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SEVENTH FRAMEWORK PROGRAMME

nanoCOPS

PMOR for Nanoelectronic Coupled Problems

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Introduction

The models for nanoelectronics coupled problems, such as electro-thermal (ET) coupled problems, are very large in scale. These models often include variability to guarantee quality and yield. The parameter variations can be due to material properties, system configurations, etc. We consider the simulation of two classes of electro-thermal coupled problems from industry: Power-MOS devices and ET package models as shown in Figure 1 below.

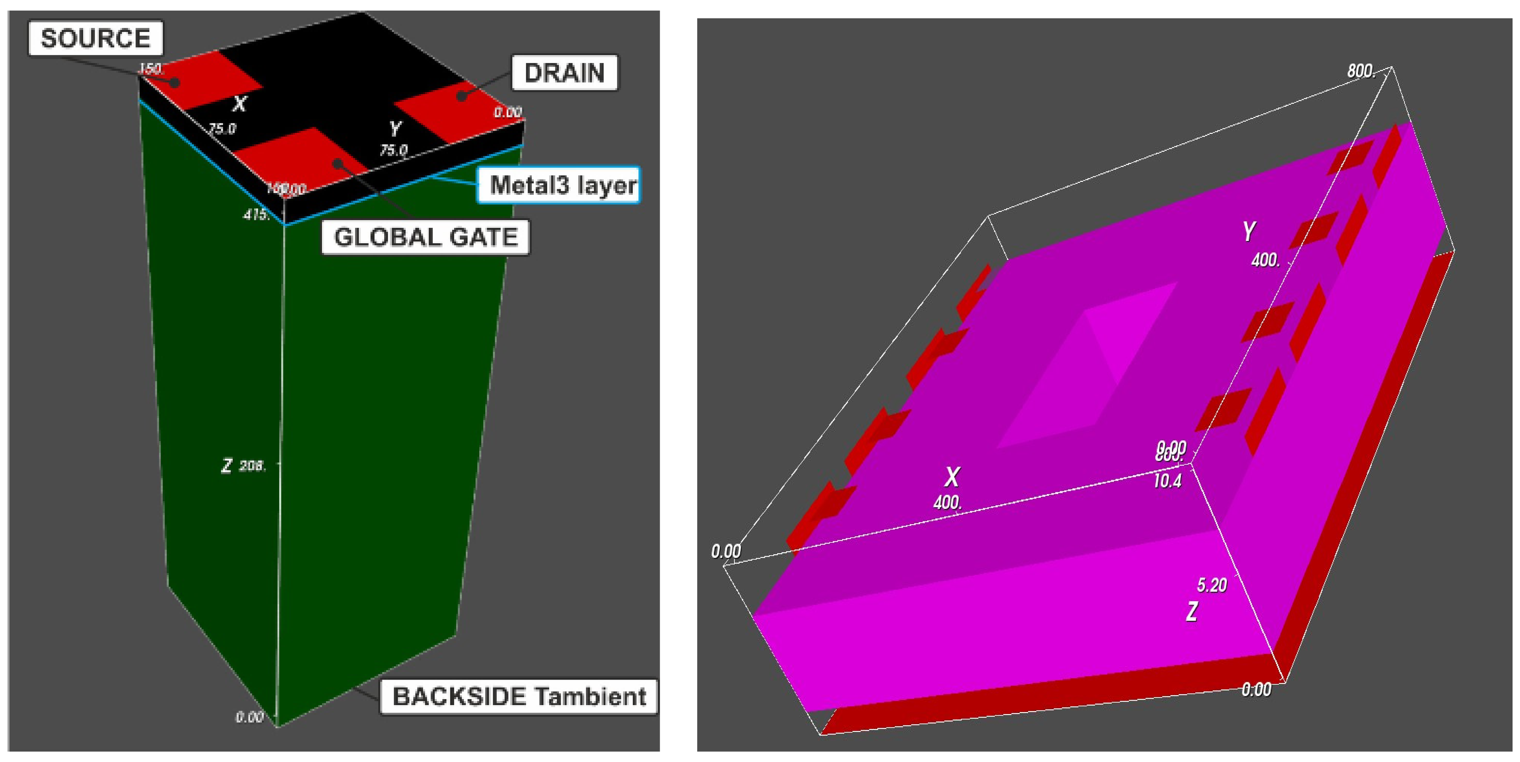

Figure 1: Power-MOS device (left); Electro-thermal package model (right).

Problem formulation

Modeling of ET problems requires heat transfer equations combined with charge-transport equations. Spatial discretization of these mathematical models yields differential-algebraic systems (DAEs), parameterized by a vector $\mu \in \mathbb{R}^d$ of the form:

$$\begin{aligned} \mathbf{E}(\mu)\mathbf{x}' &= \mathbf{A}(\mu)\mathbf{x} + \mathbf{x}^T\mathbf{F}(\mu)\mathbf{x} + \mathbf{B}(\mu)\mathbf{u}(t), \\ \mathbf{y} &= \mathbf{C}(\mu)\mathbf{x} + \mathbf{D}(\mu)\mathbf{u}(t), \end{aligned} \tag{1}$$

where

- d is the number of parameters,
- $\mathbf{x} = (\boldsymbol{x}_v^T, \boldsymbol{x}_T^T)^T \in \mathbb{R}^n$ is the state vector that includes the nodal voltages $\boldsymbol{x}_v \in \mathbb{R}^{n_v}$ and the nodal temperatures $\boldsymbol{x}_T \in \mathbb{R}^{n_T}$,
- the matrix $\mathbf{E}(\mu) \in \mathbb{R}^{n\times n}$ is singular for every parameter $\mu \Rightarrow$ parametrized DAE,
- $\mathbf{u} = \mathbf{u}(t) \in \mathbb{R}^m$ and $\mathbf{y} = \mathbf{y}(\mathbf{t}, \mu) \in \mathbb{R}^\ell$ are the inputs (excitations) and the desired outputs (observations), respectively.

The tensor $\mathbf{F}(\mu) = \left[\mathbf{F}_1^T(\mu), \ldots, \mathbf{F}_n^T(\mu)\right]^T$ is a 3-D array of n matrices $\mathbf{F}_i(\mu) \in \mathbb{R}^{n\times n}$. Each element in $\boldsymbol{x}^T\mathbf{F}(\mu)\boldsymbol{x} \in \mathbb{R}^n$ is a scalar $\boldsymbol{x}^T\mathbf{F}_i(\mu)\boldsymbol{x} \in \mathbb{R}$, $i = 1, \ldots, n$. We assume that the matrix coefficients and the tensor $\mathbf{F}(\mu)$ in (1) can be written as an affine parameter dependence, that is, $\mathbf{M}(\mu) = \mathbf{M}_0 + \sum_{i=1}^{m} f_i(\mu)\mathbf{M}_i$, where the scalar functions f_i determine the parametric dependency, and can be nonlinear functions of μ, and $\mathbf{M}_i$ can either be a constant matrix or a constant tensor. For example, in Figure 1, the ET models have the following parameter dependence:

- for the device model: $\mathbf{M}(\mu) = \mathbf{M}_0 + \mu\mathbf{M}_1$, where μ is the electrical conductivity,
- for the package model: $\mathbf{M}(\mu) = \mathbf{M}_0 + \mu\mathbf{M}_1 + \frac{1}{\mu}\mathbf{M}_1$, where μ is the thickness of the top layer.

The scientific challenges are to develop efficient and robust techniques for fast simulation of strongly coupled systems that exploit the different dynamics of sub-systems, and that can deal with signals that differ strongly in the frequency range and parameter domain.

In practice,

- $n \gg m, \ell \Rightarrow$ parameterized model order reduction (PMOR) techniques.
- To guarantee the reliability of ET coupled models $\Rightarrow$ uncertainty quantification (UQ) techniques.

However, direct application of the standard PMOR techniques to the ET coupled models of the form (1) may lead to inaccurate or unsolvable reduced-order models (ROMs) due to increased index and the mixing of field variables, and may not be able to capture the electro-thermal couplings.

Parametric model order reduction

We propose to apply projection based moment-matching methods to (1) because of its flexibility. In general the projection based PMOR is done by constructing a projection matrix $\mathbf{V} \in \mathbb{R}^{n\times r}$ which is valid for all parameters μ in the desired range, and for arbitrary inputs $\boldsymbol{u}(t)$ of interest. The reduced-order model of the system in (1) is written in the form

$$\mathbf{E}_r(\mu)\boldsymbol{x}_r' = \mathbf{A}_r(\mu)\boldsymbol{x}_r + \boldsymbol{x}_r^T\mathbf{F}_r(\mu)\boldsymbol{x}_r + \mathbf{B}_r(\mu)\boldsymbol{u}, \tag{2a}$$
$$\boldsymbol{y}_r = \mathbf{C}_r(\mu)\boldsymbol{x}_r + \mathbf{D}_r(\mu)\boldsymbol{u}, \tag{2b}$$

where $\mathbf{E}_r(\mu) = \mathbf{V}^T\mathbf{E}(\mu)\mathbf{V}$, $\mathbf{A}_r(\mu) = \mathbf{V}^T\mathbf{A}(\mu)\mathbf{V}$, $\mathbf{B}_r(\mu) = \mathbf{V}^T\mathbf{B}(\mu)$, and $\mathbf{F}_r \in \mathbb{R}^{r\times r\times r}$. However, this approach, if applied to DAEs, may produce parametric reduced-order models (pROMs) with index higher than the original model which might be inaccurate or are very difficult to solve. In [1], a PMOR method for ET coupled models was proposed as illustrated in Figure 2. It involves first decoupling the system (1) into algebraic (electrical) and differential (thermal) parts, then standard PMOR techniques such as PMOR based on implicit moment-matching [2] can be used to reduce both parts.

PMOR for nanoelectronic coupled problems

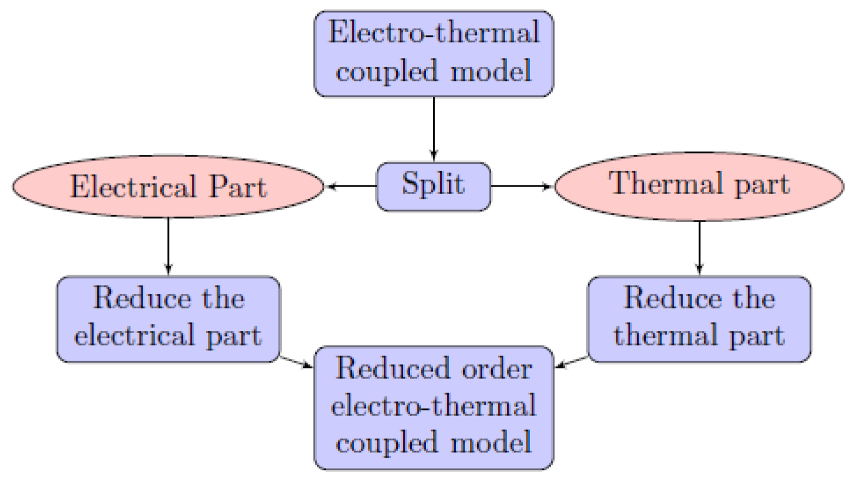

Figure 2: PMOR for ET coupled problems

- Standard PMOR techniques can be used to reduce both parts.
- A posteriori error bound can be used to automatically generate the pROMs [3], which leads to reliable and accurate parameterized reduced-order models (pROMs) for the ET coupled problems.

Decoupling of ET coupled model

Taking advantage of the natural structure of the matrices $\mathbf{E}(\mu) = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_T(\mu) \end{pmatrix}$, $\mathbf{A}(\mu) = \begin{pmatrix} \mathbf{A}_v(\mu) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_T(\mu) \end{pmatrix}$ and the tensor $\mathbf{F}(\mu)$ in (1), we obtain the following decoupled electrical and thermal parts,

$$\mathbf{0} = \mathbf{A}_v(\mu)\boldsymbol{x}_v + \mathbf{B}_v(\mu)\boldsymbol{u}, \tag{3a}$$
$$\mathbf{E}_T(\mu)\boldsymbol{x}_T' = \mathbf{A}_T(\mu)\boldsymbol{x}_T + \boldsymbol{x}_v^T\mathbf{F}_T(\mu)\boldsymbol{x}_v + \mathbf{B}_T(\mu)\boldsymbol{u}, \tag{3b}$$
$$\boldsymbol{y} = \mathbf{C}_v(\mu)\boldsymbol{x}_v + \mathbf{C}_T(\mu)\boldsymbol{x}_T + \mathbf{D}(\mu)\boldsymbol{u}, \tag{3c}$$

where $\mathbf{E}_T(\mu) \in \mathbb{R}^{n_T\times n_T}$ is a nonsingular matrix for every parameter μ, $\mathbf{A}_v(\mu) \in \mathbb{R}^{n_v\times n_v}$, $\mathbf{B}_v(\mu) \in \mathbb{R}^{n_v\times m}$, $\mathbf{A}_T(\mu) \in \mathbb{R}^{n_T\times n_T}$, $\mathbf{B}_T(\mu) \in \mathbb{R}^{n_T\times m}$, $\mathbf{C}_v(\mu) \in \mathbb{R}^{\ell\times n_v}$, $\mathbf{C}_T(\mu) \in \mathbb{R}^{\ell\times n_T}$, and $\mathbf{F}_T(\mu) \in \mathbb{R}^{n_v\times n_v\times n_T}$ is the nonzero part in the tensor $\mathbf{F}(\mu)$. We note that the total order of the decoupled system (3) is equal to the order of (1), that is, $n = n_v + n_T$.

PMOR for ET coupled model

In order to derive PMOR for the ET coupled model (1), we use the matrices from the decoupled system (3) to construct the projection matrices $\mathbf{V}_v \in \mathbb{R}^{n_v\times r_1}$ and $\mathbf{V}_T \in \mathbb{R}^{n_T\times r_2}$ using the PMOR method in [2]. This leads to a pROM of system (1) given by

$$\begin{aligned} \mathbf{0} &= \mathbf{A}_{v_r}(\mu)\boldsymbol{x}_{v_r} + \mathbf{B}_{v_r}(\mu)\boldsymbol{u}, \\ \mathbf{E}_{T_r}(\mu)\boldsymbol{x}_{T_r}' &= \mathbf{A}_{T_r}(\mu)\boldsymbol{x}_{T_r} + \boldsymbol{x}_{v_r}^T\mathbf{F}_{T_r}(\mu)\boldsymbol{x}_{v_r} + \mathbf{B}_{T_r}(\mu)\boldsymbol{u}, \\ \boldsymbol{y}_r &= \mathbf{C}_{v_r}(\mu)\boldsymbol{x}_{v_r} + \mathbf{C}_{T_r}(\mu)\boldsymbol{x}_{T_r} + \mathbf{D}(\mu)\boldsymbol{u}, \end{aligned}$$

where $\mathbf{A}_{v_r}(\mu) = \mathbf{V}_v^T\mathbf{A}_v(\mu)\mathbf{V}_v \in \mathbb{R}^{r_1\times r_1}$, $\mathbf{B}_{v_r}(\mu) = \mathbf{V}_v^T\mathbf{B}_v(\mu) \in \mathbb{R}^{r_1\times m}$, $\mathbf{E}_{T_r}(\mu) = \mathbf{V}_T^T\mathbf{E}_T(\mu)\mathbf{V}_T$, $\mathbf{A}_{T_r}(\mu) = \mathbf{V}_T^T\mathbf{A}_T(\mu)\mathbf{V}_T \in \mathbb{R}^{r_2\times r_2}$, $\mathbf{B}_{T_r}(\mu) = \mathbf{V}_T^T\mathbf{B}_T(\mu) \in \mathbb{R}^{r_2\times m}$, $\mathbf{C}_{v_r}(\mu) = \mathbf{C}(\mu)\mathbf{V}_v \in \mathbb{R}^{\ell\times r_1}$, $\mathbf{C}_{T_r}(\mu) = \mathbf{C}_T(\mu)\mathbf{V}_T \in \mathbb{R}^{\ell\times r_2}$, and $\mathbf{F}_{T_r}(\mu) \in \mathbb{R}^{r_1\times r_1\times r_2}$. Here $r_1 \ll n_v$ and $r_2 \ll n_T$ is the reduced order of (3a) and (3b), respectively. Hence the order of the pROM is given by $r = r_1 + r_2 \ll n$. We note that either (3a) or (3b) might be non-parametric even though the original system is parametric. In such a situation one can use MOR to the part with no parameters.

Numerical experiments

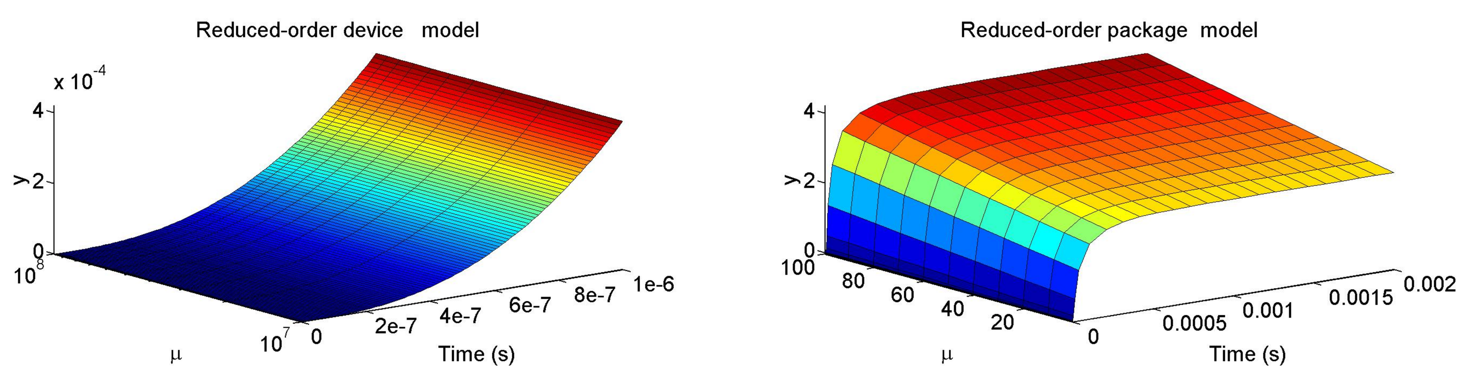

Figure 3: Output solutions of the pROMs for the device and the package models with error below 10^{-3}

Table 1: Comparison of computational cost

	Original model			Reduced-order model	Speedup factor
Models	Order (n)	inputs (m)	Outputs (ℓ)	Reduced-order (r)	$S = \frac{T_{Orig}}{T_{Red}}$
Package	9193	34	68	380	3
Device	13216	6	12	35	15

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