Reduced Basis method applied to large scale non linear multiphysics problems

Christophe Prud’homme
Christophe Trophime
1 Institut de Recherche Mathématique Avancée, Université de Strasbourg, France, 2 Laboratoire National des Champs Magnétiques Intenses, CNRS Grenoble, France

Large scale facility:
High magnetic field: > superconductors (24T)
Grenoble: Continuous fields (→ 36 T)
Toulouse: Pulsed fields (→ 90 T)

Applications:
(Bio-)chemistry
Magnetosence
Applied superconductivity

Reduced Electro-thermal Model

Electro-thermal model

\[ V : \text{Electrical potential [V]} \]
\[ T : \text{Temperature [K]} \]
\[ \begin{aligned}
    \sigma(T) &= \frac{\sigma_0}{1 + \alpha(T - T_0)} \\
    k(T) &= L T \sigma(T)
\end{aligned} \]

Material properties
Non-linearity

\[ \begin{aligned}
    V = 0 \text{ (bottom)} \\
    V = V_0 \text{ (top)}
\end{aligned} \]

\[ \begin{aligned}
    -\sigma(T) \nabla V \cdot \mathbf{n} &= 0 \text{ (electrical insulation)} \\
    -k(T) \nabla T \cdot \mathbf{n} &= h(T - T_w) \text{ (water cooling)}
\end{aligned} \]

Efficient Offline/Online strategy

\[ w(u, \mathbf{x}; \mu) = \sum_{m=1}^{M} \beta_m(u; \mu) q_m(x) \]

Non-linearity → Iterative fixed-point methods

Electro-thermal model

\[ u = (V, T) \text{ with } \mu = (\sigma_0, \alpha, \mathbf{V}_0, T_w) \]

RB approx. \[ : \sigma_M \approx \sigma(T), k_M \approx k(T), Q_M \approx \sigma(T) \nabla V \cdot \nabla V \]

Sensitivity Analysis

Quantities of interest

- Mean temperature
- Magnet power
- Magnetic field on a point
- Field homogeneity
- Mean displacements \((x, y, z)\)
- Mean constraints

Towards a full 3D non-linear multi-physics reduced model