IS “WALRAS’ LAW” REALLY WALRAS’S ORIGINAL LAW?

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Abstract: This article shows that “Walras’ Law,” which is one of the crucial foundations of modern economic theory as formulated by Lange, and modified by the modern authors, differs essentially from Walras’s own original laws. These two versions of “Walras Law” are disconnected from real economics; nevertheless, unfortunately, they replaced Walras’s original laws which are compatible with a hypothetical regime of perfectly free competition.

Key words: “Walras’ Law”; Walras’s own laws; Walras; Lange; Arrow and Debreu; General Equilibrium Theory

1. Introduction

“Walras’ Law” has played a significant role in modern mathematical economics. It is one of the crucial components used when proving the existence of equilibrium, which is one of the main achievements of the modern General Equilibrium Theory (GET) (Arrow 1989; Ingrao and Israel 1990; Weintraub 1985). Furthermore, this law is more and more frequently used in macroeconomic theory (Patinkin 1989; Barro 1997; Mas-Colell et al. 1995). Clower used it “to demonstrate” Keynes’s contribution to economic theory in his influential paper (Clower 1965, see also Greenfield 1986; Rhodes 1984; Yeager and Rabin 1997). Moreover, Morishima (1977: 48) alleged that Walras’s General Equilibrium Theory does not recognize “Walras’ Law.”

It is important to note that Walras outlined not only one law, but a system of laws in his book *Theory of Pure Economics*. These laws have an evolutionary

dimension, from a low level economy such as exchange to a high level economy, such as circulation and money. Unfortunately, neither Lange, who was the first economist, to our knowledge, who suggested using the term "Walras' Law" in economic literature, nor other economists, including modern authors, took into account this system of laws, thereby, in our view, causing much harm to economic science (see below). Moreover, even in the specialized writings on Walras's theory his laws are not mentioned at all, and if they are mentioned this is in the meaning of "Walras' Law" (Walker 1996; Van Daal and Jolink 1993).

Therefore, in this article we will discuss the relationship between Walras's original laws and "Walras' Law"—both Lange's version and the version used by modern economists. In particular, the economic interpretation of the latter will be considered.

In the following section, Walras's common method and his approach's relevant assumptions are briefly considered. In the third section, Walras's original system of laws is discussed from various angles when it is used in the exchange economy. The fourth section will take a general look at the way "Walras' Law" is implemented in the other three economies. The fifth section describes Lange's interpretation of "Walras' Law." The way modern authors interpret "Walras' Law" is considered in the sixth section. Finally, conclusions are presented.

2. Walras's Method and Assumptions in Walras's Approach

2.1 Walras's method

Walras used a common method of equilibrium establishment and re-establishment of equilibrium (variation of prices) for all four types of economies: (i) exchange, (ii) production, (iii) capital formation and credit, and (iv) circulation and money. Each economy is characterized by its specific markets, but the markets of previous levels of the economy are included in the market system of the economy in question. Walras's method is characterized by the following. First, he discussed the problem of equilibrium establishment for given basic data such as utility functions for each individual economy and the available quantities of goods and services. The determination of demand and offer of goods and services for each individual economy for a random price system starts the process. By the aggregation of the results of the solution of the individual models, the total demand and the total offer of goods and services are obtained. In this stage, Walras first formulated a model, a simultaneous equation system, for the equilibrium state. He then described the process of equilibrium establishment of the working (disequilibrium) model, which he did by using equation systems where the number of unknowns is larger than the number of equations. This was accomplished by means of his well-known algorithm—*tâtonnement*, which transforms the initial disequilibrium model into

a final equilibrium model; and which is characterized by the specific properties of each type of economy.

In all processes for each economy Walras concentrated on the *Law of Equilibrium State Establishment*. However, *the Law of the economy in question relates only to the new markets at this level of economy, and the laws of the previous levels of the economy are automatically included*. For example, the law of capital formation and credit relates only to new capital goods, saving, investment and the rate of income. But the laws of equilibrium for the consumers' goods and services from the previous types of economy (exchange and production economies) are integrated within the law of this type of economy, namely, the capital formation and credit economy.

Finally, Walras discussed the problem of a variation of prices, or the problem of the re-establishment of equilibrium, as a result of changes in the given basic data for any individual or any group of individuals and consequently formulated *the law of equilibrium re-establishment for all types of economy*.

2.2 Relevant assumptions in Walras's approach

Walras's general equilibrium approach seems to be "conveniently" characterized by the full employment of services (Morishima 1977: 58; Negishi 1979: 17). This is the result of the incorrect claim of post-Walras authors since Pareto that Walras reduced the problem of equilibrium existence to the counting and comparison between the number of equations and unknowns. Moreover, they interpreted his approach as if it was characterized by the clearing of markets. However, this is incorrect since Walras's approach assumes that at equilibrium state, there might be a voluntary unemployment of services in economies:¹ production, capital formation and credit, and circulation and money, and unsold goods in an exchange economy (Davar 1994: 51–52; 2002b). These latter statements are based on a careful analysis of Walras's approach. Walras assumed, fortunately, a number of assumptions in his *Elements* (1954), in addition to his well-known assumptions: free competition and uniform prices, which until today have been ignored by post-Walras economists and which enable us to achieve an equilibrium state with voluntary unemployment and positive prices. Let us, consider the relevant assumptions and definitions in his theory:

1. Walras assumed that the total demand function for commodities—as well as demand functions for individuals—is a strictly decreasing function (Walras 1954: 466); and the offer function of services first strictly increases and then strictly decreases (Walras 1954: 467).
2. Walras assumed that demand and supply curves for an individual may be either continuous or discontinuous, while for total demand and supply curves, they should always be continuous (Walras 1954: 95).

3. Walras determined the effective supply and effective demand as "a definite amount of a commodity at a definite price" (Walras 1954: 84, 85). This means that for both demand and supply, for a particular quantity, there is only one price, and vice versa.
4. Walras determined the state of equilibrium by comparing the effective demand and offer of a commodity: "We have now to make three suppositions according as the demand is equal to, greater than, or less than the offer." In the first case "The market is in a stationary state or in equilibrium," while in the second and third cases "the market is in disequilibrium" (Walras 1954: 85).
5. Walras stated that the demand and the offer curves are bound by an available quantity as mentioned above in both cases: for the individual and the total (Walras 1954: 166, 171). This means that, at equilibrium, if it exists, demand and offer always has to be less or equal to the available quantity for all commodities and services.

3. Walras's Original System of Laws for the Exchange Economy

3.1 Individual (micro) economics models of exchange economy

Before describing Walras's model of individual's economy in modern terms (mathematical programming) let us clarify the following points. First, Walras stated that necessary and sufficient data for the establishment of equilibrium position in an exchange economy is: "(1) the traders' utility or want equations for commodities, which can generally be represented by curves and (2) the initial quantities of the commodities in their possession" (Walras 1954: 173). This means that each individual beforehand knows the available quantities of commodities ($q_1, q_2, \dots, q_m$), which he might exchange with other individuals and thus formulates utility functions for every commodity separately ($\Phi_1(q_1), \Phi_2(q_2), \dots, \Phi_m(q_m)$). It is necessary to point out that this data *does not change* during the whole process of equilibrium's establishment. In these conditions, the goal of each individual is the maximum satisfaction of wants by the exchange of commodities for their given prices ($p_1, p_2, \dots, p_m$). In addition Walras assumed that every product might be either offered (negative) or demanded (positive) depending on its available quantity. However, product cannot be offered if an individual does not hold it. So, Walras assumed that every commodity might be either demanded (d_i) or offered (o_i), so that the total value of demand must be equal to the total value of offer. This condition is known as *the budget constraint for individuals* (Walras 1954: 165).

Furthermore, the result of the exchanges is a quantity of commodities which remains with the individual and which is computed as the available quantities plus its demand or the available quantity minus its offer. Also, the offer quantity

for each commodity is bound by its available quantity, which means that the offer quantity is either less or equal to the available quantity for each commodity. So, Walras considered for a certain commodity four types of quantitative magnitude: initial, either demand or offer, and final. It must be stressed, that for the individual economy Walras did not determine excess demand or excess offer. This is because it is impossible to determine them because a certain commodity is either demanded or offered.

Now, let us formulate the model for each individual in the modern terms (in the terms of mathematical programming) for the exchange economy:

$$\text{maximize } \sum_{i=1}^m \Phi_i(x_i), \quad (3.1-1)$$

subject to

$$x_i - d_i + o_i = q_i, \quad (i = 2, 3, \dots, m), \quad (3.1-2)$$

$$x_1 + \sum_{i=2}^m d_i p_i - \sum_{i=2}^m o_i p_i = q_1, \quad (3.1-3)$$

$$x_i, x_p, d_i \text{ and } o_i \geq 0, \quad (i = 2, 3, \dots, m), \quad (3.1-4)$$

where

- x_i is the quantity of commodity i which remains to the individual by the result of exchange and it is calculated as either $(q_i + d_i)$ or $(q_i - o_i)$;
- conditions (3.1-2) indicates that the offer of a certain commodity cannot be more than its available quantity;
- condition (3.1-3) is the budget constraint for an individual, which means that either the offer of or demand for a commodity used as the numéraire (the first, according to Walras's approach) depends on the balance between the total value of demand

$$\sum_{i=2}^m d_i p_i \text{ (expenditure)}$$

and the total value of offer

$$\sum_{i=2}^m o_i p_i \text{ (income)}$$

of the commodities not used as the numéraire. If

$$\sum_{i=2}^m d_i p_i > \sum_{i=2}^m o_i p_i$$

then *the money commodity is offered in order to pay for the excess of the total value of demand* and it is determined as

$$o_1 = (\sum_{i=2}^m d_i p_i - \sum_{i=2}^m o_i p_i).$$

Also the latter cannot be more than its available quantity q_1 . And, if

$$\sum_{i=2}^m d_i p_i < \sum_{i=2}^m o_i p_i$$

then *the money commodity is demanded in order to store (reserve) excess of the total value of supply* and it is determined as

$$d_1 = -(\sum_{i=2}^m d_i p_i - \sum_{i=2}^m o_i p_i).$$

The solution of the system (3.1-1)–(3.1-4), if exists and is unique, allows us to obtain demand quantities of certain commodities and offer quantities of others, which guarantee the maximum utility satisfaction for each individual by the additional conditions (Walras 1954: 164). These results, demand quantities of certain commodities and offer quantities of others, allow us to obtain, consequently, the final quantity of each good (x_i), i.e., the final endowment. Due to the fact that the latter does not take place in the process of equilibrium establishment, it will be omitted in the following discussion. It follows from the structure of the model that if either o_i or d_i is positive, the other would equal zero; as both have the same price and both influence the utility functions indirectly by the final endowment x_i (Hiller and Lieberman 1995: 586–591). In other words, a certain commodity cannot be offered and demanded simultaneously by the same individual. In addition, the offered quantities of each commodity are bounded by the holding quantities (condition (1-2)), when the demand quantities are bounded by the holding quantities of all commodities, i.e., by the budget constrains (condition (1-3)). Therefore, the magnitude for demand (or offer) is a function of prices, available quantities and parameters of utility. This means that the derived (generally) demand and offer function are the results of the theoretical solution of the model of individual economy, that is, the praxis can be termed in consistent and coherent theoretical terms. Equilibrium conditions of the model are identical with Walras's one, namely, the marginal utility of a certain good is equal to the marginal utility of the numéraire multiplied by the price of the good in question (Walras 1954: 165).

So, the derived demand or the derived supply is obtained for each commodity (Davar 2005):

$$o_i = f_i(p_2, p_3, \dots, p_m); \quad (i=2, 3, \dots, m) \quad (3.1-5)$$

$$d_i = f_i(p_2, p_3, \dots, p_m); \quad (i=2, 3, \dots, m) \quad (3.1-6)$$

Such results for all individuals allow the determination of the total quantities of demand and offer for every commodity separately, which are dependent on the prices of all the commodities which are calculated by the means of the aggregation of them:

$$D_i(p_2, \dots, p_m) = \sum_{r=1}^R d_i^r \text{ is the total demand of a commodity } i \text{ (} i=2,3, \dots, m \text{)} \quad (3.1-7)$$

$$O_i(p_2, \dots, p_m) = \sum_{r=1}^R o_i^r \text{ is the total offer of a commodity } i \text{ (} i=2,3, \dots, m \text{)} \quad (3.1-8)$$

$$D_1 - O_1 = -(\sum_{i=2}^m D_i p_i - \sum_{i=2}^m O_i p_i) \quad (3.1-9)$$

where

R is the number of all individuals ($r=1,2,\dots,R$);

These results enable us to discuss problems of equilibrium establishment, which will be considered in the next section.

3.2 Whole economy's (macro) model of exchange economy

In section 3.1 Walras's model for an Individual economy was considered for an exchange economy. This model allows us to determine either demand or supply for each commodity so that a certain individual is guaranteed the maximum satisfaction of his wants. These results for all individuals allow us to determine the total quantities of demand and the total quantity of offer for every commodity separately, which are obtained for certain prices of all commodities by means of the aggregation of them. On the basis of this latter data Walras formulated the model (macro) of the whole economy of the exchange economy (Walras 1954: 168–170). *Walras formulated two versions of a macro model: first, the model of equilibrium state; and second, the model of disequilibrium state.* In the following, Walras showed how the first (equilibrium state) is obtained from the second (disequilibrium state) by the iterative process (algorithm) called *tâtonnement* (see below).

On the basis of essential assumptions (see section 2.2) and what was mentioned above, which states that in an equilibrium state the total effective demand equals the total effective supply, Walras formulated equation systems (macro model) for the determination of equilibrium prices as:

$$D_i(p_2, \dots, p_m) = O_i(p_2, \dots, p_m), \text{ (} i=2,3, \dots, m \text{)} \quad (3.2-1)$$

$$D_1 - O_1 = \sum_{i=2}^m D_i p_i - \sum_{i=2}^m O_i p_i = 0, \quad (3.2-2)$$

But this equilibrium situation is obtained by the iterative process of solution. It is because at the beginning of the process when the total demand and the total supply

are obtained for a random price system there might not be equilibrium state, namely, it might be disequilibrium at least for several number of goods. In general there are:

$$D_i(p_2, \dots, p_m) \neq O_i(p_2, \dots, p_m), (i=2, 3, \dots, m) \quad (3.2-3)$$

$$D_1 - O_1 = \sum_{i=2}^m D_i p_i - \sum_{i=2}^m O_i p_i \neq 0, \quad (3.2-4)$$

Now, if by chance there is equality for all $m-1$ commodities in (3.2-3) for any price system then there is also equality for the numéraire (3.2-4) and the equilibrium is established. However, if even for one certain commodity there is an inequality (either $>$ or $<$), at the beginning of the process of establishment of equilibrium, there are a lot of commodities for which there are inequalities; it poses the question as to how Walras achieved this equilibrium situation. Walras used his famous *tâtonnement* (iterative process) for establishing equilibrium state (Jaffe 1981; Patinkin 1989; Bridel and Huck 2002).

3.3 *Tâtonnement* and the existence of equilibrium in the case of exchange economy

In the section 3.2 it was shown that in a macro model of exchange economy the number of unknowns ($2m-2$) is more than the number of equations ($m-1$), so in order to establish an equilibrium state ($m-1$) unknowns must be given in advance. Therefore, Walras assumed that the prices p_i of ($m-1$) commodities in terms of numéraire (commodity (1)) are stated at random (Walras 1954: 169). Thus, for these given prices each individual determines his demand or offer of all commodities by the micro model where they "were reached by the mathematical solution of the system of equations of demand and offer and of maximum satisfaction subject to suitable constraints" (Walras 1954: 169). As it was shown in the section 3.2, there would be an equilibrium situation if the total demand equals the total offer, obtained by the aggregation of demands and offers of all individuals for each ($m-1$) commodity, and consequentially for commodity (1) too. But, in general, there might be three situations for each commodity: (i) when the total demand is greater than the total offer; (ii) when the total offer is greater than the total demand; and (iii) when the total demand equals the total offer.

In the first situation, in the real market, the price of commodity will increase, but for the second situation, the price will decrease and in the third situation the price will not change. Walras used these *rules* of changes of prices of the real market in his theoretical solution for the equilibrium establishment (Walras 1954: 170).

According to this process of equilibrium establishment, *tâtonnement*, in the exchange economy proof of equilibrium's existence might be divided into two parts: (i) the existence of equilibrium for one certain commodity, which will be

called *partial* equilibrium; and (ii) the existence of equilibrium for all commodities simultaneously, which will be called *general* equilibrium.

The necessary and sufficient conditions, in order to establish partial equilibrium for a certain good in an exchange economy by means of Walras's rule, are: essential assumptions (section 2.2) plus the additional requirement of Walras, i.e., that the total demand curve and the total offer curve (functions) have *at least one intersecting point* (Walras 1954: 108, 171). This means that Walras's rule allows us to establish the partial equilibrium if it exists at all and we can conclude that Walras's *tâtonnement* allows us to prove *rigorously* the existence of the partial equilibrium if it exists.

The existence of partial equilibrium for all commodities *separately* does not guarantee the existence of a general equilibrium, i.e., equilibrium for all commodities simultaneously. It is necessary to stress that Walras stated that the establishment of the general equilibrium has a probable character. This means that Walras indeed did not prove rigorously that the general equilibrium exists. The iterative process of adjustment (*tâtonnement*) suggested by Walras, however, allows the achievement either of equilibrium state or at least an approximate solution.

In addition, if we take into account the fact that the offer of each commodity for any individual cannot be greater than its available quantity for it, we can conclude that in the equilibrium state the total equilibrium quantity $Q_e (=D_e=O_e)$ cannot be greater than its total available quantity Q_0 . This means that

$$Q_{ie} \leq Q_{i0}, \quad (i=1,2, \dots, m), \quad (3.3-1)$$

Therefore, we can conclude that in an equilibrium state there might be at least some of commodities for which $Q_{ie} < Q_{i0}$. If we combine this statement with the previous statement we can conclude that *in Walras's approach, in the equilibrium state of the exchange economy there might be unsold quantities ($Q_{ie} - Q_{i0}$) of some commodities but with positive prices for sold quantities*. This contradicts the modern authors' two statements: first, in this situation the *price must equal zero* (see below); and second, the *market is cleared* in Walras's approach.

3.4 Walras' Law of the establishment of equilibrium and Walras' Law of re-establishing of equilibrium in exchange economy

On the basis of the process of the establishment of equilibrium state described in the previous section, Walras stated (1964: 172):

We are now in a position to formulate the law of the establishment of equilibrium prices in the case of the exchange of several commodities for one another through the medium of a numéraire:

Given several commodities, which are exchanged for one another through the medium of a numéraire, for the market to be in a state of equilibrium or for the

price of each and every commodity in terms of the numéraire to be stationary, it is necessary and sufficient that at these prices the effective demand for each commodity equal its effective offer. When this equality is absent, the attainment of equilibrium prices requires a rise in the prices of those commodities the effective demand for which is greater than effective offer, and fall in the prices of those commodities the effective offer of which is greater than the effective demand.

This law is fundamental for Walras's equilibrium theory. It allows us to conclude that Walras's original *tâtonnement*, the iterative process of equilibrium establishment, is the *theoretical version* of the process of equilibrium establishment of the real markets.

Walras divided the study of economy, as has been mentioned above, into two stages: first, the law of equilibrium establishment for certain data; and second, the law of re-establishing of equilibrium when data are changed. Walras's followers, unfortunately, missed the second stage. Moreover, they blamed Walras for not discussing it at all (Hicks 1965: 61; Ingrao and Israel 1990: 112). Furthermore, Walras asserted that applied economics deals with a situation when conditions of equilibrium (Walras's own laws) are destroyed (Davar 2006).

The necessary and sufficient data in Exchange Economy are: (i) an individual's utility functions for each product separately; and (ii) the initial quantities of the products in their possession. Therefore, they are also the primary causes and conditions of equilibrium's destruction, and in consequence of the variation of equilibrium quantities and prices.

On the basis of demonstration of his approach Walras formulated the Law of variation of commodity prices (or re-establishment of equilibrium) in Exchange Economy (Walras 1954: 180):

Given a state of general equilibrium in a market for several commodities where exchanges take place with the aid of a numéraire, if the utility of one of these commodities increases or decreases for one or more of the parties, everything else remaining equal, the price of this commodity in terms of the numéraire will increase or decrease.

If the quantity of one of the commodities in the hands of one or more holders increases or decreases, all other things remain equal, the price of this commodity will decrease or increase.

At the same time Walras pointed out that there might be changes in the utility and the quantity when the price remains unchanged (Walras, 180).

Walras combined the *law of the variations of equilibrium prices* with the *law of the establishment of equilibrium prices* and called it the *Law of Supply and Demand* (Walras 1954: 180–181). Walras claimed that this fundamental law has hitherto been stated either erroneously or in a form devoid of meaning. Let us not repeat

Walras's interesting discussion about it (Walras 1954: 181), but only stress that the situation has not changed for more than a century.

4. Walras's Original System of Laws for Other Economies

4.1 Walras's original system of laws for production economy

Walras introduced, in the production economy, services (labor, capital and land), that are used not only for the production of commodities, but also for the consumption of individuals. Therefore, the necessary and sufficient data are also extended. First, additional utility functions of services are formulated for each individual. Second, the initial available quantities of services are determined. At the same time, Walras assumed that the initial available quantity of commodities is absent. Therefore, Walras assumed that in a production economy the demand for commodities, obtained from their demand curves, determines their supply. The total demand for any commodity, therefore, is limited by the initial available quantity of services. Also, the demand functions (curves) of services do not exist and their demand are determined by the demand for all commodities, for which these services are employed. Taking into account these facts, Walras generalized the law of the equilibrium establishment for the production economy (Walras 1954: 253–254):

Given several services by means of which various products can be manufactured and assuming that these services are exchanged for their products through the medium of a numéraire, for the market to be in equilibrium, for the prices of all services and all products in terms of the numéraire to be stationary, it is necessary and sufficient (1) that the effective demand for each service and each product be equal to its effective supply at these prices; and (2) that the selling prices of the products be equal to the cost of the services employed in making them. If this twofold equality does not exist, in order to achieve the first it is necessary to raise the prices of those services or products the effective demand for which is greater than the effective supply and to lower the price of those services and products the effective supply of which is greater than the effective demand; and, in order to achieve the second, it is necessary to increase the output of those products the selling price of which is greater than the cost of production and to decrease the output of those products of which the cost of production is greater than the selling price.

And Walras also generalized the law of re-establishment of equilibrium for the production economy in the case of a change in initial data (Walras 1954: 260).

4.2 Walras's original system of laws for capital formation and credit economy

The production economy is extended by two additional categories: first, net income (saving) and its price; and second, new capital goods (investment) and their prices,

in order to pass to the capital formation and credit economy. Therefore, the law of equilibrium establishment here has two additional parts: (1) the equilibrium magnitude of the rate of *income* is determined by the relationship between the effective demand for the new capital goods (the total investment) and the total saving; and (2) the equilibrium quantities of the new capital goods are determined by the relationship between their selling prices and costs of production (Walras 1954: 294–295):

Let there be given several services, from the prices of which it is possible to deduct an excess of income over consumption to be transformed into new capital goods proper, and which can be exchanged against various consumer's goods and various new capital goods through the medium of a numéraire. Then for the market for capital goods to be in equilibrium, or for the prices of all new capital goods in terms of numéraire to be stationary, it is necessary and sufficient: (1) that at selling prices equal to the ratio of net incomes to the current rate of net income, the effective demand for these new capital goods be equal in terms of numéraire to their effective supply; and (2) that the selling prices and costs of production of the new capital goods be equal. When these two equalities do not exist, in order to achieve the first equality, it is necessary to raise the selling prices by lowering the rate of net income in case effective demand is greater than effective supply or to lower the selling prices by raising the rate of net income, in case effective supply is greater than effective demand; and, in order to achieve the second equality, it is necessary to increase the output of those new capital goods the selling price of which is greater than their cost of production and to decrease the output of those new capital goods of which the cost of production is greater than the selling price.

And the law of re-establishment of equilibrium here is also divided into two parts, respectively. The law for the variation of the rate of net income (Walras 1954: 307) and the law of variation of capital goods prices (Walras 1954: 309–310).

4.3 Walras's original system of laws for circulation and money economy

Firstly, Walras formulated the Law of equilibrium establishment for circulation in the case when money is neither a commodity nor anything that can serve as numéraire, i.e., fiat money (Walras 1954: 327): "*The price of the service of money is established through its rise or fall according as the desired cash balance is greater or less than the quantity of money.*"

And then, Walras formulated the Law of equilibrium establishment in the case that the commodity serves as a numéraire and as money (Walras 1954: 331–332):

In the case of a commodity that serves both as money and as numéraire the uniform and identical price of its service as circulating capital and as money is

established by a rise or fall according as the demand is greater or less than the [total existing] quantity; and this price is maintained [the same in both uses] by minting or melting according as the price of its service as a money is greater or less than the price of its service as circulating capital.

Walras discussed the law of variation of the price of money services regarding solely fiat money, assuming that for commodity money which serves both as a numéraire and as money in circulation capital the law is identical to the laws for other commodities, which were established in the previous economies (Walras 1954: 328–329).

5. Lange and “Walras’ Law”

Lange was the first economist, to our knowledge, who, in his well-known paper “Say’s Law: A Restatement and Criticism,” suggested using the term “Walras’ Law” in economic literature. Lange, unfortunately, did not take into account the system of laws formulated by Walras himself (see above) and therefore, in my view, caused huge harm to economic science on various levels. First, Lange’s “Walras’ Law” generally describes disequilibrium and therefore it is an intermediate result of the original Walras’ Law. Leijonhufvud (1981: 99) asserts that “Lange’s terminological innovations—including, in particular, the entirely superfluous term Walras’ Law—somehow have taken root in macroeconomics.” Second, his interpretation is irrelevant to economic reality. Third, his study of Walras has played a crucial role not only in the modern GET for proving the existence of equilibrium (see below), but also even in macroeconomics theory; and in consequence, finally, it “replaced” Walras’s genuine (original) laws and caused them to be unknown and forgotten. Therefore, in this section we will discuss the relationship between Walras’s original laws and Lange’s “Walras’ Law”; yet, the economic interpretation of the latter will be considered.

5.1 Lange’s understanding and interpretation of Walras’s approach in *On the Economic Theory of Socialism*

Lange as well as Walras started by stating that: “It is only prices in the generalized sense which are indispensable to solving the problem of allocation of resources. The economic problem is a problem of *choice* between alternatives. To solve the problem three data are needed: (1) a preference scale which guides the acts of choice; (2) knowledge of the ‘terms on which alternatives are offered’; and (3) knowledge of the amount of resources available” (Lange and Taylor 1938: 60).

Lange’s determination of the conditions of equilibrium is generally similar to Walras (Lange and Taylor 1938: 65–66) however, there are some differences.

Lange distinguished between a theoretical solution of equilibrium establishment and its solution by trial and error, while Walras assumed that they are identical (see

above). This is Lange's main failure. Therefore, in Lange's approach a theoretical solution includes some incompatible assumptions (Lange and Taylor 1938: 68–69).

Despite that Lange did not formulate a mathematical model of the individual economy at this stage one can suppose that he considered here the exchange economy and the establishment of equilibrium is obtained by the solution of a model where all individuals and commodities are combined into one model. Let us point out Lange's two important statements: "finally, prices alone remain as the variables determining demand and supply of commodities" and "If the demand and supply schedules are all monotonic functions there exists only one set of prices which satisfies the objective equilibrium condition; otherwise, there may be a multiple solution, but some of the price sets obtained represent unstable equilibria" (Lange and Taylor 1938: 68–69). Unfortunately, Lange did not discuss a situation where there is no solution at all, as Walras did (see above). Lange demonstrated equilibrium establishment by trial and error (Lange and Taylor 1938: 70).

So, to summarize we can conclude that, in this work, Lange understood and interpreted Walras's approach on the whole correctly. Despite this he did make some errors. He incorrectly defined a free competition market and he erroneously made a distinction between the theoretical solution of equilibrium and by a solution by trial and error. Lange determined correctly the objective conditions of equilibrium stating that one of the conditions is that the equality between the quantity demanded and the quantity supplied must be required for each commodity separately, even though they are interconnected (see also Lange 1944b). Unfortunately, Lange would alter this condition when he formalized these conditions in mathematical form (see below).

5.2 Lange's version of "Walras' Law"

Lange gave the mathematical formalization of his theoretical version of equilibrium establishment from the previous section in his work "Say's Law" (1942). He started by the exchange economy and passed to production and other economies, which were described incompletely and superficially. Lange wrote (1942: 49):

Let us consider a closed system in which n commodities are exchanged, one of them—say, the n th commodity—functioning as medium of exchange as well as numéraire, i.e., as money. Denote by p_i the price of the i th commodity. We have $p_n \equiv 1$. Let $D_i = D_i(p_1, p_2, \dots, p_{n-1})$ and $S_i = S_i(p_1, p_2, \dots, p_{n-1})$ be the demand function and the supply function, respectively, of i th commodity. The equilibrium prices are determined by the $n-1$ equations

$$D_i(p_1, p_2, \dots, p_{n-1}) = S_i(p_1, p_2, \dots, p_{n-1}). \quad (i = 1, 2, \dots, n-1) \quad [(5.2-1)].$$

Some things should be noted here. The mathematical model of an individual's economy is absent, and therefore the character of the demand and supply functions

is not clear, i.e., he does not make a distinction between the primary (original) demand curve (function) and the derived (general) demand curve (function).

Lange continued:

There are only $n-1$ independent demand functions and $n-1$ independent supply functions, the demand and the supply function for the commodity which functions as money being deducible from the other ones. We have

$$\sum_{i=1}^{n-1} p_i D_i \equiv S_n \quad [(5.2-2)]$$

and

$$\sum_{i=1}^{n-1} p_i S_i \equiv D_n \quad [(5.2-3)]$$

Taking account of the last two relationships, we obtain the total demand (measured in money value) for all n commodities

$$\sum_{i=1}^n p_i D_i \equiv \sum_{i=1}^{n-1} p_i D_i + D_n \equiv S_n + D_n \quad [(5.2-4)]$$

Similarly the total supply (measured in money value) of all n commodities is

$$\sum_{i=1}^n p_i S_i \equiv \sum_{i=1}^{n-1} p_i S_i + S_n \equiv D_n + S_n \quad [(5.2-5)]$$

Therefore

$$\sum_{i=1}^n p_i D_i \equiv \sum_{i=1}^n p_i S_i \quad [(5.2-6)]$$

i.e., total demand and total supply are identically equal.

I propose to call this identity *Walras' Law* because Walras was the first to recognize its fundamental importance in the formulation of the mathematical theory of prices. It should be noted that Walras' Law does not require that the demand and supply of each commodity or of any of them be in equilibrium. The identity of (2.7) [(5.2-6)] holds independently of whether the equations (2.1) [(5.2-1)] are satisfied or not. (Lange 1942: 50; see also 52)

There are two crucial differences between Walras's and Lange's approaches. First, what Lange proposed to call "Walras' Law" is the intermediate result of Walras's own approach to equilibrium establishment for the exchange economy that, in the following, is completed by the formulation of the law of equilibrium prices (see above). Walras wrote (1954: 170):

Let us recall that we have the equation

$$X' + Y'p'_b + Z'p'_c + W'p'_d + \dots = 0,$$

which can be written

$$D'_a - O'_a + (D'_b - O'_b)p'_b + (D'_c - O'_c)p'_c + (D'_d - O'_d)p'_d + \dots = 0, \quad [(5.2-7)]$$

.... We observe that, since $p'_b, p'_c, p'_d \dots$ are positive by their nature, if some of the quantities $X' = D'_a - O'_a, Y' = D'_b - O'_b, Z' = D'_c - O'_c, W' = D'_d - O'_d \dots$ are positive, others will be negative, and conversely, if some of these quantities are negative, others will be positive. This means that if at the prices $p'_b, p'_c, p'_d \dots$ the total demand for some commodities is greater (or smaller) than their offer, then the offer of some of the other commodities must be greater (or smaller) than the demand for them.

This statement of Walras seems totally exactly with Lange's "Walras' Law" (see also Lange 1944a: 5–6). However, this is only an intermediate stage, i.e., there might be either equilibrium by chance, or more commonly there may be disequilibria. In the latter situation, therefore, Walras used the iterative process (*tâtonnement*) to establish equilibrium (see above). Also, there are some differences between Walras's intermediate result and Lange's "Walras' Law." In Walras's approach the prices are positive, whereas in Lange's approach the prices might equal zero or may even be negative. In addition, in the law of equilibrium prices in Walras's approach the equality between the effective demand and the effective supply for each commodity separately are required (see above). In fairness, it is necessary to note that Lange himself wrote about this not only in his previous work but also here (Lange 1942: 51n). So, Lange's "Walras' Law," generally, describes disequilibrium state from the point of view of the determination of equilibrium (Walras's approach).

Second, the determination of the demand and supply of the money commodity (numéraire) in Lange's approach differs from Walras's one. In Walras's approach there is equilibrium for the money commodity, i.e., the total demand for money is equal to the total supply of money as long as there is equilibrium for all other commodities separately.

Based upon this the determination of the total demand for and the total supply of the money commodity in Walras's approach differs from their determination in Lange's approach, (5.2-2) and (5.2-3) respectively. It must be stressed that from the point of view of mathematics the results are identical.

In addition, Lange's determination might lead to incorrect statements. First, Lange assumes that there is equilibrium when there exists equality between the excess demands for money with the excess supply of goods (Lange 1944a: 5–6). However,

this is the above-mentioned intermediate result, i.e. where there is disequilibrium in Walras's approach.

Second, from the above it is clear that the *equilibrium quantity of the money commodity in Walras's approach is essentially less than in Lange's approach*.

Third, Lange stated that "The total demand for commodities is equal to the total supply of commodities only in a state of monetary equilibrium," i.e., the equilibrium of all other commodities is derived from monetary equilibrium; what is opposite to Walras's approach.

It is necessary to stress that Lange also tried considering the higher stages of types of economy by an artificial ("the mechanical") expansion of the exchange economy.

6. "Walras' Law" and Modern General Equilibrium Theory

The proof of the existence of equilibrium is considered as the main achievement of modern GET (Arrow 1989; Ingrao and Israel 1990; Weintraub 1985). Unfortunately, however, this proof is based on the so-called free goods conception together with "Walras' Law." This conception means that when there is an excess supply of a commodity (factor) its price equals zero (see below). In other words, in equilibrium, if a certain factor is not fully employed, then its price is zero. For example, if unemployment exists, then wages should zero. However in this case, such a theory contradicts reality. So, if we take into account this rule of free goods then we must conclude that this achievement is meaningless.

6.1 Model of modern (Arrow-Debreu) general equilibrium

Let us consider the Arrow-Debreu model as representative of the modern GET. It is clear that over the last 40 years certain improvements in it have been made, such as the addition of taxation and inter-country trading problems. But the theoretical basis of computable (applicable) GET is still the Arrow-Debreu general equilibrium (GE) model (Arrow and Hahn 1971; Debreu 1959; Shoven and Whalley 1993).

Arrow-Debreu's basic GE model may be described as follows (Arrow and Hahn 1971: 107):

Definition 1. A price vector $\mathbf{p}^*$, a consumption allocation, $\mathbf{x}^*$, and a production allocation, $\mathbf{y}^*$, constitute a *competitive equilibrium* if

- (a) $\mathbf{p}^* > 0$;
- (b) $\sum_h \mathbf{x}_h^* \leq \sum_f \mathbf{y}_f^* + \sum_h \bar{\mathbf{x}}_h$;
- (c) $\mathbf{y}_f^*$ maximizes $\mathbf{p}^* \mathbf{y}_f$ subject to $\mathbf{y}_f \in \mathbf{Y}_f$;
- (d) $\mathbf{x}_h^*$ maximizes $U_h(\mathbf{x}_h)$ subject to $\mathbf{p}^* \mathbf{x}_h \leq \mathbf{M}_h^* = \mathbf{p}^* \bar{\mathbf{x}}_h^2 + \sum_f d_{hf}(\mathbf{p}^* \mathbf{y}_f^*)$.

Condition (a) means that prices of goods and services have to be positive, but some of the prices might be equal to zero, but not all prices [see below];

Condition (b) means that the total produced quantity plus the total existence quantity constrains the total consumption for each good and service;

Condition (c) means that each firm maximizes its profits;

Condition (d) describes a household economy and is a combination of two components whereby the households:

(1) maximize utility functions

(2) are subject to budget constraints, i.e., each household's expenditure have to be either less or equal to its revenues.

A careful analysis of this shows that the Arrow-Debreu model differs from Walras's approach in the following ways: (i) the problem of the establishment of equilibrium is solved for one general model, where all consumers and all producers are concentrated in one model. However, if there is a strong theoretical solution to the Arrow-Debreu model, it is impossible to realize practically even for a small country, as Pareto confessed; (ii) prices are obtained by the model's solution, and they are not chosen from the given interval of prices determined by the demand function for goods and supply functions for services; hence, prices might reach any magnitude, and some prices might even be zero; (3) According to Walras's approach for a situation of equilibrium it is necessary that the selling (demand) price of goods must equal its cost of production (supply price). However this condition in the Arrow-Debreu approach cannot be kept, because the cost of the production of goods is not determined directly. Because of this it is important to note that not only the price of some goods may be equal to zero, but also the price of some services might be zero; (4) the utility function for each household is described as a function of all goods and all services in one function simultaneously, which is opposite to Walras's approach where utility function is described for each good separately.

On the other hand, there are two additional positive properties of the Arrow-Debreu model: first, the maximization of profits for each firm, and second, the distribution of the profits between households that have a share in the firm. However, they become irrelevant because in the Arrow-Debreu approach, prices of some goods and services might be zero (see below).

6.2 Arrow-Debreu's proof of equilibrium's existence

Let us discuss only the relevant assumptions and the methods used to prove the existence of equilibrium in the modern GET.

The first assumption relates to the excess demand. The authors define the excess demand for good i (written z_i) as

$$z_i = x_i - y_i - \bar{x}_i \quad i = 1, \dots, n$$

Where, x_i is summation over households of x_{hi} “be decision of household h with respect to good i .” Then, if $x_{hi} < 0$, we shall say that i is a service supplied by household h ; when x_{hi} is non-negative, then i will be a good demanded by h , where this concept includes zero demand” (Arrow and Hahn 1971: 17; see also Hicks 1946; Davar 2004). Also, y_i is summation over firms of y_{fi} “as the decision of firm f with respects to good i . We regard $y_{fi} < 0$ as denoting that i is an input demanded by f , while y_{fi} non-negative means that i is supplied by f where this concept includes zero supply” (Arrow and Hahn 1971: 17). And finally, $\bar{x}_i$ is summation over households of $\bar{x}_{hi}$ “as an amount of good i owned by household h , and note that for good sense this must be a non-negative quantity” (Arrow and Hahn 1971: 18). So, consequently,

We put this formally:

Assumption 1 (F). To any $\mathbf{p}$ there corresponds a unique number $z_i(\mathbf{p})$ called the excess-demand function for i and so a unique vector of excess-demand functions $\mathbf{z}(\mathbf{p})$. We have $z_i(\mathbf{p}) = x_i(\mathbf{p}) - y_i(\mathbf{p}) - \bar{x}_i$ and call $x_i(\mathbf{p})$ the demand function and $y_i(\mathbf{p})$ the supply function. (Arrow and Hahn 1971: 18)

This is a very crucial assumption since the following assumptions and the whole process of the proof are based on this assumption. First, the excess-demand function is determined by a comparison between the demand function and the supply function plus the available quantity of the good.³ This means that the demand quantity in all cases must be equal to the whole quantity of supply when an according price is strongly positive. But, if the whole supply is greater than the demand what kind of price could there be? Second, like the demand and supply functions, this available quantity is not presented as a function of price. Such a presentation of available quantity means that trade in it does not depend on price. In other words, this means that the available quantity is completely traded for any price. Such an approach is based on the implicit assumption that the part of the available quantity that is not traded is retained for the use of its owner and is considered as if it was traded. This implicit assumption creates the illusion that the available quantity is completely traded. But, trading of the available quantity depends on price as well as the quantity produced. Therefore, it is incorrect to assume that the available quantity is traded wholly for any price. This means that to present the supply side via two separate components, a newly produced quantity depends on price and the available quantity that does not depend on price, is incorrect.

Now let us to skip to Assumption 3 (Arrow and Hahn 1971: 21):

Assumption 3 (W). For all $\mathbf{p} \in S_n$, $\mathbf{p}\mathbf{z}(\mathbf{p}) = 0$ (Walras' law).

Arrow and Hahn give arguments for the third assumption that the available quantity is completely used for any price, as if the existing quantity completely participated in the trade (*vide supra*). Despite this, only part of the available quantity generally, participates in trading. It is necessary to point out that assumption 3—called Walras' law—differs from the law formulated by Walras himself. This is because in Walras's approach, excess-demand is obtained by a comparison between effective demand and effective supply and is determined each time only for concrete situations (*vide supra*). In addition, it is formally identical with "Walras' Law" of Lange, but in the following use there is a significant difference. In order to show this let us consider the following theorem from which is derived the formal definition of equilibrium (Arrow and Hahn 1971: 23):

Theorem 1: If W and $z(p^*) \leq 0$, then $z_i(p^*) < 0$ implies that $p_i^* = 0$

From this we can conclude that there are two possibilities: when the demand is equal to the whole supply then the prices may be strongly positive, and if the whole supply is greater than the demand then the price must be equal to zero. This means that if a certain service is not wholly utilized than its price is equal to zero. In other words, if there is unemployment than the wage must be zero. However, this is not only far from real economics, but it is also incompatible with a hypothetical economics. Despite this, assume that "Walras' Law" is kept. Then, this differs from Lange's approach, where if there is excess demand for a certain good there is excess supply for another good. So, the interpretation of "Walras' Law" in proving of equilibrium existence differs from Lange's and the modern authors' approach. Moreover, they both differ essentially from Walras's own laws.

7. Conclusions

In this article it was shown that "Walras' Law," which is one of the crucial foundations of modern economic theory—both in macro- and microeconomics—as formulated by Lange, and modified by the modern authors, differs essentially from Walras's original laws. Moreover, it is an intermediate stage of Walras's own laws, and there is a significant difference also between Lange's and modern authors' versions of "Walras' Law."

First, Walras formulated two systems of law for the framework of the system of assumptions: (i) the law of equilibrium establishment for each type of economy for the initial given data, so that the law for the economy in question related only to the new markets that entered in this level of economy, and laws of the previous level of the economy are automatically included; (ii) the law of variation of prices, or the law of the re-establishment of equilibrium, as a result of changes in the given date for any individual or any group of individuals for each type of economy. In

contrast to this Lange formulated one “Walras’ Law” which is common for all types of economy.

Second, in Walras’s approach equilibrium is achieved for each commodity separately connected to equilibrium for other commodities when effective demand is equal to effective supply which is less or equal to available quantities. Therefore, in equilibrium state there might be unutilized quantity for some commodities. In Lange’s approach, however where there is equilibrium the excess supply of commodities is compensated by the excess demand for the money commodity, which is in a state of disequilibrium from the perspective of Walras’s approach.

Third, in Walras’s approach equilibrium for the money commodity is derived from equilibrium for other commodities for the exchange economy, i.e., equilibrium for the money commodity is guaranteed when equilibrium is established for all other commodities separately. Lange, however, stated the opposite situation, i.e., equilibrium of the money commodity guarantees equilibrium for other commodities.

Finally, in Walras’s approach, all prices must be strongly positive and their equilibrium magnitude obtained from the framework of the primary demand curves (functions) for goods and the supply curves (functions) for services. Whereas according to Lange’s approach, as well as the post-Walras economists’ approach, the number of prices, but not all prices, might be equal to zero, and might even be negative, because they are only obtained by the technological conditions of the model.

In addition, the interpretation of “Walras’ Law” by modern authors is formally identical to Lange’s one, but there is a significant difference. In the modern GET’s approach the excess demand (supply) function is determined as the difference between demand and new produced quantities together with available quantities, while in Walras’s approach excess demand is determined as the difference between effective demand and effective supply.

Consequently, in a modern GET determination of excess demand (supply) functions, two possibilities arise: (i) If there is a situation when excess demand for a certain commodity is less than zero (i.e., it is negative), price must be zero accordingly. But this contradicts reality. (ii) If all prices are strongly positive then all excess demand functions must be equal to zero, which also contradicts the real economics. While, in Lange’s approach, if there is excess demand for a certain good there is excess supply for another good. Therefore, the modern authors’ version of “Walras’ Law” and its function in proving of the equilibrium existence differs with Lange’s one.

These two versions of “Walras’ Law” are not only are disconnected from real economics, but also incompatible even with a hypothetical economics; and they differ from Walras’s own system of laws, which are compatible with “a hypothetical regime of perfectly free competition.” Finally, and perhaps most importantly,

"Walras' Law" replaced Walras's original laws not only in the textbooks but essentially in professional literature and has caused them to become unknown and abandoned and therefore caused huge damage to economic science. The thought of an "alternative" to Newton's laws coexisting with the original is ludicrous, yet in economics such anomalies are commonplace.

Notes

1. Keynes's definition of full employment in his *General Theory* included voluntary employment, but such a definition makes the whole theory of employment much more confusing (for more details, see Davar 2002a).
2. Originally $\bar{x}_j$.
3. Such definition of excess demand was first formulated by Hicks (1946 [1939]: 324–325; see also Davar 2004).

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