will also produce a 'language' obeying Zipf's law" (Mandelbrot, 1961), (Harremoës & Topsoe, 2006). In the musical context, meanwhile, the very ubiquity of these empirical power law relationships has led David Huron (2007) to dismiss their potential utility as an analytical tool.

In this paper, we show that due to this ubiquity, deviations from this relationship may be analytically useful. This approach more or less follows Huron's ITPRA (Imagination-Tension-Prediction-Reaction-Appraisal) theory of expectation (Huron, 2007, pp. 7-18). For our purposes, imagination involves the expected global compliance with Zipf's law, tension is the difference between the local distribution and global distribution, prediction is made of the raw data that will produce the expected Zipfian distribution, reaction is the response of local and global distribution to the new local data and appraisal is the ascription of meaning to the changes at the various levels induced by the new data.

2.2 Benford's Law

Zipf's law may be regarded as the general principle behind Benford's law, which is formulated as the relative frequency of occurrence of the leading digits of numbers in certain types of datasets. "Frequency" here refers to the broad sense of the term as a count of instances in a given window. Typically, the types of data-sets that satisfy Benford's law are naturally occurring and span many orders of magnitude. The relative frequency (or probability) is modelled by the following equation.

$$P(d) = \log_b \left(1 + \frac{1}{d} \right), d = 1, 2, 3, \dots, d - 1, (1)$$

where $P(d)$ is the probability of occurrence of the leading digit d , and b is the base (greater than or equal to 1). A base of ten, for example, denotes the decimal number system and the equation will give the probability of digits 1 to 9, respectively, being the leading digit. This gives a probability of 0.301 for 1 as the first digit through 0.176, 0.124, 0.097, 0.079, 0.067, 0.058, 0.051, 0.045 for 2 to 9 as the first digit, respectively (see Figure 1).

The law is connected to the logarithmic distance between two subsequent numbers and is both scale- and base-invariant (Hill 1995). Scale invariance implies that the logarithmic distances will remain unchanged, whether you are counting in 1s (1,2,3,4,...), in 2s (2,4,6,8,...) or in 10s (10,20,30,40,...), etc. Whether one counts in 1s, 2s or 10s, the logarithmic distance between the first two numbers (1 and 2, 2 and 4, 10 and 20) is always 0.301.

Benford (1938) observed that many natural processes produce events according to some geometric (multiplicative) rather than arithmetic (summative) sequence. Taking this view, the arithmetic sequence can be interpreted as the ranking (or ordering) of subsequent observations and not an indication of their true magnitude (or value).

Figure 1 The distribution of leading digits according to Benford's law. The logarithmic distance between 2 and 3 added to the logarithmic distance between 3 and 4, again gives us 0.301, which is the logarithmic distance between 1 and 2. The same applies to the sum of distances between numbers 4 to 8 and 8 to 16, etc.

2.3 Zipf's law

Zipf's law is a proportional relationship, where the frequency of a data point of rank r is formulated to be inversely proportional to its rank (Piantadosi, 2014).

$$F(r) \propto \frac{1}{r^\alpha} \quad (2)$$

For this equation, when $\alpha = 1$, the frequency of a data point of rank r is the direct reciprocal of that rank, a process that, when iterated, results in the harmonic sequence (1/1, 1/2, 1/3,...). The logarithmic distances between these frequencies are the same as the logarithmic distances between the natural numbers (1,2,3,...) that we explored in Section 2.2. Hence, the connection between Zipf's law (with $\alpha = 1$) and Benford's law is established via the concept of logarithmic distances.

Plotting the logarithm of the rank against the logarithm of the harmonic frequencies results in a plot with a slope of exactly -1 (see Figure 2). This translates to a constant distance of -1 between subsequent frequencies on the log-log scale. If $\alpha = 2$, the sequence (1/1, 1/4, 1/9,...) results. The logarithmic distances between these frequencies are twice that of the harmonic sequence, and when a log-rank, log-frequency plot is drawn, a slope of -2 exactly results (see Figure 2, where the log-log plot for $\alpha = 0.5$ is also drawn).

The parameter α can tell us how rank-frequency distributions are scaled away from the harmonic sequence. For example, if a line is fitted to the log-rank, log-frequency of a dataset, and the absolute value of the slope is greater than 1 ($|\alpha| > 1$) it indicates a larger difference between observational frequencies than would be observed for the baseline harmonic sequence. For a dataset of fixed size, a large difference between observational frequencies can only result if a few observations dominate. The converse holds for $|\alpha| < 1$, where events would be closer in frequency than suggested by the harmonic sequence.

Figure 2 Zipf's law for various values of α .

3. INTERPRETATION

One of the most important applications of Benford's law is in that it can be used to (weakly) infer intentional action when it is violated (Nigrini 2015), i.e. when the books—in the accounting sense—fail to follow the expected distribution. This is largely a result of the greater probability of any numerical account of any phenomenon to follow a Zipfean distribution, which means that the more unconnected sources of data are drawn from, the less likely it would become that all of them fall in an unlikely non-Zipfean distribution. This logic applies to any data that is influenced by many confounders with complex relationships.

The upshot of all of this is that the -1 slope of Zipf's law can be used as a conceptual neutral ground in analysis in this way that recalls Schenker's motto "Semper idem, sed non eodem modo" (always the same, but not always in the same way) (Schenker, 1921/2004). Schenker did consider arpeggiation as being an abbreviation of the harmonic series "for the purposes of art" (Schenker, 1935/1979). Similar reasoning from statistical likelihood is employed in justifications of the proposed role of entropy in the perception of the arrow of time (Penrose, 2004) or in statistical hypothesis testing, where the occurrence of an event unlikely to have occurred by chance is used to draw inferences about likely causality.

It may be somewhat counterintuitive, then, that a practice such as music, which is characterized by intentional manipulation of sound events, should obey the law in the first place, as Richard Voss and John Clarke (1975) found in a seminal paper for the modern interest in the phenomenon with regards to music. The apparent paradox vanishes, though, when it is recognized that any numerical representation that can be graphed constitutes a dimensional reduction of a complex object that can never truly be visualized. It is this representation that obeys laws such as Zipf's and Benford's law,

In this paper we take the position that, while the distribution as a whole may be balanced in the stochastic sense, the choices of where exactly the large deviation from the central tendency occurs is very a conscious decision of the composer, and that these deviations can therefore be treated as informative. We therefore explore the robustness of the connection between musical analysis and the underlying structural nature of the different length musical unit as expressed in terms of power laws using independent analyses of the score to provide a baseline. This should not be understood as uncritical acceptance of these interpretation, but rather as a yet further set of conceptual neutral grounds against which the idea that deviations from Zipf's law reflect some musically relevant aspects may be explored. Known objects held up in front of a camera to test the precision of the lens. Thus, we examine the question of how faithfully these structural aspects mirror musical analyses, read naively in this way, from several angles: (1) In terms of deviation of data-points from the regression line (i.e. the residuals), (2) comparisons between actual and modified data, and (3) deviations of the slope of the regression from the expected value of -1.

3. ANALYTICAL EXAMPLE

The slope value for fitting Zipf's law is arrived at by first abstracting out temporal sequence information through the process of ranking. The present study is restricted to pitch information arrived at by breaking up a given piece of music into time-slice units. For an example, see Figure 3 for a piece on a BACH theme that has a Bb for two beats, an A for a half a beat, a C for one beat and then a B for one and a half beats. Thus, the unordered set BACH = {2,0.5,1,1.5} corresponds to the ordered set: (2,1.5,1,0.5), i.e. ranked in order of size.

Figure 3 The "BACH" and "BACHBACH" themes

Note that the scale-invariance property means that the choice of beat unit is irrelevant to the final calculation, so doubling the value of a beat unit would change the data set to (4, 3, 2, 1), but this change would

not affect the slope value. Similarly, the abstraction of temporal sequence information means that repeating the passage as per the original beat value has the same effect on the overall count as doubling the beat value: values are simply added together over the relevant span under consideration.

It might be supposed that so much information is lost in this process as to render the technique effectively useless, but, as Matthieu Cristelli, Michael Batty and Luciano Pietronero (2012) observe: “There is more than a power law in Zipf”. Interpreting compliance with Zipf’s law as the coherence of a data-set, they make the perhaps somewhat surprising observation that unions and subsets of a set will not be described by the same law as the set. For our purposes, and for our example, splitting the theme BACH in exactly half (into BA and CH components) will result in the sets (2,0.5) and (1.5,1), while ranking the notes in BACHBACH as individual events by their respective durations will result in the set (2,2,1.5,1.5,1,1,0.5,0.5), and each of these three sets will have a different slope from the original. In principle then, the original temporal sequence information is always recoverable by exhaustive comparisons between analyses at different levels of subdivision.

The comparison between these different levels can be supplemented by pitch-class information, as the two sets in the BACH example will have different pitch-classes for the corresponding values, specifically (10,9) and (11,0) respectively, ordered by the duration of each pitch-class, while the BACHBACH example will have two iterations of (10,11,0,9). After ranking the data, the logarithm of the rank is plotted against the logarithm of the data at each rank (see Figure 4).

Figure 4 Calculating the slope of the “BACH” theme.

Note that the base of the logarithm does not affect the outcome. Thus, in the BACH example, the logarithms of (4,3,2,1), are plotted against the logarithms of the ranks (1,2,3,4) to give a slope of -0.924 (which corresponds to Zipf’s law where $\alpha = 0.924$). The individual notes can then be assessed against this linear model, either on an event-by-event basis (see Figure 5) or in terms of pitch classes (see Figure 6), with the residuals that this produces representing the distance of each data point from the trend line.

Figure 5 “BACH” theme residuals in order of event occurrence.

Figure 6 “BACH” theme residuals by pitch-class.

We can then assess whether such deviations from a trend line have any psychological or musicological significance. Another set of procedures that can be performed on the data is comparing the slopes of sections and subsections to one another and to the overall slope itself, as can be seen in the analysis of various ways of subdividing the repeated BACH theme: BACHBACH (see Figure 7).

Figure 7 Slopes of various subdivisions of the “BACHBACH” theme compared with combined and average slopes.

Local slopes can also be compared to the global and supra-local (i.e. sentence level) slopes and average slope values as well. This can be taken one step further by taking the slope of the section slope values themselves, or the slope of the difference of local slope values from local and global slopes and averages. In this way a vast array of different perspectives of the same data can potentially be generated, and analytical approaches that re-arrange the source material, such as paradigmatic analysis, can only further

increase the scope. Rigorous hypotheses about musical tension and segmentation may then be formulated, which can then be tested in subsequent investigations. These might take such forms such as “performers tend to emphasize the note with the largest residual”, “composers put phrase marks where the Zipfian analysis predicts segmentation to be” or “listeners react to momentary excursions from Zipfian distributions”.

Even at this stage, though, it is worth noting that the difference between the individual section slopes and both the combined slope (obtained by joining all the data points into a single slope) and average slope (obtained by finding the mean of the slopes of the subdivisions) is minimized at what might be considered the most “natural” subdivision: BACH | BACH.

4. CLAUDE DEBUSSY’S SYRINX

4.1 Background

Claude Debussy’s *Syrinx* (1913) is useful to analyze for several reasons. Being a major work in both the solo flute repertory as well as Debussy’s oeuvre means that there are many existing analyses available, which means that we do not have to rely on an entirely novel analysis to accompany a novel analytical approach. Its monophonic texture also circumvents the issue of how to deal with harmony and counterpoint in a Zipfian analysis of this type. Finally, it generally has fairly well-defined phrase boundaries, something that is not always the case, making it easier to compare different levels of analysis. It must be noted, though, that many of these boundaries were not actually specified by Debussy, but were possibly provided either by Marcel Moyse, who prepared the edition passed down to us, or by Louis Fleury, who first performed the work and whose handwritten performance copy Moyse’s edition was probably based upon (Ewell, 2004, pp. 1-7), henceforth referred to as the “Jobert edition” (Debussy, 1927).

In proceeding with this investigation, it is important to note that what is being analyzed here is not the experience of a hypothetical listener, instead it is perhaps better seen as an analysis of the experience of a metaphorical listener. We can follow David Lidov (Lidov, 2005, p. 93) in saying that “[t]hough score analysis [...] is insufficient to indicate the aesthetic character of an artwork, it is still possible to indicate the specificity of the work”, and extend that by observing that this specificity is embodied by the statistical relations of the score, as laid down by Debussy (and his editors). If we allow the nature of those statistical relations to act as a metaphor for the tensions inherent in the structure of the work, they can be blended with our metaphorical person to produce a metaphor for the experience of tension. This should not be read as suggesting that an auditor performs Zipfian calculations explicitly, or even that she is aware of the work in the way the analysis suggests. Indeed, to the extent that the present analysis completely omits

rubato, dynamics, articulation, timbre, and so on, means that, if anything, it may be less, rather than more, detailed than an actual experience by an actual person.

The analysis is not a claim of what the experience of Debussy's *Syrinx* is like, because those are always unique to the moment, but an acknowledgement that there are experiences of the work and if we are to understand the generality of those experiences, we have to start with the specificity of the score (and the statistics it embodies) that binds them together as experiences of *Syrinx*, rather than, say, a rainy afternoon in the countryside.

4.2 Slopes, regression analysis, and residuals

The analysis of the entire work reveals a slope of -1.038 with a coefficient of determination of 0.885 (see Figure 8), which conforms to the expectation that Zipfian distributions are common, or even ubiquitous, in music.

Figure 8 Overall rank-frequency profile (on log-log scale) and the pitch classes of residuals ranked by absolute size: *Syrinx*, by Claude Debussy.

Of specific interest here are the deviations from the calculated slope. We want to ask whether (a) there is anything meaningful about the slope value that it is not exactly -1 and (b) whether the perturbations around the slope itself (the residuals) can tell us anything of value. In Figure 8, we can observe that the most common note overall is Bflat (PC10), which, along with other cues including its metrical placement and function in cadences, identifies it as performing a tonic function. This note also has the second largest residual (i.e. second furthest from the trend-line). Meanwhile, the significance of C (PC0) (the note with the largest residual) is that it plays a major role in the middle section, which stands apart from the outer sections in terms of themes, phrasing and tessitura. This is the only occurrence of PC0 for any significant duration in the work. As an absolute pitch it is both the lowest note of the work and the flute. Novelty and rarity often have both a psychologically significant but contradictory effect on creative tasks such as segmentation (De Jonge, Rietzschel, & Van Yperen, 2018).

While Bflat might be identified as the tonic, *Syrinx* is not tonal in the diatonic sense, being instead chromatic in a way that Laurel Ewell describes as a double-tonic complex comprised of the two whole tone scales, which represent for her a metaphor for the “coexistent relationship between the conscious and

subconscious will” (Ewell, 2004, p. 30). It is possible to use this contention as a way to segment the overall data so that it can be used to explore the hypothesis that Zipfian analysis provides evidence of intentional behavior in keeping with the canonical application of Benford’s law in detecting fraud. In fact, the overall principle that some inferences about tonality can be made on the basis of Zipf’s law has already been shown by (Manaris, Vauhan, Wagner, Romero, & Davis, 2003), who found, using a more complex set of metrics than those used here, statistically significant slope differences between different genres within their selection of works, specifically finding a very low slope figure for twelve-tone works (-0.813), the lowest for all genres investigated. Since we know that such serial techniques deliberately avoid tonality through various devices, the slope can be read as being informative in this sense. For present purposes, though, the data is broken down to show the slopes of each of these whole tone scales individually (see Figure 9) which reveals a slope of -1.373 and -1.028 for the pitch-class sets {0,2,4,6,8,10} and {1,3,5,7,9,11} respectively, henceforth WT1 and WT2.

Figure 9 Profiles of the two component whole tone scales over the span of the whole of Syrinx.

If the deviation from the -1 slope for WT1 is indeed significant, then it may be indicating that something significant has been done (by Debussy) to WT1 to cause a deviation. Of course, we do know that Debussy did do something intentionally by composing the work in the first place in the way that he did, but we are looking for evidence that that intention has manifested itself as an intentionality of the metaphorical person of the work itself, or what might be informally described as what the work means.

4.3 Deviation

The principal method for performing such a test is to modify the work in some way that might conform to some expectation that Debussy subverts in his score. Here, the first two thirds of the piece (up to the recapitulation starting on the upbeat to bar 26) do indeed indicate a preference for the Bflat, which is by far the most abundant note even if the total count for each whole tone scale (i.e. the sum of the number of occurrences of each pitch) is very similar. The final third veers strongly towards the Dflat, with the WT2 construction having twice as much duration devoted to it here than WT1, suggesting that the “tonic” should

be Dflat instead, what in another context might be labelled the relative major. This re-alignment is reinforced by the fact the work ends by explicitly spelling out WT2 in its final cadence, the only time that one of the two scales is laid out directly in such a manner in the entire work.

The realignment to Dflat is also supported by the fact that the slopes for WT1 and 2 are -1.165 and -1.037 for the first two sections respectively (see Figure 10), while the last section has slopes of -1.37 and -1.351 respectively instead (see Figure 11).

Figure 10 Sections 1 & 2 broken down by whole tone scale.

Figure 11 Section 3 broken down by whole tone scales. Notice the clear separation of WT2 and its linear ordering, matching the closing descending passage.

This supports a reading that something “has been done” to the whole tone scales in the last section. We can further test the idea that a deviant slope indicates deliberate manipulation by attempting a clumsy reversion of Debussy’s realignment in the last phrase to maintain WT1 as the primary construction. This is achieved here by simply replacing all the PC1’s with PC10’s, with no attempt to justify the process musically. The result is that, while the overall slope is brought slightly closer to -1 than Debussy’s original (-1.018 in Figure 12 compared to -1.038 in Figure 8), the component whole scales show a dramatic deviation from Zipf’s law, with a slope of -1.533 for WT1 and -0.746 for WT2, altered from the original -1.373 and -1.028 respectively. This can be interpreted as the deliberate change we made in an attempt to restore the Zipfian profile (and our quasi-tonal expectations) resulting in component slopes showing significant deviation, which in turn suggests that a -1 slope is not sufficient evidence of a lack of deliberate manipulation, and that a significant deviation from the -1 slope, as in Debussy’s original slope for the WT1 construction,

can be treated as weak evidence of deliberate (intentional) signification (in a way that conforms to the usage found in Collins, 2017).

Figure 12 Overall result of altering final phrase, replacing PC1's with PC10's.

The concept can be generalized by saying that composers can create signals in music by subverting statistical regularities, and that those regularities may derive from perceptual, cognitive or physical universals such as Zipf's law, or may be the result of structures laid down in the course of the work itself. The significance of a signal can thus be understood in terms of the conflict between expectations and regularities in different domains. For example: Lifting one's arm outside of the expected range of motion to indicate an important point in time (i.e. a downbeat) to follow. The significance of the downbeat is established not by its own character, but by the exceptional nature of the action that precedes. In this way, the imputation of meaning occurs in the negative space by balancing the action, restoring the previous regularity.

4.4 Residuals

Analyzing smaller-scale structures in this way can often become problematic due to artefacts of the technique itself. This can be seen in the analysis of the very first phrase, where the set of pitch classes {1,4,5,6,9,11} all share the same duration. In such cases, there is no basis for selecting the order of these pitches in the analysis, but whichever one is picked as first and last would have higher residuals. The actual residual for the whole set would be the absolute average residual for this set. This must be taken into consideration, even if in real performances a performer can emphasize or modify the inter-onset interval such that no two notes would ever be exactly equal in any real sense. Residuals in general may be considered in relation to altered versions as in Figure 13, where the interruption of the downward chromatic passage is eliminated in favor of continuing the passage to what might be considered an interrupted cadence.

This change alters the residuals quite dramatically, without doing much to the slope. Such changes can be good targets for psychological investigations into whether slope values have any direct impact on the perception of a work, guided by music-theoretical insights. For example, while the slope value of section 1 is -1.293 (see Appendix), changing the second-to-last note from an Eflat to an F to effect a perfect rather than plagal cadence changes the slope to -1.230 , which might be interpreted as suggesting that slopes are related to concepts in perception such as tonal closure.

Figure 13 Zipfian profile and residuals for phrase 1 (top), altered (bottom) to continue the chromatic line to the cadence.

5. THEORY BUILDING: ANALYSIS OF SEGMENTATION AND TENSION

5.1 Segmentation and cohesion

Given that slopes and regressions can differ considerably depending on the scale and segmentation of the data, it can become difficult to conclusively justify one analytical scale over another. The challenge of determining basic semantic structures can be approached in several different ways, Juan Perotti and Orlando Billoni (Perotti & Billoni, 2020) The rather unique origin of *Syrinx*, as a work passed down to us as a performance copy, means that we have phrase marks already marked in, but we also know that the composer had different ideas about these things, marking only breaths at bars 2, 4 and 14 (Ewell, 2004, p. 6). The phrasing used for this study follows the breaths in the Jobert edition precisely; meaning that for section 1 there is one additional phrase mark in bar 5.

We can now ask the question of whether this phrasing matches a Zipfian analysis of the same. The results, seen in Figure 14 show the slope changing by half-beat increments, adding up the total duration of each pitch class up to a given point in half beat increments (the cumulative slope) and subtracting the slope

from the average of cumulative slopes for the whole of section 1. Subtracting from the average of cumulative slopes works here because the behavior is fairly cyclical around the average, meaning that it makes sense to treat that value as a baseline to compare other values to. In this case, the average of cumulative slopes of -1.203 is quite similar to the end cumulative slope (i.e. the slope for the whole section) of -1.208 .

Figure 14 Difference between the average of the cumulative slopes and cumulative slopes in half beat units for section 1. Numbers above columns indicate bar numbers.

The key thing to note here is the points where Debussy marked breaths (at the end of bar 2 and the middle of bar 4) are also points where the difference is minimized. If deviations from the average slope are considered to indicate tension, the cadences (as defined by the breath marks) therefore represent points of repose in relation to the section as a whole.

While a listener would not need to use the slopes to identify cadential points, they could use the fact that cadential points are associated with certain slopes to develop an expectation that future cadences would do the same. The fact that phrases then do proceed to follow this pattern would contribute to a sense of cohesion. If this interpretation is accepted it would not make sense to put a breath mark where the phrase actually turns in the middle of bar 6. Debussy's breath marks can therefore be interpreted as phrase marks, where the player should naturally breathe in one or both of the rests in bars 5 and 6 without breaking the phrase. The phrasing mark in bar 4 would then be an indication to avoid the grouping, suggested by the motives, of phrases 2 and 3 and to group phrases 3 and 4 instead. This has the effect of turning the rests in the first sections from moments of repose into tense, sobbing gestures.

5.2 Difference from Zipf's law as tension graph

The question of whether the difference slope of -1 itself has any psychological significance is one that may be amenable to empirical testing. Figure 15 shows the absolute result of subtracting the cumulative values up to each point from -1 plotted against inter-onset-intervals for twelve recordings of performances.

Each chart is smoothed by taking the successive averages of bar-length units. An alternative method for smoothing slopes not employed here is taking the slope of larger windows of data. The result will correlate

with the moving average, but not exactly. The divergence between these measures can then be used as a measure itself. Because power laws are scale and base invariant, the differences in analytical outcomes between approaches such as these can be used to optimize the signal to noise ratio but are highly resistant to significant structural distortion.

Power law analysis similarly resist variation in empirical method. For example, the chart omits the timings of the pauses, since these were highly variable between the per-formers. Other dimensions amenable to analysis are the relationship between the lengths of the pauses to the overall length and to the slope of inter-onset-intervals by each performer. Changing parameters like these can enhance focus and fit, without necessarily changing broad structural details. This opens the door for methods, such as Granger causality, that can yield concrete comparisons between results obtained from different parameter settings. While this would not allow one to conclusively attribute analytical outcomes to parameter selections, it does allow for precise specifications and well-defined comparisons to be made.

Figure 15 Comparison between the difference between the cumulative slope per beat and average inter-onset interval of twelve performances of the opening section of Debussy's *Syrinx*.

5.3 Comparative analysis

Aside from directly analyzing performances, another way to overcome the need for relying on personal reflection is to consider a large body of independent analyses. One such application is found in the analysis of the second original breath mark in *Syrinx* in bar 14 (before phrase 8, phrases being defined as the breath marks in the Jobert edition).

In his meta-analysis of eleven segmentations, (Meyer, 2017) found that there was a universal consensus that section 3 starts after bar 25-2 and near universal consensus that the first section ends after bar 8, with Julia Ann Larson (1990) being the sole dissenter, making the first division only after bar 15 (i.e. at phrase 8). The middle section, though, is much less clearly segmentable. Of the group, only three authors chose to treat section 2, from bar 9 to bar 25, without segmentation (de Natale, 1996), (Seraphin, 1962), (Dobrinski, 1981), while a further four segmented section 2 after bar 15 (Austin, 1966), (Borris, 1969),

(Mahlert, 1986), (Nicolet, 1967). Two authors made a large number of segmentations in this section, with one including the segmentation at bar 15 among these (Cogan & Escot, 1976) while the other (Tenney & Polansky, 1980) concurs with (Whitman, 1977) in placing the segmentation after bar 13 instead, making that the only one in the middle section. The clearest agreement of Zipfian analysis with the majority position of segmentation at bar 15 (phrase 8) can be seen in Figure 16, showing the cumulative slopes by phrase. This graph is also produced by adding the total number of times that each pitch-class occurs cumulatively, this time at the coarser phrase-by-phrase level. It shows a clear shift in regime with a large movement precisely on phrase 8, though with the caveat that this technique has a bias in showing greater variability towards the start of a time series.

Figure 16 First differences of successive cumulative phrase slopes show a marked divergence at the transition between phrases 7 and 8.

Figure 17, meanwhile, shows the absolute difference of the full slope (-1.038) from slopes at each phrase. Plotted against this is the residuals of the actual slope of each phrase against the absolute difference of the actual phrase slope from the overall average slope at each point. These higher order interpretations agree with each other to a remarkable extent. Notably, the two largest intervals in the entire work also occur where the first differences in both measures maximize: at the transition from phrase 9 to 10, with an octave leap in phrase 9 (bar 17), which is also the shortest phrase per the segmentation by breath marks, followed by a major ninth in phrase 10 (bar 19). The disagreement between Figure 16 and 17 reflects the disagreement between the various analysts, specifically in the segmentation of the middle section, and shows that (paradoxically) such a failure to reach commonality may actually be a good reflection of the underlying information-theoretic structures inherent in the score.

Figure 17 Absolute difference between full slope and phrase slopes plotted against the residuals of each phrase slope

7. CONCLUSION

There are a number of observations that can be drawn on the basis of the foregoing reading. Perhaps, most notable among these is that power law analysis is fecund, which means that it can generate a considerable variety of different interpretations depending on how many layers of abstraction one is prepared to entertain. Each of these interpretations is testable against a wide variety of other analytical devices, which can range from comparing the opinions of analysts to observing audience reactions and beyond. This, combined with a general coherence (harmony) between different levels of analysis, means that the numerical analysis can be used to generate well-defined and testable theories about the musical substrate, as embodied in the score. Musical insights can be used to generate insights into the possible meaning and significance of power law analysis itself, which can be applied to further musical analysis. It is unlikely, though, that there will ever be a conclusive one-to-one Zipfian method of musical interpretation. Indeed, if anything, the above investigation suggests that the chief advantage of the method is to fix interpretations numerically, which means that we can meet Popper's demands for precision while making falsifiable hypotheses about the power law analysis itself in terms of independent tests, whether they be music-theoretically or psychologically based.

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APPENDIX

Syrinx - La flûte de Pan

Claude Debussy

Overall

Section 1

Section 2

Un peu mouvementé (mais très peu)

Phrase 5

Phrase 6

Phrase 7

The musical score for Section 2, Phrases 5, 6, and 7, is shown. The tempo is marked 'Un peu mouvementé (mais très peu)'. The score includes dynamic markings *p* and *mf*, and a measure number 13.

Phrase 11

Phrase 12

Phrase 13

Section 3

Phrase 14

Phrase 15

Phrase 16

Phrase 17

Phrase 18

Phrase 19