Wittgenstein and Tufte on Thinking in 3D: 'Escaping Flatland'

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Before the philosopher Ludwig Wittgenstein became Bertrand Russell's protégé at Cambridge, his formal education was directed towards shaping the mind of a sophisticated research engineer. The engineer's ability to visualise inventions and solve design problems by creatively altering configurations of their elements calls for constructive, spatial, synthetic thinking. Training in engineering drawing builds visual thinking skills and teaches engineers how to represent visualised three-dimensional objects on a two-dimensional surface. Visualisation involves seeing projective relations in the mind's eye, which plays a critical role in the *Bild* theory of the *Tractatus*, Wittgenstein's early philosophical classic. He asked himself 'What is the ground of our---certainly well-founded---confidence that we shall be able to express any sense that we like in our two-dimensional script?' As Edward Tufte says, 'Even though we navigate daily through a perceptual world of three dimensions...the world displayed on our information displays is caught up in the two dimensionality of the endless flatlands of paper and video screens...Escaping this flatland is the essential task of envisioning information.' I will explore how Wittgenstein's *Tractatus* relates to Tufte's theories of envisioning information and could contribute to understanding the principles underlying three-dimensional visualisation.

Wittgenstein. Tufte. Visualisation. Tractatus.

One time, we were discussing something – we must have been eleven or twelve at the time – and I said, 'But thinking is nothing but talking to yourself inside.'

'Oh Yeah?' Bernie said. 'Do you know the crazy shape of a crankshaft in a car?'

'Yeah, what of it?'

'Good. Now tell me: how did you describe it when you were talking to yourself?'

So I learned from Bernie that thoughts can be visual as well as verbal.

(Feynman1988)

1. INTRODUCTION

Ludwig Wittgenstein presents his famous picture (*Bild*) theory of language in the *Tractatus Logico-Philosophicus*. Thinking it through in his notebooks, he asked, 'What is the ground of our---certainly well-founded---confidence that we shall be able to express any sense that we like in our two-dimensional script?' (Wittgenstein 1961). As Edward Tufte says In *Envisioning Information*, 'Even though we navigate daily through a perceptual world of three dimensions...the world

displayed on our information displays is caught up in the two dimensionality of the endless flatlands of paper and video screens...Escaping this flatland is the essential task of envisioning information' (Tufte 1990). In the *Tractatus*, Wittgenstein develops a law of projection to translate between states of affairs and their representations, understanding his *Bilder* as standing in a 'projective relation' to reality. His method of projection calls for the visual thinking of the engineer and the descriptive geometer.

Visualisation involves seeing projective relations in the mind's eye, and this kind of visual thinking and representation plays a critical role in the *Bild* theory of the *Tractatus*. Could principles developed in the *Bild* theory help us understand visualisation in ways that could contribute to the development of professional disciplines in which visual thinking and representations are central? In this paper, I will explore how principles laid out in Wittgenstein's philosophy relate to Edward Tufte's work on visualising information. I argue that Tufte's visual displays can be understood as exemplars of Wittgenstein's propositional signs, functioning according to principles laid out in the logic of depiction in the *Tractatus*. To frame the argument, I will first give an account of Wittgenstein's

background, and then I will present the *Bild* Theory of language.

2. RUSSELL'S GERMAN ENGINEER

Wittgenstein was an iconic philosophical figure, perhaps most famously as a dominating presence in philosophy at Cambridge University. But, in 1911, when he showed up unexpectedly in Bertrand Russell's rooms in Cambridge, Russell did not know what to make of him. Very soon, he felt Wittgenstein had learned all that he could teach him and gone further. In the Cambridge community, he was Russell's designated philosophical heir.

Wittgenstein's prior education had been technical and scientific, however. For the youngest son of one of the wealthiest and most cultured families in Vienna, the prestige of an elite gymnasium, then university, would have been normal. His father, Karl Wittgenstein, had financed Olbrich's Secession building, paid for Klimt's famous university mural, and had the young Casals and Brahms perform in his magnificent palais. The family's cultural life was deeply felt and intensely lived, particularly in their devotion to music. For generations, Wittgensteins had patronised important musicians and performed music to exacting standards themselves. But Ludwig had shown an aptitude for the practical, building a small sewing machine when he was only ten. His father, a self-made industrialist trained as an engineer, supported his son's practical bent.

Thus, Ludwig attended an *Oberrealschule*, a technical high school, and received his engineering certification from the *Technische-Hochschule* in Berlin, then the MIT of Europe. Engineering training there focused on mathematical drawing, hands-on drafting, and experimental experience. Wittgenstein was trained to think like a design engineer.¹ Brian McGuinness wrote

He liked to think with the machine – to understand every detail of its functioning – and this accounts both for his interest in the older and more perspicuous types of mechanism . . . and for his success in repairing mechanisms that had gone wrong. Always it was achieved by the most careful observation of the machine from every side and deep and concentrated reflection until he had internalised the principle of the machine. It was thus that he had observed the distrustful sempstress in the Allee-gasse and it was thus that he would examine the searchlight or boiler or whatever it was, sometimes rather to the irritation of onlookers, only intervening . . . when the course to be followed was absolutely clear to him (McGuinness 1988).

Wittgenstein retained his love of machines, as well as a practical engineering ability in experimental

research, throughout his life. Professor G. H. von Wright, Wittgenstein's close friend and literary executor, told Allan Janik that it was important to remember 'firstly, that he was a Viennese and, secondly, that he was an engineer with a thorough knowledge of physics,' and this important insight was at the heart of their brilliant account of Wittgenstein's life and philosophical work as rooted in his Central European heritage, *Wittgenstein's Vienna* (Janik and Toulmin 1973).

After this first-class formal engineering education, he spent four years pursuing aeronautical engineering research at the University of Manchester, designing and patenting an innovative propeller.² Experimental data used in research and design in this area was mostly obtained from wind tunnel experiments with scale models, but such data presented serious difficulties for designers due to problems with scale effects. Did the experimental situation in the wind tunnel faithfully reflect the flying conditions of the actual airplane? How do you translate from the scale models of the engineer to the real life, full-scale engineering problem? Accurate translation between scale model experiments and real life situations depended on dimensional analysis, and propeller research was thus highly mathematical. Shifting his interests to the foundations of mathematics, he read *The Principles of Mathematics*, encountering the challenge to solve Russell's paradox. He then turned up in Lord Russell's rooms, too nervous to speak English, and Russell's 'German engineer' soon became the leading light in Cambridge philosophy.

What kind of changes would have taken place in Wittgenstein's mind as he learned to think like a design engineer? As Eugene Ferguson writes,

Many features and qualities of the objects that a technologist thinks about cannot be reduced to unambiguous verbal descriptions; therefore, they are dealt with in the mind by a visual, nonverbal process. The mind's eye is a well-developed organ that not only reviews the contents of a visual memory but also forms such new or modified images as the mind's thoughts require. As one thinks about a machine, reasoning through successive steps in a dynamic process, one can turn it over in one's mind. The engineering designer, who brings elements together in new combinations, is able to assemble and manipulate in his or her mind devices that as yet do not exist (Ferguson 1992).

The engineer's ability to visualise an invention, to solve design problems by creatively altering configurations of its elements, calls for a kind of constructive visualisation that is also important for scientific practice. As Brook Hindle argues, 'synthetic-spatial thinking is, of course, involved in

most intellectual activity including science, but in technology it has to be central' (Hindle 1981). Engineering drawings are representations of these thoughts, communicating the engineer's ideas from the level of tentative creativity in thinking sketches to the precision of technical blueprints for the construction of a completed mechanism. Wittgenstein brought this mindset to the *Tractatus*.

3. THE *BILD* THEORY

The *Tractatus* is an unusual, brilliant, and profoundly difficult little book. The major work of the early Wittgenstein, in contrast to the later Wittgenstein of the *Philosophical Investigations*, it is the only book he published during his lifetime. It was written under the duress of life on the Eastern Front in WWI, and it was a deeply personal statement, the product of years of intense reflection. A profound influence on European philosophy between the wars, it has been a continuing source of philosophical debate, controversy, and insight to this day.

The structure is bewildering at first sight. It consists of carefully arranged numbered propositions, and it has been aptly described as a series of aphorisms.

The decimal numbers assigned to the individual propositions indicate the logical importance of the propositions, the stress laid on them in my exposition. The propositions *n.1*, *n.2*, *n.3*, are comments on propositions no. *n...* (Wittgenstein 2010/1922)

It is elegant, spare, and impossible to understand without careful study. Ordinary words are used in ways that will perplex intelligent readers outside their professional disciplinary ambit. It is not even a case of knowing their technical meanings in the context of the field. His use of words, development of propositions, and presentation of their connections repay careful study as much as the information in any dense Tuftean visual display. Indeed, I would argue that in many ways, the *Tractatus* is a superb, dense, Tuftean visual explanation. It is certainly 'self-exemplifying' in the manner of Tufte's books, as the object itself also embodies the ideas written about. As it is impossible to deal with the depth and richness of the work of either of these thinkers adequately in this brief space, I will consider some of the important principles of visual, analytical thinking developed in their work, showing critical connections between them. In-depth consideration must await space for a more complete, detailed analysis.

The first set of propositions in the *Tractatus* presents an ontology that describes the structures

of the world as we know it according to his theory. He asserts that 'the world is all that is the case'(1), specifying that 'the world is the totality of facts, not of things'(1.1). His definition of states of affairs specifies that 'What is the case – a fact – is the existence of states of affairs'(2), and 'a state of affairs (a state of things) is a combination of objects (things)''(2.01). Thus, it is not objects standing alone, unrelated to each other, that constitute the world, but objects as they exist in combination, related to one another in states of affairs. Existing states of affairs make up what is the case, or the world as we know it.

What makes it possible for things to combine in states of affairs? 'The possibility of its occurring in states of affairs is the form of an object'(2.0124).

2.0271 Objects are what is unalterable and subsistent; their configuration is what is changing and unstable.

2.0272 The configuration of objects produces states of affairs.

2.03 In a state of affairs objects fit into one another like the links of a chain.

2.031 In a state of affairs objects stand in a determinate relation to one another.

2.032 The determinate way in which objects are connected in a state of affairs is the structure of the state of affairs.

2.033 Form is the possibility of structure.

2.034 The structure of a fact consists of structures of states of affairs.

The form created by the configuration of objects combined with one another can be the structure of a state of affairs. To determine what actual structure exists in the world, one must check to see which of all the possible combinations of objects did, in fact, occur.

Returning to Wittgenstein's engineering training, using models of working parts of machines to teach machine construction to engineers was standard. Polhem's 'mechanical alphabet' represented mechanical movements necessary for the design of complex machines. The five 'powers' of Hero of Alexandria: the lever, the wedge, the screw, the pulley and the winch, were the vowels of his mechanical alphabet. No 'machine limb (could) be put into motion without being dependent on one of these' (Ferguson 1992) This alphabet of objects, whose configurations produced various working inventions, gave engineers a tacit understanding of the component parts of machines and the principles underlying the forms of machines,

allowing them to visualise these elements combined in new configurations.

Figure 1a: Polhem's Mechanical Alphabet

Figure 1b: Reuleaux Model

of Machine Element (Hamilton 2001, 71 and 72)

Wittgenstein's coursework used models of machine elements intensively. A famous and widely used set of kinematic models was actually created by Professor Franz Reuleaux of the *Technische Hochschule* in Berlin. Such machine parts have also been beautifully represented historically in engineering drawings, perhaps most famously in the engineering notebooks of Leonardo DaVinci.

In terms of the *Bild* theory, I ask you to imagine, to visualise in your mind's eye, individual machine parts. See what combinatorial possibilities exist between them. Obviously, the forms of the parts determine how they can stand in relation to one another in a machine that will work. Obviously, again, there are many possible, yet predetermined (as only certain combinations are possible), structures that can exist. The forms of the parts (objects) determine the possibilities of different

Figures 2a and 2b: Leonardo da Vinci's Engineering Drawings (Hamilton 2001, 74 and 81)

structures of machines (possible states of affairs). The actual combination of parts (configuration of objects) realised depends on which of those possibilities was realised in the construction of the machine. 'If I know an object, I also know all its possible occurrences in states of affairs. (Every one of these possibilities must be part of the nature of the object)'(2.0123). 'If all objects are given, then...all possible states of affairs are also given'(2.0124).

Engineers can 'see' the structure of a projected machine; they actually think through how the principles will work in the mind's eye in a nonverbal

fashion. The role of spatial thinking, seeing the form or configuration of a structure in space, is crucial for understanding the possibilities of the structure being designed. Analyzing Morse's invention of the telegraph, Hindle developed this thought:

His great strength remained a quality of mind that permitted him to manipulate mental images of three-dimensional telegraph components as well as complete telegraphic systems, altering them at will and projecting various possibilities for change and development. Although he had used this mode of thinking in his art, his telegraph in no way depended upon his art. Conspicuous success in each, however, absolutely required conspicuous ability in spatial thinking (Hindle 1981).

As Morse himself wrote, 'painting and her sister arts of design rely upon form displayed in space' (Hindle 1981). Leonardo da Vinci is perhaps the best example of the combination of artistic and practical talents that David Pye feels 'are really different expressions of one potentiality' (Pye 1978). Certainly, 'form displayed in space' is an excellent characterization of propositional structure in the *Tractatus*.

Seeing machine elements in various possible configurations would have been second nature to Wittgenstein given his engineering training. Viewing the *Bild* theory from this perspective is unusually concrete, but 'that utterly simple thing, which we have to formulate here, is not a likeness of the truth, but the truth.... (Our problems are not abstract, but perhaps the most concrete that there are)'(5.5563).

Moving to what Tufte characterises in Galileo's words as 'what 'the eye of the mind' envisions,' Wittgenstein asserts 'we picture facts to ourselves'(2.1). (*Wir machen uns Bilder der Tatsache*). David Stern warns

Wittgenstein used the German word 'Bild' to talk about the model, a term usually translated as 'picture'; as a result, the theory of meaning it inspired is generally known as the picture theory. While both words cover such things as images, film frames, drawings, and paintings, the idea of a three- dimensional model is more readily conveyed by the German 'Bild' than the English 'picture...it is important not to be misled: the theory involves generalizing from what models, pictures, and the like are supposed to have in common, and treats two-dimensional pictures as just one kind of Bild' (Stern 1995, 35-36).

'A picture presents...the existence and non-existence of states of affairs'(2.11). 'A picture is a model of reality'(2.12), and 'in a picture objects

have the elements of the picture corresponding to them'(2.13). 'In a picture the elements of the picture are the representatives of objects'(2.131). 'What constitutes a picture is that its elements are related to one another in a determinate way'(2.14).

Most importantly, thinking of a state of affairs means modelling it to yourself. 'A *Bild* is a model of reality as we imagine it (4.01). 'A logical *Bild* of the world is a thought.' We communicate our thoughts. 'We use the perceptible sign of a proposition (spoken or written, etc.) as a projection of a possible situation. The method of projection is to think of the sense of the proposition'(3.11). 'A proposition is a propositional sign in its projective relation to the world'(3.12). It has that projective relation because of our thinking it, modelling it to ourselves. 'A propositional sign applied and thought out is a thought'(3.5), and 'a thought is a proposition with a sense'(4). 'The totality of propositions is language'(4.0001).

Propositions are put together 'by way of experiment'(4.031). Instead of 'this proposition has such and such a sense, we can say, this proposition represents such and such a situation.'(4.03311) 'One name stands for one thing, another for another thing, and they are combined with one another. Names as signs for concrete objects, objects such as tables, chairs, and books standing in relation to one another, for the essence of a propositional sign is very clearly seen if we imagine one composed of spatial objects (such as tables chairs and books) instead of written signs. Then the spatial arrangement of the things will represent the sense of the proposition' (3.1431).

Wittgenstein's insight into the *Bild* theory famously came in the context of his reflections on the use of a scale model of an accident in a Parisian law court. Professor Von Wright recounted the story as Wittgenstein described it to him.

It was in the autumn of 1914 on the Eastern Front. Wittgenstein was reading in a magazine about a lawsuit in Paris concerning an automobile accident. At the trial, a miniature model of the accident was presented before the court. The model here served as a proposition, that is as a description of a possible state of affairs. It had this function owing to a correspondence between the parts of the model (the miniature houses, cars, people) and things (houses, cars, people) in reality. It now occurred to Wittgenstein that one might reverse the analogy and say that a proposition serves as a model or picture, by virtue of a similar correspondence between its parts and the world (Von Wright 1984).

He thought this through in the Notebooks as follows:

The general concept of the proposition carries with it a quite general concept of the co-ordination of proposition and situation: The solution to all my questions must be *extremely* simple.

In the proposition a world is as it were put together experimentally (As when in the law-court in Paris a motor-car accident is represented by means of dolls, etc.) [Cf. 4.031.]

This must yield the nature of truth straight away (if I were not blind).

Let us think of hieroglyphic writing in which each word is a representation of what it stands for. Let us think also of the fact that *actual* pictures of situations can be *right* and *wrong*. [Cf. 4.016.]

If the right-hand figure in this picture represents the man A, and the left-hand one stands for the man B, then the whole might assert, e.g.: 'A is fencing with B'. The proposition in picture writing can be true and false. It has a sense independent of its truth or falsehood. It must be possible to demonstrate everything essential by considering this case.

It can be said that, while we are not certain of being able to turn all situations into pictures on paper, still we are certain that we can portray all *logical* properties of situations in a two-dimensional script.

This is still very much on the surface, but we are on good ground...

It can be said that in our picture the right-hand figure is a representation of something and also the left-hand one, *but* even if this were not the case, their relative position could be a representation of something. (Namely a relation.)

Figure 3: Fencers graphic (Wittgenstein 1961)

In a proposition, we combine names (representing objects) to represent concretely three-dimensional states of affairs. To put propositions together experimentally, we construct their sense and then test them against reality to see if we have a match. They answer yes or no, true or false, in order to represent reality. This is their truth-value, and a *Bild* that captures a possible logical form that exists, that is realised as a structure of a state of affairs, has represented something true about reality. 'It reaches right out to it' (2.1511) . . . 'it is laid against reality like a measure' (2.1512).

Figure 4: 'When a Cyberstar Is Born' the first image in *Beautiful Evidence* (Tufte 2006, 12)

4. TUFTE AND WITTGENSTEIN: 'ESCAPING FLATLAND'

The mapping relation is central to understanding and representing in Tufte's theory of visual displays. His data displays function as propositional signs in Wittgenstein's sense. Elements in propositional signs map onto reality as names representing objects and are related to one another in determinate ways, accurately representing complex states of affairs. For Wittgenstein, propositions are complex. They are constituted of many layers capable of ever more concrete analysis, and they must have a complete and determinate analysis, down to the level of the elementary propositions. Tufte would say they are contextual and multivariate. Propositional signs accurately represent actual states of affairs, with all their complexity and density of information. They map to reality as true or false, and that establishes their truth-value. Understanding propositions thoroughly requires careful analysis.

For Tufte, accuracy, high-resolution density of represented information, depth of insight into the core structures and relationships at the heart of the represented situation/state of affairs, and clarity of communication of ideas are some of the key characteristics that drive the quality and effectiveness of evidence presentations. They can make powerful and perspicuous arguments, and they can deepen our understanding of situations as disparate as the spread of the Sars virus or the tragic Napoleonic campaign of 1812 as portrayed by Marey. In that beautifully mapped graphic, for example, one can both see at glance the tragic loss of so many lives, and reason back to the causes. A simple comparison of the width of the bands representing the size of the army at the beginning and end of the campaign captures the pathos of the

ruthless destruction of human life, and that only begins to tell the story embedded in this data-rich

Figure 5: E. J. Marey, *La Methode Graphique dans les Sciences Experimentale* (Paris, 1878), (Tufte 2006, 123-24).

design. Serious visual displays give that level of insight to careful consumers. Above all, they help us think through the data to understand the represented state of affairs, for 'the point of evidence displays is to assist the thinking of producer and consumer alike.' Thus, another powerful metaphor is important in Tufte's thought: 'The metaphor for evidence presentations is analytical thinking' (Tufte 2006). The rules of analytical thinking are universal, and they cover the logic of all of our languages.

Wittgenstein would have liked Tufte's questions.

The world is complex, dynamic, multi-dimensional; the paper is static, flat. How are we to represent the rich visual world of experience and measurement on mere flatland? To envision information...is to work at the intersection of image, word, number, art...and the standards of quality are those derived from visual principles that tell us how to put the right mark in the right place (Tufte 1983).

Putting the right mark in the right place, and using only those marks necessary to communicate data, is critical to the quality and truthfulness of visual displays. The marks are the elements of evidence displays as propositional signs. Presenting the relationships among those marks as accurately as possible constitutes the truth-value of data displays.

This insight informs Tufte's principle of data-ink ratio. *Data-ink* is the non-erasable core of a graphic, the non-redundant ink arranged in response to variation in the numbers represented.

$$\text{Data-ink ratio} = \frac{\text{data-ink}}{\text{total ink used to print the graphic}}$$

Figure 6: Data-ink ratio (Tufte 1983)

The goal is to 'maximise the data-ink ratio, within reason' and 'erase non-data ink, within reason' (Tufte 1983). Denser and more perspicuous information enhances the truth-value of a visual display. Observing this principle calls for the inclusion of more high-resolution, significant data and less chartjunk masquerading as data. This is critical to the quality of the visual display as evidence. Sins against this and related principles, such as distorting data, presenting it in misleading ways, cherry picking data, or marring communication with non-representing chartjunk diminish the truth-value of data displays.

Science plays a prominent role in both of their theories. It is the paradigm for both Wittgenstein's propositions and Tufte's evidence presentations. Tufte's gold standard for data display is the best scientific journals, particularly *Nature* or *Science*. He would say that scientists and engineers deal with facts, and no one gets their own facts. One of his best sources of elegantly crafted visual arguments is the work of Galileo. Galileo embedded arguments concerning sun spots, Jupiter's satellites, mountains and craters on the moon etc., in his drawings. According to Tufte,

Figure 7: Galileo's Scientific Drawings (Tufte 1990)

Galileo displayed his arguments and evidence; he showed their truth. Wittgenstein's most famous doctrine from the *Tractatus* is the importance of what can only be shown and not said. 'The proposition is a picture of reality, for I know the state of affairs presented by it, if I understand the proposition. And I understand the proposition, without its sense having been explained to me'(4.021). 'The proposition shows its sense. The proposition shows how things stand, if it is true. And it says, that they do so stand'(4.022). In Tufte's words, 'Graphics reveal data' (Tufte 1983).

In a parallel instance, Tufte considers the function of 'Links and Causal Arrows,' analysing the diagram on the cover of the 1936 exhibition on *Cubism and Abstract Art*. Alfred Barr created a flow chart for the cover that acted as a 'didactic Art History genealogy...paths of artistic influence---the verbs of the analysis--- are depicted by 51 arrows'

mapped on a grid of time, with the arrows representing causal claims of influence.

Tufte removes the arrows in the course of his analysis to make a point, impoverishing the information display. Scattered, unconnected nouns are left, and the sense of the original evidence presentation is lost, along with the significance of the flow chart. As Wittgenstein says, 'states of affairs can be described but not named. (Names resemble points; propositions resemble arrows, they have sense.)' (3.144).

Figure 8: Comparison of Cubism and Abstract Art flow charts (Tufte 2006, 66)

As the conclusion to this discussion of links and arrows, Tufte asks how Galileo would have handled linking lines. 'This 1610 drawing shows carefully articulated links, as detailed annotation describes and delineates most links. We should do as well.' (Tufte 2006)

Figure 9: Theorica specula concavi sphaerici by Galileo (Tufte 2006)

You can think and write in pictures, quite literally. Galileo did, and so did Wittgenstein, integrating image and text in the body of their respectively scientific and philosophical investigations. A sophisticated development of this incorporation of image into text is the *dataword*, for which Tufte coined the term sparkline. Tufte's sparklines are 'small, high-resolution graphics, usually embedded in a full context of words, numbers, images. Sparklines are *datawords*' (Tufte 2006)

ta imperfezione dello strumento, o dell'occhio del riguardante, perché facendo la figura di Saturno così , come mostrano alle perfette ville i perfetti strumenti, dove manca tal perfezione apparisce così non si distinguendo perfettamente la separazione, e figura delle tre itelle; ma io che mil-

Figure 10a: Galileo's visual noun (Tufte 2006)

Figure 10b: Wittgenstein's graphic (Wittgenstein 1914)

'The wordlike qualities of sparklines create the wonderful possibility of *writing with data graphics*.' (Tufte 2006) Elements of data displays, such as the whiskers in the following sparkline/*dataword* where upward whiskers represent wins and downward whiskers represent losses, are elements of the propositional sign, showing the outcomes of the course of the baseball season.

Figure 11: Sparkline for 'win/loss sequence for the season's first 39 games' (Tufte 2006)

The virtues or vices of such evidence presentations can change the world for the better or worse, for the results of failure can be serious. The truth can be liberating, increasing our understanding and ability to navigate in the world, but flawed, dishonest, or misleading evidence displays can lead to serious errors of judgment on the part of those making decisions. Failure of visual representations can be tragic in practice, as Tufte's critique of the thin, misleading data displays in PowerPoint presentations that crippled communication during the Columbia space shuttle tragedy shows. Underlining the realism of mathematical drawings in 'Wittgenstein and the Mind's Eye,' I wrote

What is important for the Tractatus...is the concrete character of the manner in which points on the surfaces of figures...are connected to real world figures. When the subject of these drawings is a sophisticated invention that can fail, or a working bridge, the realism of these elegant pictures is immediately obvious. A faulty invention won't work. Wittgenstein once drew sketches for a machine based on a principle that had occurred to his father when he was a young boy. The principle was faulty, and the machine would not have moved. When force is added, as in graphical statics, it is even more concrete. Mistakes might mean that someday a bridge could collapse with your car on it (Hamilton 2001).

This principle is even more poignant in Tufte, given his critique of PowerPoint and the space shuttle Columbia tragedy. The failure to visualise and communicate effectively contributed to tragically flawed decisions and the death of the Columbia astronauts. As Tufte argues,

There are right ways and wrong ways to show data, there are displays that reveal the truth and displays that do not. And, if the matter is an important one, then getting the displays of evidence right or wrong can possibly have momentous consequences' (Tufte 1997).

An example of a flawed evidence presentation/*Bild* needing rethinking comes from the heart of Wittgenstein's thought. The physicist Heinrich Hertz's *The Principles of Mechanics* was an important influence on Wittgenstein's thinking. One particular quote from it resonated for him philosophically throughout his life.

we have accumulated around the notions 'force' and 'electricity' more relations than can be reconciled with themselves . . . We have an obscure feeling of this and want to have things cleared up. Our confused wish finds expression in the confused question as to the nature of force and electricity.... It is not by finding out more and fresh relations it can be answered; but by removing the contradictions between those already known (Hertz 1899).

Hertz was troubled by the obscurity surrounding the notion of force in the canonical axiomatizations of mechanics. More relations had accumulated around the notion of force than could be reconciled. In Tufte's terms, there was too much confusing non-representing data ink. Wittgenstein could quote this famous passage by heart his entire life: 'When these painful contradictions have been removed...our minds, no longer vexed will cease to ask illegitimate questions' (Hertz 1899). In the *Principles*, Hertz constructs his own *bildliche Darstellungen* of mechanics, eliminating those flaws in the representations of mechanics.

As the foundation of the physical sciences, mechanics describes reality, however indirectly. Wittgenstein expresses this as follows:

6.341...Mechanics determines the form of the description of the world by saying: all propositions in a description of the world must be capable of being got in a given way from a number of given propositions---the axioms of mechanics. In this way it supplies the stones for building up natural science and says: Whatever building you want to erect you must construct it somehow with these and only these stones.

He uses a mesh analogy to describe mapping the propositions of mechanics to reality, mirroring Tufte's discussion of the effectiveness of grid maps.

6.341 Newtonian mechanics, for example, imposes a unified form on the description of the world. Let us imagine a white surface with irregular black spots on it. We then say that whatever kind of picture these make, I

can always approximate as closely as I wish to the description of it by covering the surface with a sufficiently fine square mesh, and then saying of every square whether it is white or black. In this way I shall have imposed a unified form on the description of the surface. The form is optional, since I could have achieved the same result by using a net with a triangular or hexagonal mesh. Possibly the use of a triangular mesh would have made the description simpler: that is to say that we might be able to describe the surface more accurately with a coarse triangular mesh than with a fine square mesh (or conversely), and so on. The different nets correspond to different systems for describing the world...

Analyzing grid-square maps and mesh maps, Tufte notes the improvement in the quality of information conveyed by these more fine-grained, perspicuous maps in contrast to conventional blot maps. This is shown by the following two maps representing the relative densities of the Japanese population.

Figure 13a: Grid-square map of population census results in Japan (Tufte 1990) **Figure 13b:** Mesh map of population census results in Japan (Tufte 1990, 41)

These examples exemplify the principles of seeing and knowing with which we are concerned. Tufte reflects that

Science and art have in common the intense *seeing*, the wide-eyed observing that generates empirical information. *Beautiful Evidence* is about how seeing turns into showing, how empirical observations turn into explanations and evidence. The book identifies excellent and effective methods for showing evidence (Tufte 2006).

These principles are universal, dealing with the logic of all our languages, for 'principles of evidence display are derived from universal principles of analytical thinking.' The quality of our visual thinking and representations affects the truthfulness and integrity of our understanding, and both of these thinkers are deeply concerned with the integrity of thinking and representing.

Making an evidence presentation is a moral act as well as an intellectual activity. To maintain standards of quality, relevance, and integrity for evidence, consumers of presentations should insist that presenters should be held

intellectually and ethically responsible for what they show and tell (Tufte 2006).

Wittgenstein famously described the *Tractatus* as an 'ethical deed.' Janik and Toulmin argue that the *Tractatus* was part of the Krausian critique of language, indeed that Wittgenstein's work reflects the crisis of integrity of all our means of representation in *fin-de siècle* Vienna. These ideas have deep moral implications.

If the *Tractatus* is not understood as a whole, its point will be lost. This is Wittgenstein's view in the famous passage in his letter to Ludwig Ficker.

The book's point is an ethical one. I once meant to include in the preface a sentence which is not in fact there now, but which I will write out for you here, because it will perhaps be a key to the work for you. What I meant to write, then, was this: My work consists of two parts: the one presented here plus all that I have not written. And it is precisely this second part that is the important one. My book draws limits to the sphere of the ethical from the inside as it were, and I am convinced that this is the ONLY rigorous way of drawing those limits (Janik and Toulmin 1973)

The important part is what was not said. This makes the *Tractatus* self-exemplifying, as the book embodies the ideas written about. "There are, indeed, things that cannot be put into words. They make themselves manifest'(6.522). They can only be shown.

Given the accumulated force of these examples and arguments, and many more could be given, I would argue that Tufte's information displays are propositional signs in Wittgenstein's sense. What would Wittgenstein think of a proposition sign as a visual display, say a chart or a diagram? It is perhaps best to look to the *Tractatus* to answer that question. One of Wittgenstein's most famous and frequently used contributions to logic is the truth table, first presented in the *Tractatus*.

4.442 For example, the following is a propositional sign:

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Figure 14: Truth table as a propositional sign

There are quite a few tables and diagrams presented as propositions in the *Tractatus*. Wittgenstein immediately uses the visual display of 4.462 to analyze other representations for thinking about the material, and to explain why one of Frege's 'logical marks' is unnecessary, eliminating more non data ink.

The following is a gallery of Wittgenstein's visual displays as propositions in the *Tractatus*.

- 4.31 The truth-possibilities can be presented as schemata of the following kind ("T" means "true", "F" false). The rows of T's and F's under the row of the elementary propositions show their truth-possibilities in an easily intelligible symbolism.

P	Q	R	P	Q	R
T	T	T	T	T	T
T	T	T	T	T	F
T	T	T	T	F	T
T	T	T	T	F	F
T	T	F	T	T	T
T	T	F	T	T	F
T	T	F	T	F	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	T	T	T	F
T	F	T	T	F	T
T	F	T	T	F	F
T	F	F	T	T	T
T	F	F	T	T	F
T	F	F	T	F	T
T	F	F	T	F	F

- 4.4 A proposition is the expression of agreement and disagreement with the truth-possibilities of the elementary propositions.
- 4.41 The truth-possibilities of the elementary propositions are the conditions of the truth and falsehood of the propositions.
- 4.411 It seems possible even at first sight that the introduction of the elementary propositions is fundamental for the comprehension of the other kinds of propositions. Indeed the comprehension of the general propositions depends *wholly* on that of the elementary propositions.

Figure 15: Truth table as propositional sign

- 5.101 The truth-conditions of every number of elementary propositions can be written in a schema of the following kind:

TTTT(p,q,r) translates	if p then q and if q then q = (p → q) & (q → q)
FTTT(p,q,r) translates	Not both p and q, [~(p & q)]
TTFT(p,q,r) "	If q then p, [q → p]
TTTF(p,q,r) "	If p then q, [p → q]
TTFF(p,q,r) "	p & q, [p & q]
FTFT(p,q,r) "	Not q, [~q]
FTTF(p,q,r) "	Not p, [~p]
FTFF(p,q,r) "	p & q, but not both, [(p & q) & ~(p & q)]
FFTT(p,q,r) "	If p then q and if q then p, [(p → q) & (q → p)]
FFTF(p,q,r) "	p
FFFT(p,q,r) "	q
FFFT(p,q,r) "	Neither p nor q, [~(p & q) & ~(q & p)]
FFTF(p,q,r) "	p and not q, [p & ~q]
FFFT(p,q,r) "	q and not p, [q & ~p]
FFFT(p,q,r) "	p and q, [p & q]
FFFT(p,q,r) "	Characterization: p and not p and q and not q, [(p & ~p) & (q & ~q)]

These truth-possibilities of the truth-arguments, which with the proposition, I shall call its *truth-grounds*.

Figure 16: A schema as propositional sign

- 5.6361 For the field of sight has not a border like this:

Figure 17: Image integrated in the text of the *Tractatus*

- 5.6363 In order to recognize a number as such, we run, in cases in which no sign of generality occurs in the language, make use of the following intuitive method: I write instead of "p", "q", "r", etc., " $T_1 p T_1$ ", " $T_2 p T_2$ ", " $T_3 p T_3$ ", etc. The truth-conditions I express by brackets: r, q

and the co-ordination of the truth or falsity of the whole proposition with the truth-conditions of the truth-arguments is done in the following way:

This sign, for example, would therefore present the proposition $p \supset q$. (Now I will proceed to inquire whether such a proposition as $\sim(p \supset q)$ [The Law of Contradiction] is a tautology. The letter " $\sim$ " is written in our notation.)

the form " $\sim(p \supset q)$ " thus:

Figure 18: Graphic as propositional sign

- 5.63611 The Knutian problem of the right and left hand which cannot be made to cover one another already exists in the plane, and even in one-dimensional space where the two congruent figures a and b cannot be made to cover one another without moving them out of this space. The right and left hand are in fact completely congruent. And the fact that they cannot be made to cover one another has nothing to do with it.

A right-hand glove could be put on a left hand if it could be turned round in four-dimensional space.

Figure 19: dataword

Hence the proposition $\sim(p \supset \sim q)$ thus thus:

Figure 20: Propositional sign

This perhaps explains that the figure

can be seen in two ways as a cube and all similar phenomena. For we can see two different facts.

At first we see first on the corner and only glance at the corners in front and behind and vice versa.

Figure 21: Figure as propositional sign (5.5423)

5. BEAUTIFUL EXPLANATIONS

In conclusion, the argument presented from 4.01 to 4.015 should now fall into place. For, 'A proposition is not a blend of words----(Just as a theme in music is not a blend of notes)' (3.141). Identifying propositions with musical themes is significant, for this is a Wittgenstein, whose relationship to music was integral to his life, using music to display principles at the heart of the logic of depiction.

4.01 A proposition is a picture of reality. A proposition is a model of reality as we imagine it.

4.011 At first sight a proposition---one set out on the printed page, for example---does not seem to be a picture of the reality with which it is concerned. But neither do written notes seem at first to be a picture of a piece of music, nor our phonetic notation (the alphabet) to be a picture of our speech. And yet these sign languages prove to be pictures, even in the ordinary sense of what they represent.

4.012 It is obvious that a proposition of the form 'aRb' strikes us as a picture. In this case the sign is obviously a likeness of what is signified...

4.014 A gramophone record, the musical idea, the written notes, and the sound waves, all stand to one another in the same internal relation of depicting that holds between language and the world. They are all constructed according to a common logical pattern.

4.0141 There is a general rule by means of which the musician can obtain the symphony from the score, and which makes it possible to derive the symphony from the groove on the gramophone record, and, using the first rule, to derive the score again. That is what constitutes the inner similarity between these things which seem to be constructed in such entirely different ways. And that rule is the law of projection which projects the symphony into the language of

musical notation. It is the rule for translating this language into the language of gramophone records.

4.015 The possibility of all imagery, of all our pictorial modes of projection, is contained in the logic of depiction.

The law of projection enables us to translate from the gramophone record, to the musical idea, to the written notes, to the sound waves in the air, because what is projected is the logical form, through the internal relation of depicting. The most beautiful instantiation of this principle I have encountered was in Tufte's workshop, where the notes of Chopin's 'Berceuse' floated through the air as the sound waves were dynamically projected in a diagram on the screen.

Frédéric Chopin, *Berceuse*, opus 57, depicted by Stephen Malinowski's excellent Music Animation Machine

Figure 22: 'Frederic Chopin's *Berceuse*, opus 57, depicted by Stephen Malinowski's excellent music animation machine' another projected propositional sign of the musical theme (Tufte 2006, 46)

6. REFERENCES

- Ferguson, E. (1992) *Engineering and the Mind's Eye*. Cambridge, Mass. MIT Press.
- Feynman, R. (1988) *What Do You Care What Other People Think?* W. W. Norton, New York.
- Hamilton, K. (2001) 'Some Philosophical Consequences of Wittgenstein's Aeronautical Engineering Research.' *Perspectives on Science*, Vol. 9, No. 1, Pages 1-37, MIT Press, Cambridge.
- Hamilton, K. (2001) 'Wittgenstein and the Mind's Eye.' In James Klagge (ed.), *Wittgenstein: Biography and Philosophy*. Cambridge University Press, Cambridge.
- Hamilton, K. (2002) 'Darstellungen in *The Principles of Mechanics* and the *Tractatus*: The Representation of Objects in Relation in Hertz and Wittgenstein.' *Perspectives on Science*, Vol. 10, No. 1, Pages 28-68, MIT Press, Cambridge.
- Hertz, Heinrich. (1899). *The Principles of Mechanics*. Translated by D. E. Jones and J. T. Walley. Dover Publications, Inc., New York.

- Hindle, B. (1981) *Emulation and Invention*. New York University Press, New York.
- Janik, A. and S. Toulmin. (1973) *Wittgenstein's Vienna*. Simon and Schuster, New York.
- McGuinness, B. (1988) *Wittgenstein: A Life*. University of California Press, Berkeley.
- Pye, D. (1978) *The Nature and Aesthetics of Design*. Herbert Press, London.
- Stern, D. (1995) *Wittgenstein on Mind and Language*. Oxford University Press, Oxford.
- Tufte, E. (1983) *The Visual Display of Quantitative Information*. The Graphics Press, Cheshire, Connecticut.
- Tufte, E. (1990) *Envisioning Information*. The Graphics Press, Cheshire, Connecticut.
- Tufte, E. (1997) *Visual Explanations: Images and Quantities, Evidence and Narrative*. The Graphics Press, Cheshire, Connecticut.
- Tufte, E. (2006) *Beautiful Evidence*. The Graphics Press, Cheshire Connecticut.
- Von Wright, G. (1984) *A Biographical Sketch*. In

Norman Malcolm (ed.), *Wittgenstein: A Memoir*. Oxford University Press, Oxford.

Wittgenstein, L. (1961) *Notebooks 1914–1916*. Edited by G. E. M. Anscombe and G. H. von Wright. Basil Blackwell, Oxford. University of Florida online edition.

Wittgenstein, Ludwig. (1922/2010) *Tractatus Logico-Philosophicus*. Routledge, London and Gutenberg Project online.

ENDNOTES

1. For much greater detail, including an in-depth account of his schedule of courses and contemporary German engineering education, see my 'Wittgenstein and the Mind's Eye.'
2. For a detailed account of this research community and Wittgenstein's aeronautical research, see my 'Some Philosophical Consequences of Wittgenstein's Aeronautical Research.'
3. For a detailed account of this argument, see my 'Darstellungen in *The Principles of Mechanics* and the *Tractatus*: The Representation of Objects in Relation in Hertz and Wittgenstein.'