FRG analysis of a multi-matrix model for 3d Lorentzian quantum gravity

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Introduction

The application of the functional Renormalization Group (FRG) [1] in the discrete quantum gravity context facilitates a background-independent form of coarse graining where the number of degrees of freedom serves as a scale for an RG flow. This program was initiated by analyzing matrix and tensor models for Euclidean quantum gravity in $d \geq 2$ [2, 3]. Results indicate (also using other techniques [4]) that apart from $d = 2$ their continuum limits so far only lead to geometrically degenerate configurations.

The impact of Lorentzian signature and causality for such models as well as the analysis of their phase structure has largely remained an open problem. The success of the causal dynamical triangulations approach (CDT) [5] suggests that additional structure enforcing micro-causality is needed to recover higher-dimensional physical continuum geometries from such models. Indeed, there exists a proposed correspondence between a Hermitian two-matrix model with ABAB interaction [6] with CDT in $2 + 1$ dimensions which leads to a variant of CDT also allowing for degeneracies of the local geometry [7]. The essential idea of this model is that causality can be imposed on a matrix model by introducing a second matrix that ultimately enables a distinction of space-like and time-like edges in the dual triangulation. Here, we present this models' phase structure based on our FRG analysis in [8].

CDT and the matrix model

The ABAB matrix model [6] is defined by the partition function

$$Z = \int dA dB e^{-\text{Tr} \left(\frac{1}{2} AA + \frac{1}{2} BB - \frac{\alpha_1}{4} AAAAA - \frac{\alpha_2}{4} BBBBB - \frac{\beta}{2} ABAB \right)},$$

where A and B are Hermitian $N' \times N'$ matrices. Its Feynman diagrams are ribbon diagrams with two distinct types of lines such that the duals of the three types of vertices correspond to three distinct squares, cf. Fig. 1. The presence of the ABAB term allows to encode $2 + 1$ dimensional structure [7].

Figure 1: Distinct squares correspond to blue/continuous (type A^4), red/dashed (type B^4) and bi-colored/continuous-dashed (type ABAB) 4-vertices in the dual ribbon graph.

In CDT, one only considers geometries with Lorentzian signature which admit a global foliation in proper time t and disallow topology change such that micro-causality is rigidly maintained. The basic building blocks are pyramids and tetrahedra with distinct time-like and space-like edges, cf. Fig. 2. The correspondence with the ribbon graphs of the ABAB matrix model arises when considering the spatial hypersurface at $t + \alpha/2$ therein, where the three distinct types of squares dual to the vertices of the matrix model appear, as in Fig. 1. These squares form equilateral quadrangulations dual the ribbon graphs of the matrix model.

Figure 2: The three fundamental building blocks of CDT in $2 + 1$ dimensions which interpolate between two consecutive spatial hypersurfaces at integer times t and $t + \alpha$. The numbers in brackets refer to the number of vertices in the quadrangulations at t and $t + \alpha$.

In this way, any CDT configuration can be encoded in terms of a Feynman diagram of the matrix model, which could motivate setting the CDT partition function equal to the free energy of the matrix model, as usual in the correspondence between triangulation and matrix models. However, the number of CDT configurations is smaller than that generated by the matrix model since the latter generically generates disconnected subgraphs which in the dual triangulation picture lead to branched trees of spherical bubbles. Such pathologies of the local geometry are disallowed by construction in CDT [5]. As discussed in [7], one can identify phases of this model which can be characterized by the dominance or statistical irrelevance of these degeneracies.

FRG analysis of the model

The scale derivative of the effective average action Γ_N satisfies the exact flow equation

$$\partial_t \Gamma_N = \frac{1}{2} \text{STr} \left[\left(\Gamma_N^{(2)} + R_N \right)^{-1} \partial_t R_N \right],$$

wherein $\partial_t = N \partial_N$ and STr denotes a sum over the available matrices A, B and indices thereof. $\Gamma_N^{(2)}$ is the Hessian of the effective average action and R_N is the regulator function closely modelled after Litim's cutoff [1] which allows to integrate out matrix entries larger than N .

Working in the simplest truncation deduced from the above-given partition function, employing the $\mathcal{PF}^{-1}$ expansion and after appropriate rescaling to allow for a $1/N$ expansion of the beta functions, we yield for the latter [8]

$$\beta_{\alpha_1} = (2\eta_A + 1) \alpha_1 - \frac{4}{5} (5 - \eta_A) \alpha_1^2$$

$$\beta_{\alpha_2} = (2\eta_B + 1) \alpha_2 - \frac{4}{5} (5 - \eta_B) \alpha_2^2$$

$$\beta_\beta = (\eta_A + \eta_B + 1) \beta - \frac{2}{5} ((5 - \eta_A) + (5 - \eta_B)) \beta^2$$

with the anomalous dimensions $\eta_{A/B} = -\partial_t \ln Z_{A/B}$ given by

$$\eta_{A/B} = \frac{8\alpha_{1/2}}{-3 + 2\alpha_{1/2}}.$$

The three beta functions reflect the exchange symmetry $A \leftrightarrow B$. In Figs. 3 and 4 we give the phase structure in the symmetric and asymmetric cases, respectively.

Phase structure in the symmetric case

Figure 3: Phase diagram with $\alpha_1 = \alpha_2 = \alpha$. 4 fixed points are shown. The black dots (B, D) denote saddle points, the red dot (A) corresponds to the GFP while the blue dot (D) marks a NGFP with two relevant directions. $\alpha = \beta$ gives the pink line. Arrows point towards the IR, i.e., small N .

Phase structure in the asymmetric case

Figure 4: Phase diagram with $\alpha_1 \neq \alpha_2$. 8 fixed points are shown. The black dots (B, C, D, E, F, G) denote saddle points, the red dot (A) corresponds to the GFP while the blue dot (H) marks a NGFP with three relevant directions.

As discussed in [7, 9] the line connecting C and D in Fig. 3 (equivalent to that connecting H and G in Fig. 4) agrees with that phase of $2 + 1$ d CDT which corresponds to extended three-dimensional spacetimes [5].

Conclusion and outlook

Our results look strikingly similar to those obtained using other methods [6], constituting a strong test of the FRG technique. Improvement on the numerical accuracy is expected from enlarging the truncation.

To characterize the emergent geometries solely from within the FRG method, the composite-operator renormalization construction [1] has to be carried over to the discrete quantum gravity context.

Our study paves the way for future work on multi-field models that encode causality in the interaction structure.

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