Reduced Basis method applied to large scale non linear multiphysics problems

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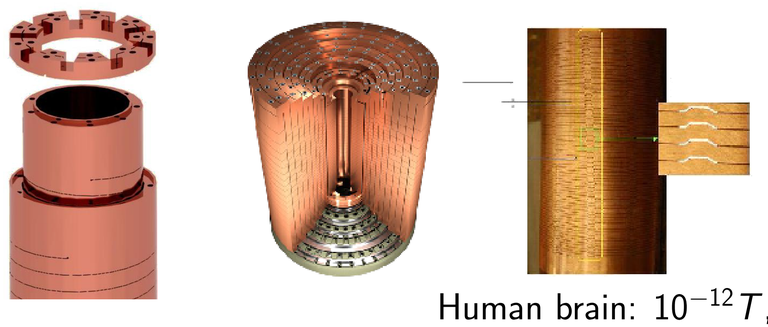
LNCMI : French high magnetic field laboratory



Large scale facility :
- High magnetic field : > superconductors (24 T)
- Grenoble : Continuous fields ($\longrightarrow$ 36 T)
- Toulouse : Pulsed fields ($\longrightarrow$ 90 T)

Applications :
- (Bio-)chemistry
- Magnetoscience
- Applied superconductivity

Human brain: $10^{-12}T$, Earth : $5.10^{-5}T$, Pace-maker : $10^{-3}T$, IRM : $1T$, Superconductor : $24T$

Reduced Electro-thermal Model

Electro-thermal model

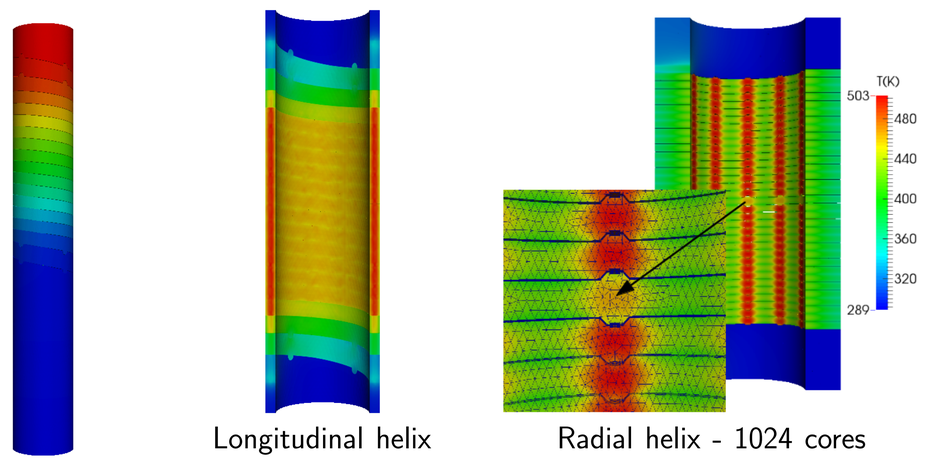
V : Electrical potential $[V]$ $\qquad$ T : Temperature $[K]$

$$\begin{cases} -\nabla\cdot(\sigma(T)\nabla V) = 0 \text{ dans } \Omega \\ -\nabla\cdot(k(T)\nabla T) = \sigma(T)\nabla V\cdot\nabla V \text{ dans } \Omega \end{cases}$$

$$\sigma(T) = \frac{\sigma_0}{1+\alpha(T-T_0)} \xleftarrow[\text{Non-linearity}]{\text{Material properties}} k(T) = LT\sigma(T)$$

$V = 0$ (bottom) $\qquad$ $-\sigma(T)\nabla V\cdot\mathbf{n} = 0$ (electrical insulation)

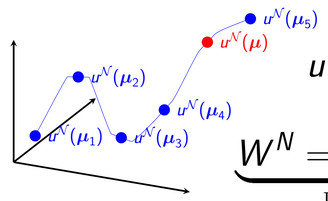
$V = V_D$ (top) $\qquad$ $-k(T)\nabla T\cdot\mathbf{n} = h(T-T_w)$ (water cooling)



Elec. potential $\qquad$ Longitudinal helix $\qquad$ Radial helix - 1024 cores

Temperature

Reduced Basis method



$u^N(\mu) \approx u^{\mathcal{N}}(\mu)$: linear combination of FEM solutions

$$\underbrace{W^N = span\{\xi_i \equiv u^{\mathcal{N}}(\mu_i)\}_{i=1}^N}_{\text{RB approximation space}} \rightarrow \underbrace{u^N(\mu) = \sum_{i=1}^N u_i^N(\mu)\xi_i}_{10\leqslant N\leqslant 100}$$

Efficient Offline/Online strategy

Non-affinely parametrized functions $\longrightarrow$ Empirical Interpolation Method

$$w(u,\mathbf{x};\boldsymbol{\mu}) \approx w_M(u,\mathbf{x};\boldsymbol{\mu}) = \sum_{m=1}^M \beta_m^M(u;\boldsymbol{\mu})q_m(\mathbf{x})$$

Non-linearity $\longrightarrow$ Iterative fixed-point methods

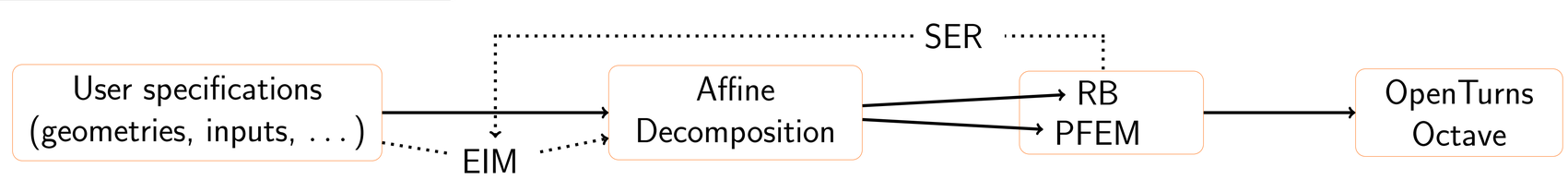
$$\sum_{i=1}^N\left[\sum_{q=1}^{Q_a}\sum_{m=1}^{M_a^q}\underbrace{\gamma_{a,m}^q({}^k u,\boldsymbol{\mu})}_{\text{online}}\underbrace{a_m^q(\xi_i,\xi_j)}_{\text{offline}}\right]{}^{k+1}u_i^N(\boldsymbol{\mu}) = \sum_{q=1}^{Q_f}\sum_{m=1}^{M_f^q}\underbrace{\gamma_{f,m}^q({}^k u,\boldsymbol{\mu})}_{\text{online}}\underbrace{f_m^q(\xi_j)}_{\text{offline}}$$

Electro-thermal model

$u = (V,T)$ with $\boldsymbol{\mu} = (\sigma_0,\alpha,L,V_D,h,T_w)$

EIM approx. : $\sigma_M \approx \sigma(T)$, $k_M \approx k(T)$, $Q_M \approx \sigma(T)\nabla V\cdot\nabla V$

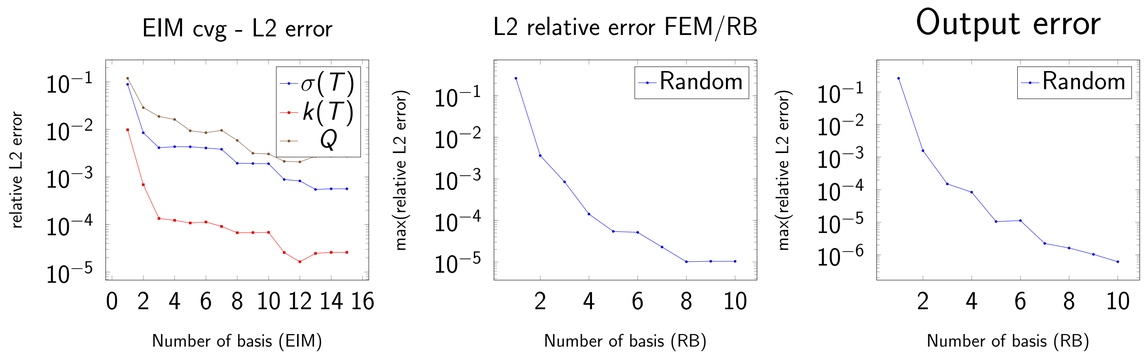
Feel++ www.feelpp.org
Reduced Basis Framework



Electro-thermal model on radial helix

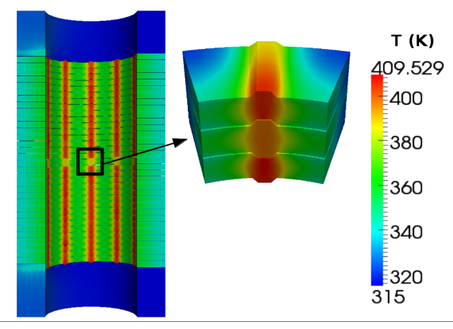
Number of dofs $\approx 5\cdot 10^5$
HPC on 12 procs
EIM : 15 basis, RB : 10 basis

FEM : 1011 seconds $\approx$ 16 min
RB (Online) : 6.7 seconds
Gain factor : 150



Sensitivity Analysis

$\sigma_0 \in [50.10^6; 50,2.10^6](S.m^{-1})$
$\alpha \in [3,3.10^{-3}; 3,5.10^{-3}](K^{-1})$
$L \in [2,5.10^{-8}; 2,9.10^{-8}]$
$U \in [0.14; 0.15](V)$ Intensity $\approx$ 25 kA
$h \in [70000; 90000](W.m^{-2}.K^{-1})$
$T_w \in [293, 313](K)$



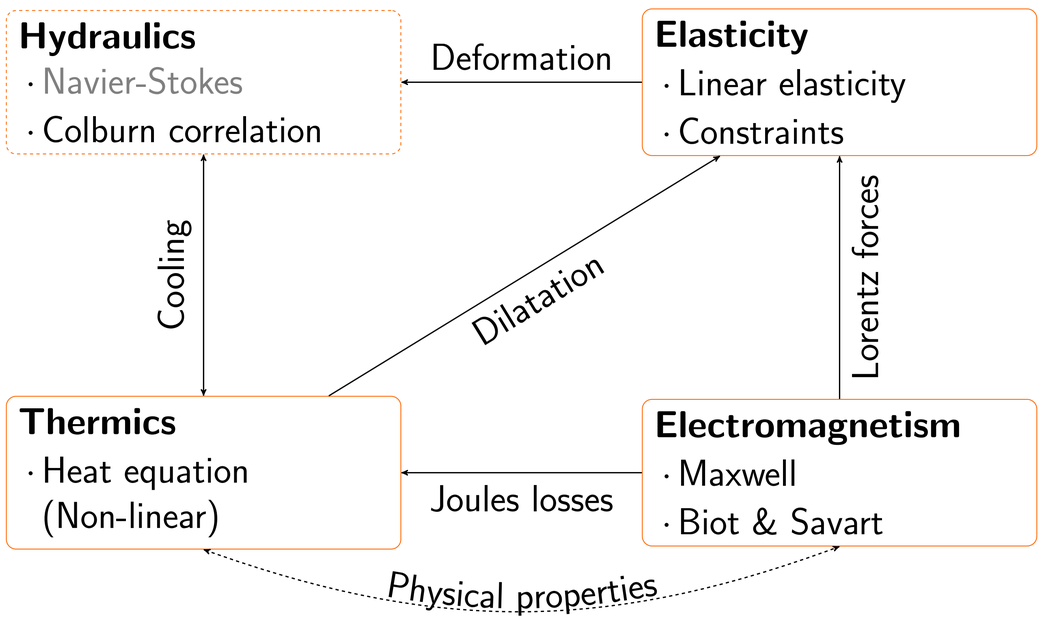
Sensitivity indices (Sobol) :

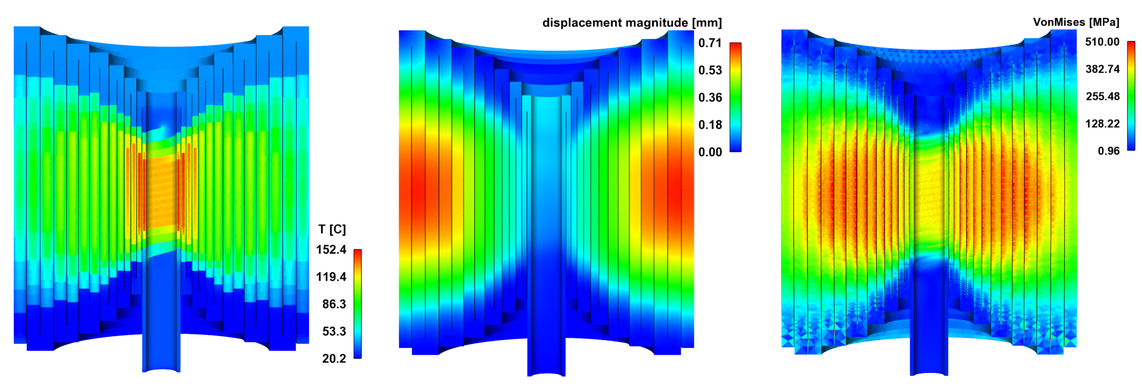
— σ_0 : 6.85e-05	— V_D : 1.28e-01
— α : 4.54e-04	— h : 2.40e-01
— L : 9.21e-03	— T_w : 6.21e-01

Quantiles :
$q(\gamma)$ such that $P(Y < q(\gamma)) > \gamma$

99.0 quantile = 380 K = 107 C
80.0 quantile = 377.5 K = 104.5 C

Towards a full 3D non-linear multi-physics reduced model





Temperature $\qquad$ Displacements $\qquad$ Constraints

Multi-physics model on 14 helices magnet

Quantities of interest

- Mean temperature
- Magnet power
- Magnetic field on a point
- Field homogeneity
- Mean displacements (x,y,z)
- Mean constraints