Model Reduction of a Nonlinear Crash Model of a Racing Kart

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Motivation

Automotive safety is a core area in the design process of new cars.

- hardware crash tests are expensive, simulation based design process
⇒ many simulation runs are needed on HPC
- nonlinear finite element equations

$$\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e + \mathbf{k}_e(\mathbf{q}_e, t) = \mathbf{h}_e \quad (1)$$

with mass matrix $\mathbf{M}_e \in \mathbb{R}^{N \times N}$, elastic stiffness vector $\mathbf{k}_e(\mathbf{q}_e, t) \in \mathbb{R}^N$ and applied forces $\mathbf{h}_e \in \mathbb{R}^N$ solved with explicit time solver (LS-DYNA)

Figure 1: Frontal crash of a 2001 Ford Taurus.

Figure 2: Essential parts of the FE kart model.

- tremendous simulation times and energy consumption due to highly detailed Finite Element models. Apply model order reduction for speedup
- for separation and understanding effects, a racing kart crashing against a pole as a surrogate model for research
- crash simulations possess three basic sources of nonlinearities in mechanics:

large deformation
nonlinear material behavior
multiple contact scenarios
internal forces $\mathbf{k}_e = \mathbf{k}_e(\mathbf{q}_e, \dot{\mathbf{q}}_e, t, \dots)$ are nonlinear

Reduction Approach Substructuring:

Separation of the model in parts with linear and parts with nonlinear behaviour [1]. Apply model reduction only to linear part with $\mathbf{k}_e = \mathbf{K}_e \cdot \mathbf{q}_e$ ($\mathbf{K}_e$ is stiffness matrix). Linearization $\mathbf{K}_e \approx \mathbf{K}_e^{\text{tang}} = \partial \mathbf{k}_e / \partial \mathbf{q}_e$ around suitable working point using implicit solver option.

Offline Step

Identification of areas with small deformation (likely to behave linear): several offline simulations with varying initial velocity and position of the pole, see Figure 3.

- similar to passenger cars in frontal crashes, the plastic deformation in the rear of the kart is rather small
- separation of the model introducing $5 \times 15 \times 6 = 420$ interface dofs

Figure 3: Nine offline simulations.

Figure 4: Separation in two parts and interface nodes at the rims.

Interface Reduction

static condensation:

$$\begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I} \\ \Psi \end{bmatrix} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix}$$

- 420 interface modes
⇒ brute-force method

interface approximation with Ω :

$$\begin{bmatrix} \mathbf{K}_{bb} & \mathbf{K}_{bi} \\ \mathbf{K}_{ib} & \mathbf{K}_{ii} \end{bmatrix} \cdot \begin{bmatrix} \Omega \\ \Psi \end{bmatrix} = \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix}$$

- extract the most important deformation patterns at the interface, [4]

Figure 5: Overall reduction scheme: After partitioning of the system, the interface is reduced. CMS-based reduction methods are preferred. Therefore, the constraint modes and the modes of the internal dynamics are calculated with advanced MOR.

Optimal Reduction of Linear Substructures

- goal: find suitable methods for subsequent nonlinear simulation by linearized analysis
- reduce both front and rear part ⇒ separate basis reduction

$$\begin{bmatrix} \mathbf{M}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_1 \\ \ddot{\mathbf{q}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{K}_1 + \mathbf{B}_{c,1} \mathbf{K}_c \mathbf{B}_{c,1}^T & -\mathbf{B}_{c,1} \mathbf{K}_c \mathbf{B}_{c,2}^T \\ -\mathbf{B}_{c,2} \mathbf{K}_c \mathbf{B}_{c,1}^T & \mathbf{K}_2 + \mathbf{B}_{c,2} \mathbf{K}_c \mathbf{B}_{c,2}^T \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{i,1} \mathbf{u}_1 \\ \mathbf{B}_{i,2} \mathbf{u}_2 \end{bmatrix}$$

- best interface reduction after analysis: global approach in combination with balanced truncation

Nonlinear Simulation with Reduced Rear Part

- Craig-Bampton: many interface nodes ⇒ high eigenfrequency, therefore small maximum time step and high calculation time (over 11 days vs. 1 hour for unreduced model)
- good match until 30 ms, phase shift after 30 ms, see Figure 6, 7
- faster element processing (970 s vs. 2210 s) amortized by additional time to calculate reduced body (1260 s)

Figure 6: Acceleration of the driver calculated with different reduction approaches.

Figure 7: Plastic strain of the deformed, reduced model (CMS-Gram with modal interface reduction) and original model (gray) for comparison.

Lessons Learned

- challenge: coupling between reduced and original model
- interface reduction is important
- orthogonalization with respect to rigid body modes

Future Steps

- reduced simulation should be much faster by our experience
- apply method to full-scaled car
- nonlinear MOR for crash

[1] J. Fehr and D. Grunert.
Model reduction and clustering techniques for crash simulations.
Proceedings in Applied Mathematics and Mechanics, 2015.
Accepted for publication.

[2] J. Fehr, P. Holzwarth, and P. Eberhard.
Interface and model reduction for efficient simulations of nonlinear vehicle crash models.
submitted to *Mathematical and Computer Modelling of Dynamical Systems*.

[3] P. Holzwarth and P. Eberhard.
SVD-based improvements for component mode synthesis in elastic multibody systems.
European Journal of Mechanics - A/Solids, 49:408–418, 2015.

[4] Philip Holzwarth and Peter Eberhard.
Interface reduction for CMS methods and alternative model order reduction.
In *Proceedings of the MATHMOD 2015 – 8th Vienna International Conference on Mathematical Modelling*, Vienna, Austria, 2015.