The Role of Modelling in Learning Formal Specification
R. Leman
The role of modelling skills in learning formal specification

Ruth Leman
Faculty of Information and Engineering Systems, Leeds Metropolitan University, Leeds, UK

Abstract

The paper describes the results of a qualitative study into the ways in which students learning formal specification understand the concepts of propositional and predicate logic. The rationale for choice of certain tasks is given, and the learning demands made by these are analysed in terms of the understanding of logic and of the process of modelling itself. The results reveal that many students were found to have as much difficulty with aspects of modelling as with logic, and the paper concludes by suggesting that these may have a central role to play in the development of students’ understanding of logic.

1 Introduction

Many students learning formal specification have difficulty even with the most basic material, and this does not seem explainable by the level of difficulty of the mathematical and software system concepts alone. Other possible factors that have been suggested may be applicable to a small minority (e.g. lack of familiarity with problem domains, lack of facility with the English language) but could not account for the scale of the problem.

Observation in the field has suggested that difficulties begin at the earliest stages of the work, are experienced by a majority of students, (not just those without mathematical qualifications) and are associated particularly with tasks that involve analysing natural language descriptions in mathematical terms. However, analysis of the learning demands made on novice students during this period of preliminary observation showed a far more complex picture than that of simply learning some basic mathematical concepts and then applying them to example systems. It quickly became evident that there were a number of other elements that were not being explicitly taught, but were a requirement of successful completion of even the simplest of the tasks, and these skills were those that could be broadly subsumed under the area of ‘knowing how to model’.
This paper puts forward the view that students’ difficulties in formal specification tasks can be understood partly in terms of their lack of modelling ability. It describes the results of an investigation of students working on simple formal specification problems which demonstrates that modelling skills not only play a key part in enabling successful completion of the tasks, but also in encouraging the development of their understanding of some of the more difficult concepts in the domain. It was found that many students showed a surprising lack of ability in areas of modelling skill, in quite fundamental areas such as use of truth tables. By gaining an understanding of the nature of their difficulties, it should be possible to build up a picture of the kind of support they may need in such activities and to plan appropriate teaching developments accordingly. It is expected that the findings will be applicable to other educational contexts, for example industrial training, as well as other areas of software engineering where modelling skills are used.

The structure of the paper is as follows. Firstly, the whole area of modelling needs to be defined. This will be done by drawing up a brief specification for modelling as a dimension of the formal methods curriculum. Then the investigation will be described, by presenting first the questions used and their demands on the students, then the types of responses given. Finally, the implications for teaching and further research will be discussed.

2 What is modelling?

That students lack the ability to model or abstract is a frequently heard lament across the whole software engineering discipline, but it is useful to ask the question, What exactly does this ability entail? Although implicit in work on formal specification, it does not appear on curriculum description documents, nor is it directly taught or assessed. The following specification of modelling has therefore been obtained by analysis of the overall learning demands of specific tasks, in the context of a software engineering ethos. It is intended to act as a working framework, and does not purport to be definitive. Some overarching modelling concepts are identified, then a number of component skills and concepts:

**Purpose of modelling**

- To understand that one of the reasons for producing a formal specification (or other model) is to improve the quality of the analysis product and that this is achieved by the process of modelling.

To appreciate the limitations of any mathematical (or other) model in capturing the real world system, due to the fundamental differences between the ‘rules’ of these two worlds.
The role of modelling skills in learning formal specification

Modelling activities

- Ability to assess the quality of information supplied by the natural language description (e.g. sufficiency and relevance)
- Awareness of the expressive limitations of natural language for the purposes of specification, (e.g. ambiguity and lack of precision)
- Understanding that there are other representations that can be used for description and analysis, what these are and when and how to use them, (Venn diagrams truth tables, finite state models etc.)
- Knowing how to apply a systematic approach to ensure a complete and accurate model, involving the following kinds of activities:
  - interpreting, reformulating, verifying, instantiating, re-representing, use of counter-example, confirming.

Clearly this list covers some significant components of a typical formal specification curriculum, being part of the means whereby the ideas in the discipline are developed and communicated, as opposed to the subject matter itself. Traditionally it has been assumed that this kind of knowledge is easily assimilated by students without any explicit teaching. However current thinking in Higher Education research suggests that this is not the case, and may be quite significant in accounting for difficulties in many disciplines. [1]

It is certainly evident that, in formal specification, the two domains are inextricably linked: for example, an awareness of the expressive limitations of natural language clearly plays an important part in developing an understanding of the meaning of the logical operators. The converse also needs to be considered: for example does a lack of awareness of the expressive limitations of natural language seem to help to prevent the development of these concepts?

Having established a framework for the consideration of modelling as an educational activity, research findings in this area need to be considered. There is no systematic study in the immediate domain, but plenty of relevant work in the area of Mathematics education, a useful reference being [2]. Research findings on problem-solving, the role of language and diagrammatic representations are all useful, and will be mentioned later in the discussion of findings.

To conclude, current thinking in this area suggests that it can be assumed that modelling plays an important role in learning formal specification, and the purpose of the investigation was therefore to analyse its operation.
3 The investigation

A qualitative methodology (phenomenography [3]) was used in order to obtain descriptions of students’ thinking and approaches to specification tasks through one-to-one interviews in which students discussed their responses to problems and to related ideas. This enabled an open approach at the data gathering stage, with the identification of appropriate perspectives being then made at the stage of analysis. The sample was self-selected on a volunteer basis, from both year one and year two cohorts (having respectively one and two modules experience with formal methods under their belt, but being interviewed in both cases several months after the end of their most recent experience), and contained a balance of age, sex, and ability, and one student with a first language other than English.

One of the initial challenges in planning this study was to find tasks which presented sufficient depth to produce some worthwhile thinking and discussion, but without over-loading the students. As mentioned above, observation had shown that most difficulty was experienced with questions involving the interpretation of English descriptions. Analysis of peer-group discussion sessions had also revealed that the students’ work on the module assignments (reading and writing a Z specification) had been characterised by a ‘strategic’ approach [4], rather than the intention to engage fully with meaning, and it was felt that very simple specification problems drawing on fewer concepts would be more likely to do this. [5] This decision was vindicated to the extent that although the interview schedule had ten such questions, at most five were covered in the hour, indicating depth of coverage achieved. The questions were piloted on fellow researchers, providing useful information on an expert model with which to contrast the student responses. The following discussion will centre around just two of the simplest of these.

Generally, examples were chosen in which there was not a straightforward mapping between the English syntax and the formal notation, as it had been found that these kinds of questions produced very shallow discussion. For the majority this was successful in that it encouraged a lot of discussion about possible meanings. However in one or two cases, the meaning of the sentences was not understood. This is in itself a significant finding, given that lecturers tend to assume an adequate knowledge of English, but this issue will not be addressed by this research.
Example 1: *How would you represent the following sentences using formal notation?*

- *I only go to Elland Road when Leeds United are playing at home.*
- *I always go to Elland Road when Leeds United are playing at home.*

This had been used as a tutorial example and it was expected that at least its familiarity would provide a comfortable lead-in to the rest of the questions. The contrast was designed to encourage precision and attention to meaning.

Example 2: *Express the following using quantifiers and logical connective as appropriate:*

*All that glisters is not gold*

This deliberately literary example was chosen again in order to stimulate a thinking process that could be captured and analysed.

Both questions caused some degree of difficulty with many of the students, and in order to understand this it is necessary to analyse the demands they are making of students’ understanding. Clearly the rationale behind the first’s inclusion in a tutorial exercise was simply to measure students’ ability to identify a) the simple propositions and b) the logical connective that links them. However, on reflection it is clear that the ability to make an instant recognition would tend to imply that an understanding of the concept was already in place, which was not realistic for many of the students at this stage. Therefore the example is also demanding other knowledge: the knowledge of how to reprocess the information in order to arrive at a correct solution when it is not immediately obvious to the learner, and this is where the skills of modelling are called for. There are two possible approaches. One is to first consider the four possibilities and decide upon their truth values, then recognise the logical connective represented by that combination. The other is to make a guess at the connective (perhaps by looking for key words in the sentence) then verify this by checking each case in turn as before.

Similarly, the second requires an understanding of the use of quantifiers and propositions in predicate logic, but also the ability to go through a process like that just described, this time clarifying first what is being asserted about which subset of objects and then translating this into the appropriate quantified predicate, verifying and reconfirming etc. Referring to the list of modelling skills given above, the process just described corresponds to the skill of knowing how to apply a systematic approach.
The role of modelling skills in learning formal specification

In the process of thinking through the example, the various modelling techniques, of instantiating, providing a counter example, etc. can be used to verify the solution. Other representations apart from the linguistic could be used for clarification, for example a truth table to model the four different scenarios in the first example, and a Venn diagram to identify the contents of the sets concerned in the second. As regards the other elements of modelling, the importance of an awareness of the ambiguity of natural language has already been mentioned. The first, ability to assess the quality of the information produced could be said to be called for in that it may be helpful to recognise the literary style of the second example. The aspects that relate to more general awareness of the nature of the activity as a whole however are not as clearly implicit, but the less successful students demonstrated a lack of such awareness through some of their comments. This area will not however be covered in this paper.

Having identified the demands made by the questions, students’ responses will now be examined.

4 Results

The data obtained from the transcripts reveal a lot about the ways in which students understand the logical concepts covered, and their understanding of modelling. The ultimate aim is to produce a detailed description of the qualitatively different ways of understanding modelling for formal specification, and the ways in which these ways support understanding of the domain itself. However, at this stage a brief summary of the most significant findings will be given.

Since the research was concerned with obtaining information about students’ understanding, success was measured by the quality of the explanation accompanying the solution. Analysis of these responses enabled several distinct ways of understanding to be identified for each of the logical concepts covered. For example, ways of understanding of material implication were (from least to most adequate): process of elimination of other more well-understood connectives (‘implication is that other one that is difficult to remember’!), reliance on syntax of the sentence, e.g. looking for if...then constructs, understanding of everyday use (i.e. logical implication). Within the third there were a number of distinctly different ways of thinking which will be explored more fully elsewhere. More than half of the sample fell into the first two.

Difficulties could therefore be ascribed to inadequate understanding of the domain concepts. However, the level at which these concepts were understood seemed to correlate with level of understanding and skill with modelling. It therefore appeared that those students with the least developed understanding of logic were
The role of modelling skills in learning formal specification

handicapped further by not having the means to work out the answer. Some more specific detail will now be
given to support this.

A typical pattern of behaviour for the majority of students in attempting to formalise these examples was
as follows:

Example 1 - The answer \( a \implies b \) was produced by the thinking in either a) or b) above, but then the student
was thrown into a state of doubt by the next sentence. This stimulated an attempt to engage much more
thoroughly with the meaning of both sentences; the problem was re-represented and rephrased a number of
times. However, when an (incorrect) answer was generated by this process, no attempt at verification was made.

In terms of the modelling activity flow chart then, they participated in only some part of the whole cycle,
jumping out before the end. This may have been because they recognised that their understanding of the
concept was too limited to cope with the demands of the task, therefore in effect were giving up. However,
when prompted for a general case “How would you check an answer to see if it represents the meaning given in
the English?” they still did not appear to have a concept of verification. It is useful to note that this behaviour
was also shown in research on low attainment in mathematical problem-solving [6].

Another finding related to the use of representation to establish the mathematical meaning of the
sentences. None of the students in the sample used any form of representation other than the linguistic. In
many cases this had the effect of making the meaning more and more elusive, and it was surprising that they
did not recognise the need to use a Venn diagram etc. when this happened. The reason became clear when
discussing their understanding of these other representations, revealing that there was a lack of understanding
of the purpose and use of these. There was an enormous amount of confusion and misunderstanding about
these at a surprisingly basic level, complaints being made that the truth tables for the connectives are not
sufficiently memorable, about the use of \( t \) and \( f \) instead of the more familiar 0 and 1, etc., and many comments
were made which displayed dislike and even fear. With the second example where a Venn diagram would have
clarified the meaning immediately, the suggestion to use one was refused in most cases, on the grounds for
example that ‘I only use them for set theory work’ , ‘I wouldn’t know where to start’ etc. Generally there was
a lack of appreciation of the semantic equivalence of different representations, their use being seen in terms of
‘either /or’.

The strength of using an alternative representation for novice students is not just that when one strategy
fails, another often succeeds. For example, to have been forced to draw up a truth table to model the behaviour
described in example one, or to represent the proposition about the set in example two would have actually encouraged the adoption of the mathematical thinking required to produce a correct solution. Work in Mathematics learning on problem solving has also suggested that the use of linguistic strategies in problem solving is less efficient than a mixture of linguistic and diagrammatic representations. Research has also been conducted on the ways in which difficulties can arise with understanding the purpose of representations and with their interpretation (especially those which use visual means to represent non-visual concepts). [6]

This account has given an indication of the scale and extent of the problems students had with using representations and adopting a systematic method for modelling, and has argued that this prevented them from adopting a mathematical approach necessary to produce an accurate model. The question arises of why this is the case, and what the implications are for teaching, and these will finally be considered.

5 Conclusion

The paper has presented some of the results of an investigation into students learning formal specification, showing how the students’ problems with modelling contribute towards difficulties when engaging even in simple specification problems. It has focused the discussion on two particular areas of modelling which seemed to be the most significant in the given examples, the following of a systematic method and the use of alternative representations, while recognising the importance of all the skills and concepts described in the specification for modelling. Furthermore the results show that the adoption of a purely linguistic approach to producing a formal specification does not best address the needs of novices, where the mathematical concepts are not yet well developed, and that this approach could be seen as preventing the process of development of these concepts, by encouraging the use of syntactic mapping instead of engagement with meaning.

What are the implications of this for teaching? One is that there may need to be a more explicit teaching of these modelling skills. In considering why the students in the sample seemed to have developed them so little, it may be that the activities they have engaged have not encouraged their development (in other words questions need to be quite carefully structured to encourage engagement with meaning rather than reliance on guesswork). Also it is possible that the more expert model (i.e. the lecturer or tutor’s behaviour) is inappropriate as a model for learners, in that for questions at the novice level, the expert can dispense with the use of diagrammatic representation and most of the techniques described above, and move easily between the linguistic and the formal representation. Seeing this use of exclusively linguistic strategies may confirm the
idea that there is a straightforward linguistic mapping between the words and symbols, and prevent the development of the concept of the inherent limitations of natural language for specification, which is so central to the work. Similarly, by missing out stages in the modelling process, lecturers are in danger of giving the impression that solutions are ‘plucked from the air’, leading to a sense of powerlessness and frustration when engaging in formal specification activities. This investigation has served to provide an initial framework for the further exploration of these ideas.

6 References:

1. Laurillard, D. Rethinking University Teaching Routledge 1993


3. Marton, F. Phenomenography - describing conceptions of the world around us Instructional Science 10

4. Entwistle, N. and Ramsden P. Understanding student learning Croom Helm 1983
