Battiston, S., Puliga, M., Kaushik, R., Tasca, P. & Caldarelli, G. : Supplementary Information to: "DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk". Sci. Rep. 2, 541; DOI:10.1038/srep00541 (2012)

DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk.

SUPPLEMENTARY INFORMATION to *Sci. Rep. 2, 541 (2012)*

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1 The FED Emergency Programs

The Federal Reserve System (FED) is the central bank system of the United States, put into effect on December 23, 1913 with a Federal Act. The FED is composed of the Board of Governors which is a federal agency and 12 other regional Reserve Banks. Among its scopes of activities, the FED serves as lender of last resort not excluding emergency lending facilities as, for example, the *discount window*. In addition to the discount window, the FED during the credit crisis of 2007, put in place several emergency programs to assist individual institutions for instance the support to the AIG, and the help extended to JP Morgan in acquiring Bear Sterns.

Generally, operations between the FED and individual institutions are kept confidential with the argument that disclosure of such information could trigger bank runs upon the financial institutions. Bloomberg News petitioned to have the data disclosed under the Freedom of Information Act, on account of the sheer volume of public money that was utilized to bailout the banking system. The supreme court of the United States ruled in favour of Bloomberg news and data was subsequently released to the public in batches on Dec. 1, 2010, March 31 and July 6 of 2011.

The data is especially relevant to systemic risk because (1) it covers specifically the emergency loans, i.e., those loans for which FED is the lender of last resort, and (2) it allows to obtain insights into the financial fragility of many key players in the US, as well as in the global financial system.

1.1 Data

In this section we describe how we have parsed the FED original dataset, that can be obtained for research purposes with a formal request to the FED (filling the form for FOIA requests). From this dataset we created a fine-grain relational database at the level of individual loans. Details and samples are available from the authors upon request. Bloomberg did similar and also additional work (see below) and released the data aggregated by institution in a set of Excel files the 23 Dec of 2011, see (Bloomberg-News, 2011). In this work we used the Bloomberg dataset (after transforming it into a relational database).

1.2 The FED dataset

The original FED dataset consists of a collection of 30 thousands pdf pages where each page accounts for each transaction under various programs. Each entry reports the name of the borrowing institution, the credit channel used for financing, the origination date and the maturity date of each loan. In addition to transactional data there are several other kinds of documents that include emails, internal notes, and reports.

We created a database by parsing the pdf files with the following procedure: a) convert the pdf's to simple text respecting the page layout; b) parse the text with regular expressions to extract the desired information. Many of these documents, even though they are marked as pdf, are essentially pictures that cannot be parsed by a text extraction mechanism.

Bloomberg		**FED ledger database**	
outstanding days	1004	number of days covered	761
number of aggregated institutions	407	number of non aggregated institutions	1984
maximum outstanding debt per institution	USD 107 billions	maximum amount for individual loan	USD 61 billions
peak of the FED outstanding exposure	USD 1204 billions (07/12/2008)	Average daily amount	USD 29 billions
maximum number of institution exposed in a day	259 (22/04/2009)	Tot. transaction no.	31713

Table 1: Comparison of the FED original parsed dataset (based only textual entries) and the Bloomberg's parsing effort (that also includes non textual documents).

The parsed database structure and its fields are derived from the pdf entries: (1) *Institution*. It is a unique name describing the borrowing institution. In this case we are not adjusting for alliances, holding companies and subsidiaries; (2) *Amount*. It is the amount of the individual lent to the institution; (3) *Origination* date. It is the day when the loan was extended; (4) *Maturity* date. It is the day when the loan matures and the amount must be paid back in full; (5) financial *District*. It is one of the twelve FED's regional reserve banks; (6) *Channel*. It denotes is the funding facility utilized by the borrowing institution (see Section 5.1). In addition in our database we reported the name of the collateral used in the funding channel, e.g. the Asset Backed Commercial Papers that appear in most of the JP Morgan transactions. For the reader convenience a glossary can be found in 5.2. Moreover unlike Bloomberg, our dataset has not been aggregated, it carries the origination and maturity dates of all the loans on a regional basis. Such a disaggregation has the benefit of being utilized in studies that might want to correlate geographical regions and loan tenures.

1.3 The Bloomberg dataset

Bloomberg analysed the FED dataset releasing a set of excel files on Dec 23, 2011. Data consists of a set of 407 daily timeseries' of outstanding debt and market capitalizations. Each timeseries refers to a single institution that can be represented in the FED data with several conflicting names, e.g. JP-Morgan and JPM among others. The Bloomberg team aggregated the institution names by their respective parent entities. When possible, the emergency loans were further disaggregated by type according to the FED funding programs such as CPFF, TAF etc. They also parsed other materials released in emails and internal reports by the FED. Thus, unlike our database, the number of days covered in the Bloomberg database covers a period of 1004 days as opposed to 761. Table 1 compares the main facts of the two datasets.

2 Measures of credit portfolio concentration and risk

In this section we list the events captured in the graphs as peaks and trends (see Fig. 1 a)). We use as reference, for each event, the Crisis timeline of the Federal Reserve Bank of Saint Louis (`http://timeline.stlouisfed.org/`).

- **(1) 17 March of 2008** Bearn Sterns was acquired by JP Morgan Chase for USD 240 millions which was 10% of Bear Sterns's value from a week before The FED was exposed to Bear Sterns for a maximum of 30 billions.
- **(2) 18 September of 2008** Lehman Brothers defaulted after paying back an earlier loan to the FED amounting to USD 38.5 billions. The debt/market-capitalization ratio topped at 45000% few days before the default, boosting the weighted fragility of the financial system to 42 (see Methods in main paper).
- **(3) 16 October of 2008** Citigroup announced losses of USD 2.8 billions. After few weeks the debt/market-capitalization ratio topped at 600%.
- **(4) 7 March of 2009** All institutions collectively were at their minimum market capitalizations triggering a peak in the weighted fragility.

Figure 1 b) compares the decline of the FED's exposure (broken down into specific emergency programs) with the rise of the mortgage-backed-securities purchases (from January of 2008 to August of 2010) an operation carried out to help the financial institutions to offload the most problematic securities (see FCIC (2011)) with the goal of lowering the long term interest rates on mortgages. While this operation saved the financial system from the collapse, it remains to clarify who carries most of the burden. The data of the MBS purchases and other details about the FED's financial programs can be found on the statics of the FED balance sheet (`http://www.federalreserve.gov/releases/h41/`).

2.1 Debt Distribution

In this section, we investigate the distributional form of the outstanding debt of banks to the FED by means of several statistical tests. To this end, we have fitted the data distribution with the exponential and the log-normal distributions. The results for period 7 are shown in Fig. 2-right. Similar results hold

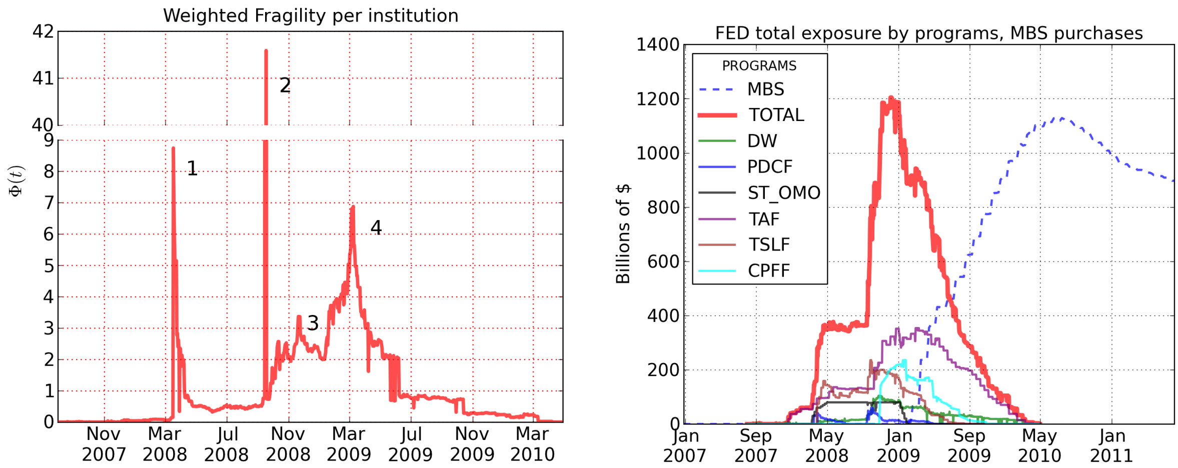

Figure 1: **Fragility peaks. FED MBS purchases a)** Weighted fragility plot where the peaks capture the major events during the credit crisis (see text). In this figure we included the Lehman Brothers Peak (not shown in the main paper). **b)** FED's exposure broken down by different kinds of programs compared with the Mortgage Backed Securities FED's purchases. Data on MBS comes from the FED balance sheet. Notice the decline in the exposure caused by the end of the emergency programs and the contemporary increase of the MBS. In FCIC (2011) this operation is reported as a measure to acquire the *toxic assets* from the institutions that in this way can consolidate the balance.

for the other periods. A visual inspection suggests that (1) the data follow, approximatively, a log-normal with a cutoff on large values of debt, and (2) the exponential does not fit well the data. The cutoff in the data can be explained by a FED policy not to lend money beyond a certain threshold, which may vary upon the general conditions of the economy. We carry out the Anderson-Darling and the Shapiro tests for the log-normality, and the Cramér Von Mises and the Kolmogorov-Smirnoff test for the exponentiality. In all cases the tests show that the data neither follow the log-normal, nor the exponential distribution at the confidence level of 95%.

As mentioned in the main paper, previous studies have found a power law distribution of debt across banks, while here we found a distribution that has a tail below the one of a power law and seems approximatively a log-normal with a cutoff. A possible explanation is that, in our case, the small institutions are under-represented in the FED database. A precise statement is the following: The fact that small institutions are under-represented in our dataset is one possible explanation why we do not observe a power law. To validate this statement we need to show that, given a random variable with a power-law distribution, it is possible to construct a sample, possibly following some not-random rule, such to obtain a distribution which resembles the one we find empirically. We thus carry out the following steps:

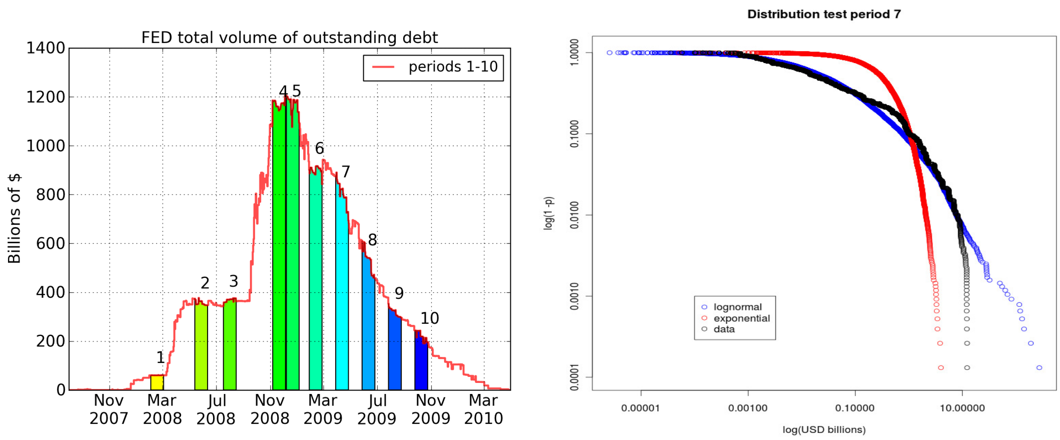

Figure 2: **Left**. The time series of the outstanding debt during the crisis, the vertical stripes are the chosen periods for the debt distribution analysis. **Right** Comparison between the fitted exponential, log-normal, and the dataset for the period 7. Similar results can be shown for every other period: the data distribution (black line) is in between the exponential (red) and the log-normal (blue) and it shows a cutoff for a high debt level according to the market conditions.

- We create a sample X of size n following a power law with parameter α.
- We fit on the sample X a log-normal distribution obtaining the log-normal parameters (mean and variance of the log-transformed data) μ and σ
- We create a sample Y of size n following the log-normal distribution with the parameters fitted from the power law sample.
- We plot the X and Y histogram in semilog scale, computing the counts within the same bin classes (Fig.3).

As we can observe in the figure, for low values of the random variable x, the counts are higher in the power law sample X than in the log-normal sample Y. Assume now that x represents debt size. We can conclude that, if small institutions are less likely to be eligible for the FED loans and if the loans are proportional to size, then even if the underlying size distribution of firm size were power law small the counts of the low values of debt would be smaller than in a power law distribution.

In the time span of the data we selected 10 representative periods on which we performed a statistical study the outstanding debt (Fig. 2-left). Here, we want to verify that the data within each period are homogeneous from a statistical point of view. To this end, we use the tool of quantile-quantile plot (qq-plot). In a qq-plot, whenever the quantiles of two datasets lay on the diagonal with a strong linear correlation, we can conclude that the two datasets have the same distributional form (although they can

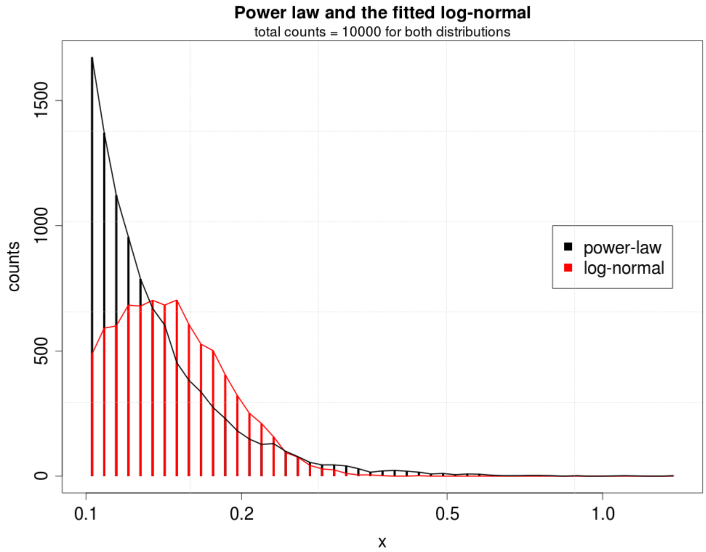

Figure 3: Comparison of counts between a power law distribution and the corresponding, fitted, log-normal. From the plot it is clear that, for low values of the x variable (in our case the debt size), the log-normal is below the power law, confirming that the small debts (and the corresponding small institutions that borrow the money from the FED) are under-represented respect to the alternative hypothesis of the debt following a power law distribution.

have different parameters, e.g. mean, variance or other moments). In Fig. 4, the qq-plots for successive periods (i.e. period 5 and period 6 or period 4 and period 6 etc.) show strongly correlated behaviour (with linear correlation coefficient r always above 0.95). When we compare non-successive periods the correlation coefficient is lower but still greater than 0.85. We conclude that all datasets are likely to come from the same statistical distribution, with parameters that may vary across periods. As we can see in Fig. 2a in the main article, the distributions of the different periods seem to cluster in groups according to 3 market phases.

A final issue to address concerns the homogeneity of the data within each period. To study the variability of the outstanding debt in each period, we used the coefficient of variation, which is defined as $cv = \frac{\sigma}{\mu}$. We find that the cv of the data varies from 0.4% (period 1) to a maximum of 9.5% (period 7). The average across periods is 3.4%, compared to the value of 102% for the entire data set. We conclude that the outstanding debt data is approximatively homogeneous within each period.

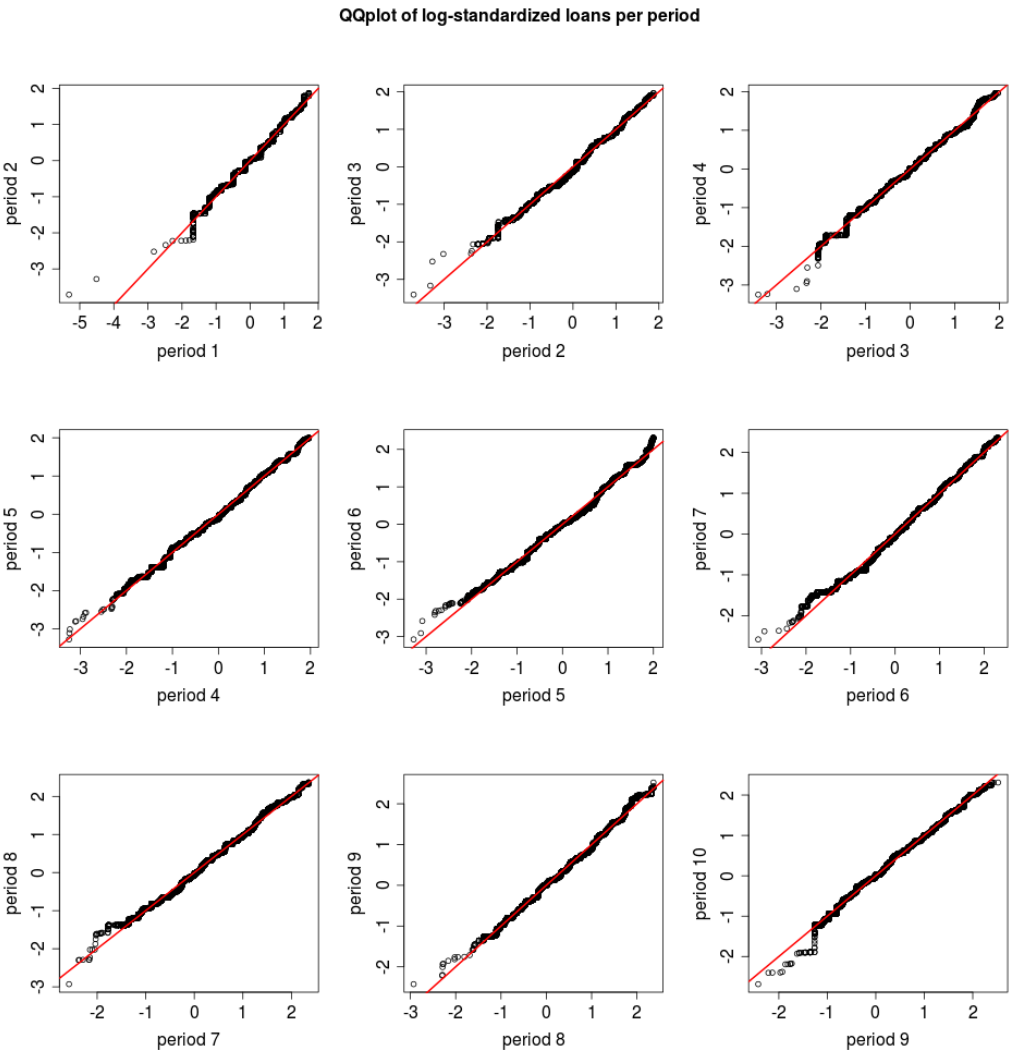

Figure 4: The qq-plots (quantile versus quantile) plot used to study the distributions of pairs within adjacent periods (i.e. period 5 and 6, period 6 and 7 and so on). Data are log-transformed: they follow a clear linear pattern whose correlation coefficient r (linear correlation coefficient) is always greater than 0.95 for adjacent periods and greater than 0.85 for non successive periods.

3 Network analysis

3.1 Measuring Systemic Importance of Financial Institutions

In the following, we use interchangeably "institutions" or "nodes", and "counterparties" or "neighbors". Because the direction of the links in this context is crucial, we distinguish between the "predecessors" of a node i, i.e. the nodes pointing to i, and the "successors" of i, i.e., the nodes pointed by i.

One way to estimate the systemic importance of nodes in a financial network is to assume a default cascade dynamics and to measure the size of the cascade of defaults caused by its initial default. This approach is used in stress-tests carried out in central banks (Cont et al., 2010; Mistrulli, 2011). When the node i defaults, the counterparty j faces a loss equal to the amount invested in i (neglecting a possible recovery rate). Then, the counterparty also defaults whenever the loss exceeds its equity (more precisely, its so-called "tier 1 capital"). This dynamics bears similarities with several dynamics typical of critical phenomena in physics such as fracture dynamics and the bundle fiber problem. In particular, many default cascade models can be mapped into three classes for which the cascade size can be studied analytically in an (heterogenous) mean-field approximation (Lorenz et al., 2009).

Notice that profitability pushes financial institutions to be highly leveraged, i.e., the value of equity are typically a small fraction of the assets (see SI for more details). This implies that even a small relative impact $W_{ij} << 1$, can cause the default of a large institution. If many connected institutions have equity values below the value of their relative exposure to the others, then the default of one can cause a large cascade of defaults. Therefore, regulators impose requirements on the minimal amount of capital. To simplify, there is an obvious way to prevent that a single default causes other defaults. For any given vector of equity values in the network, it is enough that every node dilutes its exposure to the others so much that $W_{ij} < E_i$ for all i and j. However, there are also amplification mechanisms that make the strategy to connect to the all the other nodes potentially dangerous (Battiston et al., 2012b).

An important limitation of default cascade models is that they do not account for the fact that, even when the default does not propagate from a node to a successor, there is still a propagation of distress (Battiston et al., 2012a). Indeed, the node facing the loss becomes more fragile and this makes also its counterparties more fragile. As a result, even when default cascades are small or absent there can still be a significant systemic impact. Thus the dynamics of distress propagation extends the default cascade dynamics. Here, we combine the notion of distress propagation with the one of feedback centrality, which has already found successful applications in many domains of complex networks (Nicosia et al., 2012). There are several variants of feedback centrality. Interestingly, they can also be mapped to known notions in physics. For instance, PageRank is related to the fraction of random walkers that have visited a node in the long run (see below). It can also be mapped into the time-independent Schroedinger equation (Perra et al., 2009). As another example, control in ownership networks can be mapped into the cumulative in-flow of mass through a given node (Glattfelder and Battiston, 2009).

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3.2 DebtRank

In light of the above discussion, we thus introduce a novel measure, called DebtRank, that takes recursively into account the impact of the distress of an initial node across the whole network. DebtRank is a number measuring the fraction of the total economic value in the network that is potentially affected by a node. The measure is inspired by analogous centrality measures (such as the PageRank and the corporate control (Vitali et al., 2011)) allowing to take into account the whole set of interconnections in a self-consistent way.

3.2.1 The Dynamics of Debt Rank on a Simple Example

In this section, we illustrate the dynamics of DebtRank on a simple example. The methodology and the formulas are described in the Section Methods of the main text. However, for convenience of the reader, we report here the definition of the dynamics of DebtRank already given in the main text. To each node we associate two state variables. h_i is a continuous variable with $h_i \in [0, 1]$. Instead, s_i is a discrete variable with 3 possible states, undistressed, distressed, inactive: $s_i \in \{U, D, I\}$. Denoting by S_f the set of nodes in distress at time 1, the initial conditions are: $h_i(1) = \psi \, \forall i \in S_f$; $h_i(1) = 0 \, \forall i \notin S_f$, and $s_i(1) = D \, , \forall i \in S_f$; $s_i(1) = U \, \forall i \notin S_f$. The parameter ψ measures the initial level of distress: $\psi \in [0, 1]$, with $\psi = 1$ meaning default. The dynamics is defined as follows,

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_j W_{ji} h_j(t-1)\right\}, \text{ where } j \,|\, s_j(t-1) = D, \tag{1}$$

$$s_i(t) = \begin{cases} D & \text{if } h_i(t) > 0 \;\&\; s_i(t-1) \neq I \\ U & \text{otherwise,} \end{cases} \tag{2}$$

for all i, where all variables h_i are first updated in parallel, followed by un update in parallel of all variables s_i. Notice that, the Equations above imply that any node is able to impact their successors only in the time step after it has received distress from its predecessors. This is because of the condition where $j \,|\, s_j(t-1) = D$ in the sum of Eq. 1, together with the fact that nodes go into state I in the time step after having impacted its successors. However, nodes remain able to receive impact from the others also after they have gone into state I.

As an illustration, each step of the dynamics of distress, following equations 1 and 2 above, is represented in Fig. 5. For didactical purposes, we have considered a simple network in which the matrix of impact has the same values everywhere, $W_{ij} = 0.5$ for all the links. We take as initial condition $\psi = 1$. Therefore, the values of distress transmitted downstream of node 1 are all combinations of the powers of 0.5: $0.5^2 = 0.25$, $0.5^3 = 0.125$, $0.5^4 = 0.0625$. For the economic value of the nodes we take $v_i = 1/8$ for all i.

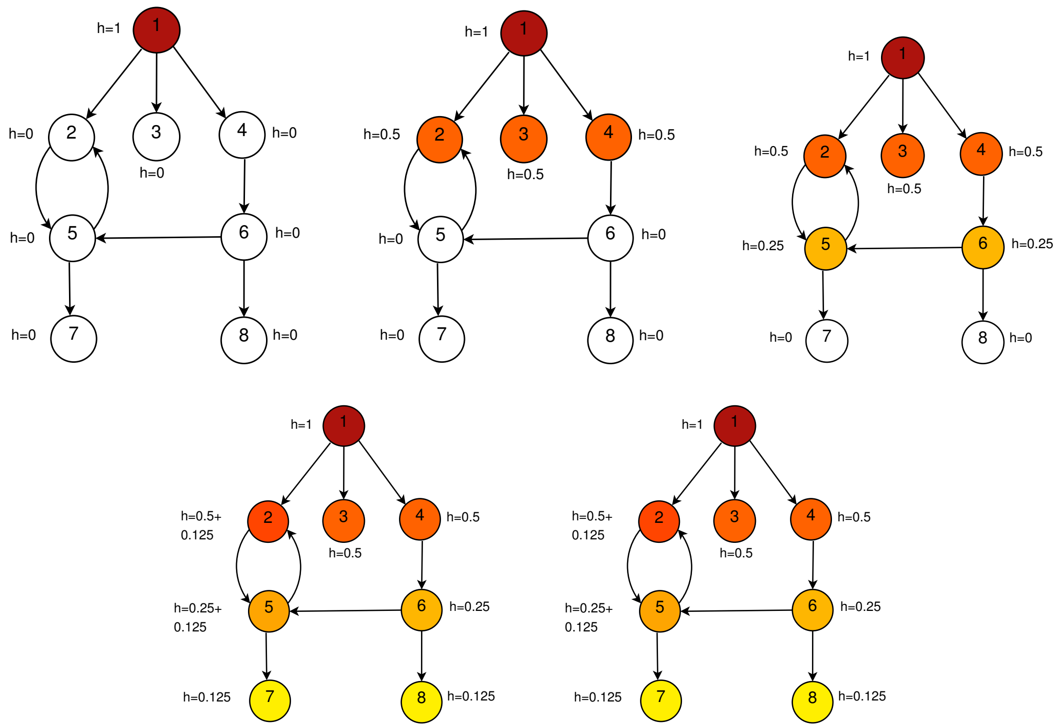

Figure 5: **DebtRank computation**. Illustration of the dynamics used for the computation of DebtRank on a simple network example. See text. The color of a node i represents the level of distress $h_i(t)$ of the node at a given time step. **(Top Left) Step 1**. Node 1 is in full distress: $h_1(1) = \psi = 1$, $s_1(1) = D$. All other nodes have zero distress and are in state U. **(Top Middle) Step 2**. Node 2 is impacted by node 1: its level of distress goes from 0 to $h_2 = 0.5 \times 1$ and its state becomes D. The same happens to node 3 and 4. Node 1 goes into state I and will not impact any node from the next time step on. **(Top Right) Step 3**. Node 5 is impacted by node 2: its distress goes from 0 to $h_2 = 0.5^2 \times 1 = 0.25$ and its state becomes D. The same occurs to node 6, impacted by 4. At the end of this step nodes 2, 3 and 4 go into state I and will no longer impact other nodes. **(Bottom Left) Step 4**. The distress of node 7 goes from 0 to $h_3 = 0.5^3 \times 1 = 0.125$ and its state becomes D, due to the impact of node 5. The same occurs to node 8, impacted by node 6. The distress of node 5 increases by $0.5^3 = 0.125$, due the impact of node 6. The distress of node 2 increases further due to the impact of node 5. Notice that it is the distress of node 5 at the previous time step that is propagated, i.e. $h_2(4) = 0.5 + 0.5^3$, and *not* $h_2(4) = (0.5 + 0.5^3) \times 0.5$. Notice that in the cycle between 2 and 5, since node 2 is already in state I it will no longer impact 5, i.e., no more reverberations. **(Bottom Right) Step 5**. The dynamics stops. We sum up the distress induced in the network by the initial distress of node 1 (excluding 1 itself). We obtain $\sum_{j\neq} 1 h_j(5) = 2.5$, which is higher than the initial distress $h_1(1) = \psi = 1$. The total distress in the system at time 5 is thus 3.5. For DebtRank, we obtain $R_1 = (1/8) \times \sum_{j\neq 1} h_j(5) = 2.5/8 = 0.3125$.

3.2.2 The Impact Matrix on a Simple Example

Figure 6 illustrates the procedure to construct the impact matrix on a simple example.

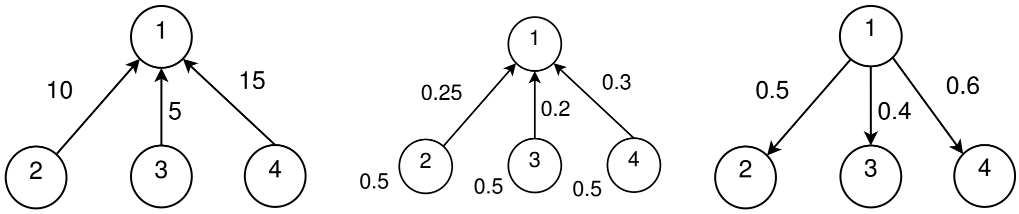

Figure 6: **Construction of the impact matrix**. **(Left)** A link from i to 1 represents the amount of the investment made by i in the funding of j. The figure represents the situation in which the investments Z_{j1} of nodes 2, 3, 4 in the funding of node 1 are, respectively, 10, 5, 15 dollars. **(Middle)**. A link from i to 1 and its weight represent now the relative exposure of i to j. Node 2 has investments in other nodes, not represented in the figure, for a total of 40 dollars. Therefore, the exposure of node 2 to 1 in terms of fraction of its total exposure, is $\frac{Z_{21}}{\sum_l Z_{2l}} = 0.25$. Similarly, node 3 and 4 have total investments of 25 and 45 dollars, respectively. This means that nodes 3 , and 4 have links of weight 0.2 and 0.3 pointing to node 1. However, the impact of node 1 on each of the other nodes can be larger than these values (see below). **(Right)**. A link from 1 to j and its weight represent in this case the ratio $W_{1,j} = Z_{j1}/E_j$ of the relative exposure of j to i over the core capital of j. This is the coefficient of the impact that 1 has on j when 1 goes in distress. In the situation considered in the figure, node 2,3,4 have core capital of, respectively, 20, 12.5, 25 dollars, which leads to impact values of 0.5, 0.4, 0.6. Notice that these numbers are larger than the relative exposures of 2,3,4 to 1. Nodes can have a very little relative exposure to any single node, but if their core capital shrinks enough they become very vulnerable. Notice that the sum of the values of the impact made by node 1 on the other nodes is larger than 1. In this way, even a single node can impact significantly many other nodes.

3.3 PageRank

The concept of PageRank has been introduced as a method to bring order to the web Page et al. (1998) by a group of students at Stanford University and lately adopted as main ingredient of the web search engine Google. The idea is that the quality of a web page can be enhanced by the endorsement it receives from the pages that are pointing to it. If the quality of every page i is measured by this PageRank PR_i, we can define such quantity on a network of N vertices in this self-consistent way

$$PR_i = \alpha \sum_{j \to i} \frac{PR_j}{k_j^o} + (1-\alpha)\frac{1}{N}. \tag{3}$$

The first term on the r.h.s. gives with probability α the PageRank inherited by pages j pointing to i. Every page j gives a proportion PR_j/k_j^o to the specific destination node i (k_j^o is the out-degree of the page j). The second term with the complimentary probability $1-\alpha$ gives a contribution of $1/N$ equal for all. This formula can be viewed as the progress of a random walker that wandering on the graph spends more time on the vertices most connected or connected with other vertices very connected. In this case the second term on the r.h.s. can be viewed as the probability of "teleportation", that is the probability to enter into the vertex from any of the other N vertices (even if not connected). This term has the function of remove the problems related to the presence of loops and dangling ends in the graph and to ensure that the set of the above equations has a solution.

Unlike DebtRank, in PageRank the matrix involved in the computation is column stochastic. Another important difference is that here cycles have infinite reverberation. Finally, the teleportation is not present in DebtRank.

3.4 Network Evolution

During the crisis the FED funded the institutions according to several emergency plans. In the following pictures (Fig. 7 - 11) we show how the exposure were distributed across institutions and how the debt impacted to the centrality of the institutions (see Methods). Moreover we show the effects on the asset size of the institutions and the average debt during all the periods.

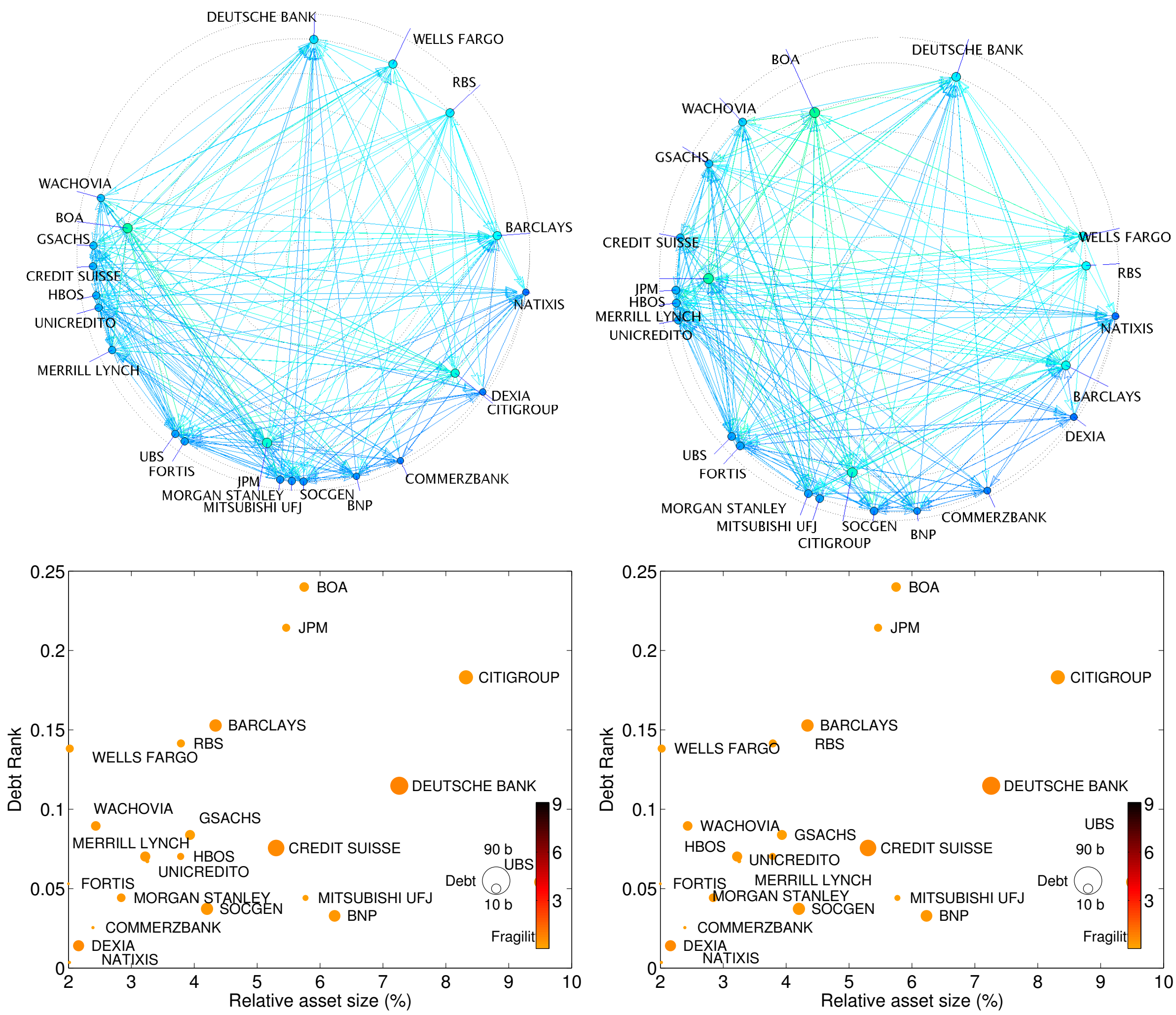

Figure 7: **Network evolution and DebtRank** Period 1 and 2. **(Top)** As Fig. 3 in main text. Nodes represent financial institutions with debt larger than USD 5 billion, computed as an average across the ten periods chosen for the analysis. Thus, the set of institutions in (Top left) and (Top right) are not all the same. Outgoing links represent the estimated potential impact of an institution to another one (see Methods, main text). The nodes are positioned within a circle of radius 1, centred in 0. The distance of each node from the center is $1 - R^{(D)_i}$, while the angle increases linearly with $R^{(D)}$ from 0 to 10π. Thus, the closer a node is to the center the higher is its DebtRank (the intuition here is its centrality). A node in the center (DebtRank = *1*) is able to put under distress the entire economic value of the network. DebtRank decreases by moving outwards and leftwards along the spiral. (text continues on Fig. 8)

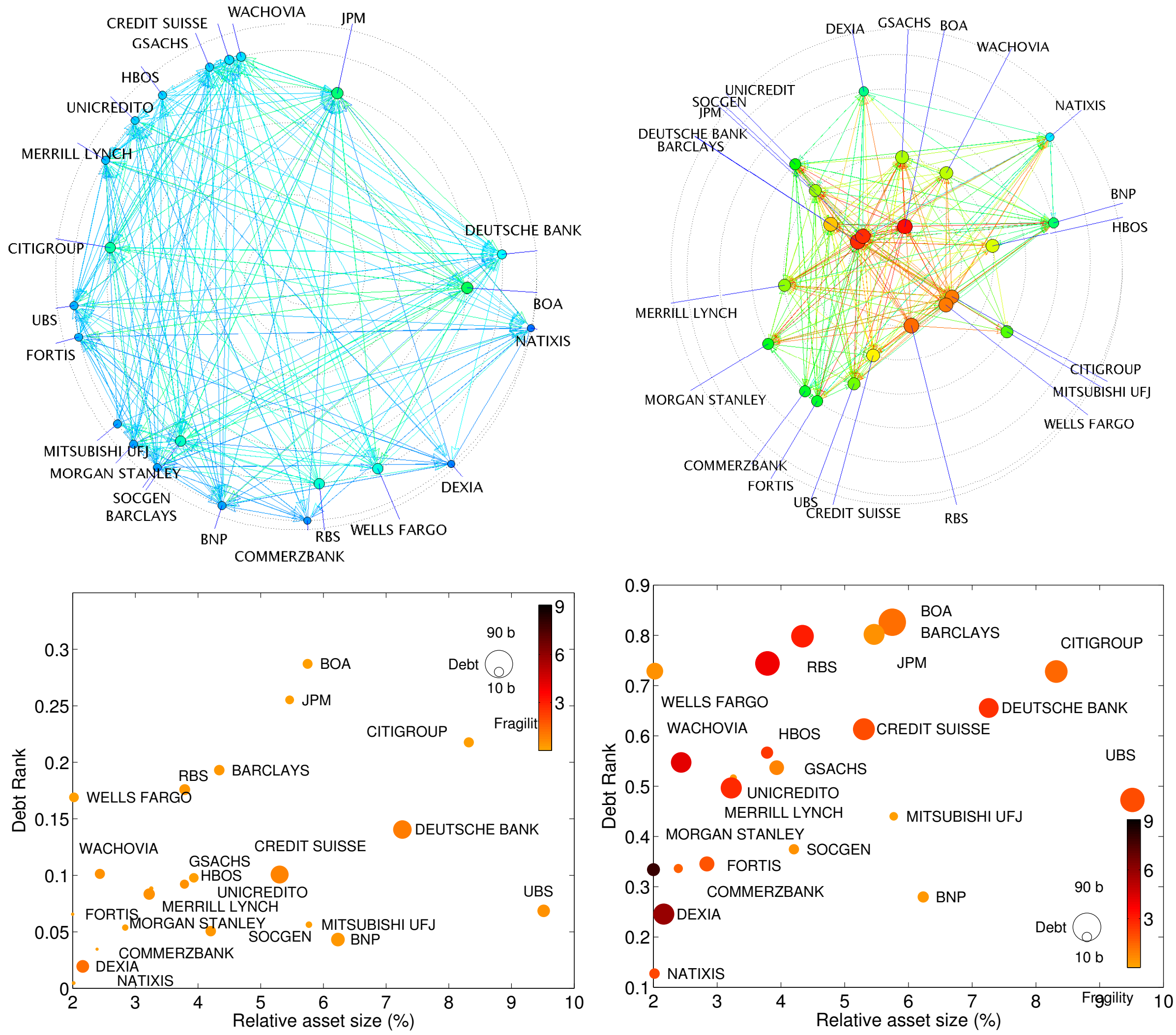

Figure 8: Same as Fig. 2 for period 3 and 4. Text continues from the Fig. 7 . The diagram allows at the same time to visualize the structure of the network and to compare the importance of any two given nodes. The size and the color of the node reflects the DebtRank value (larger and red nodes have higher DebtRank). The color of a link reflects the DebtRank of the node from which it originates (red links originate from node with high DebtRank and make high impact to the destination nodes). **(Bottom)**.Scatter plot of DebtRank versus asset size of each institution (as % of the total across institutions). The color and size are set as in Fig. 4 in main text.

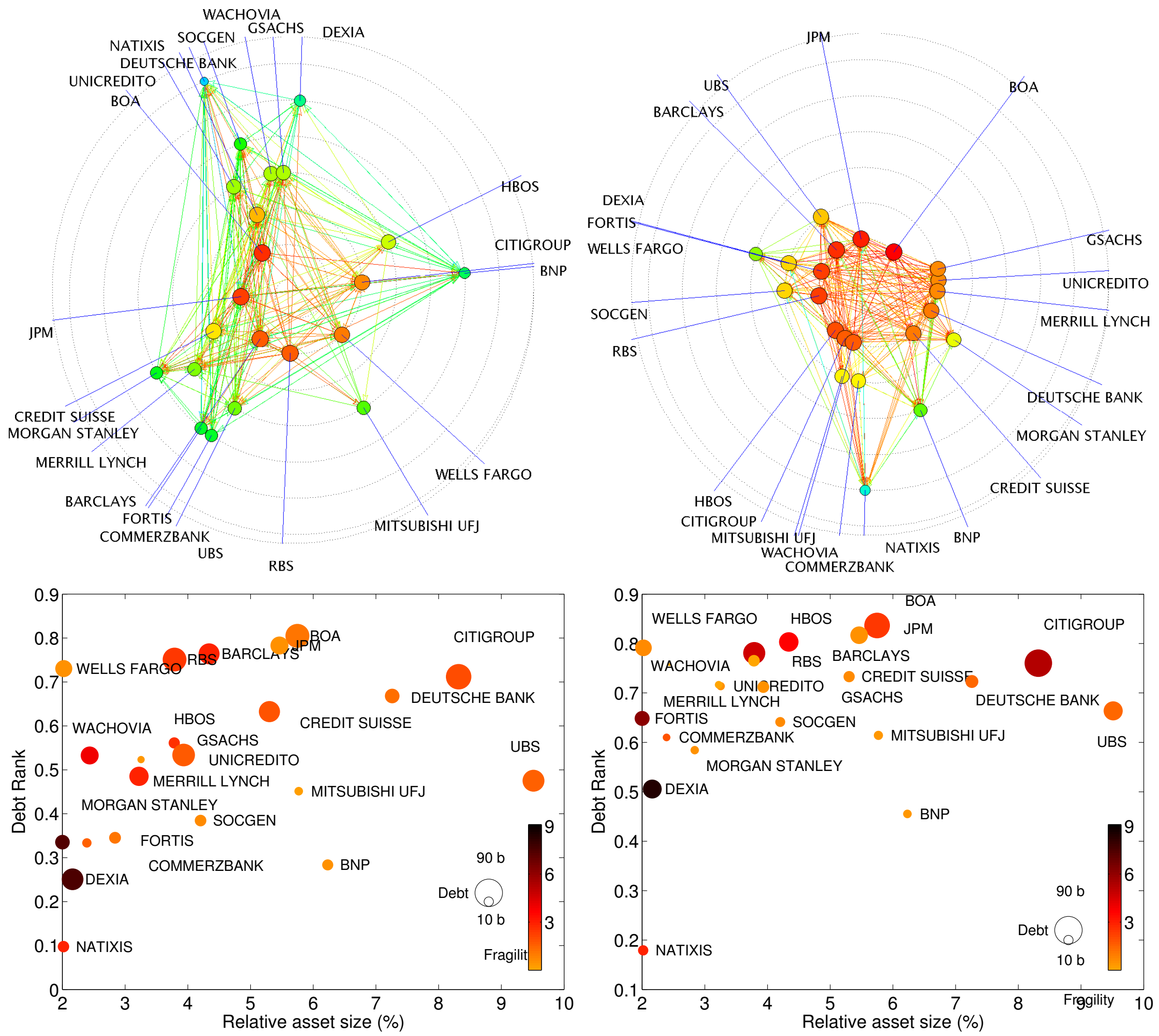

Figure 9: Same as Fig. 2 for period 5 and 6

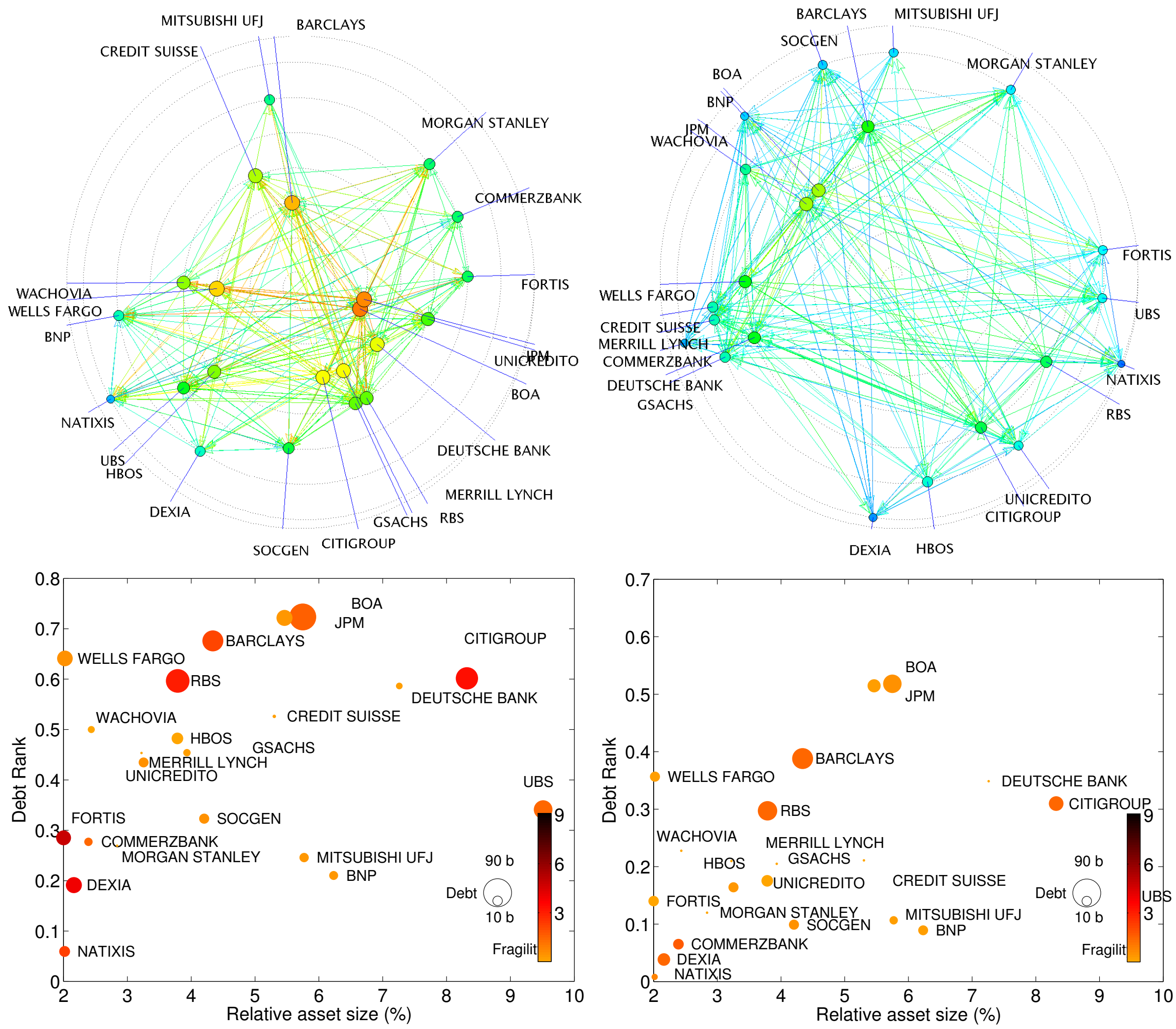

Figure 10: Same as Fig. 2 for period 7 and 8

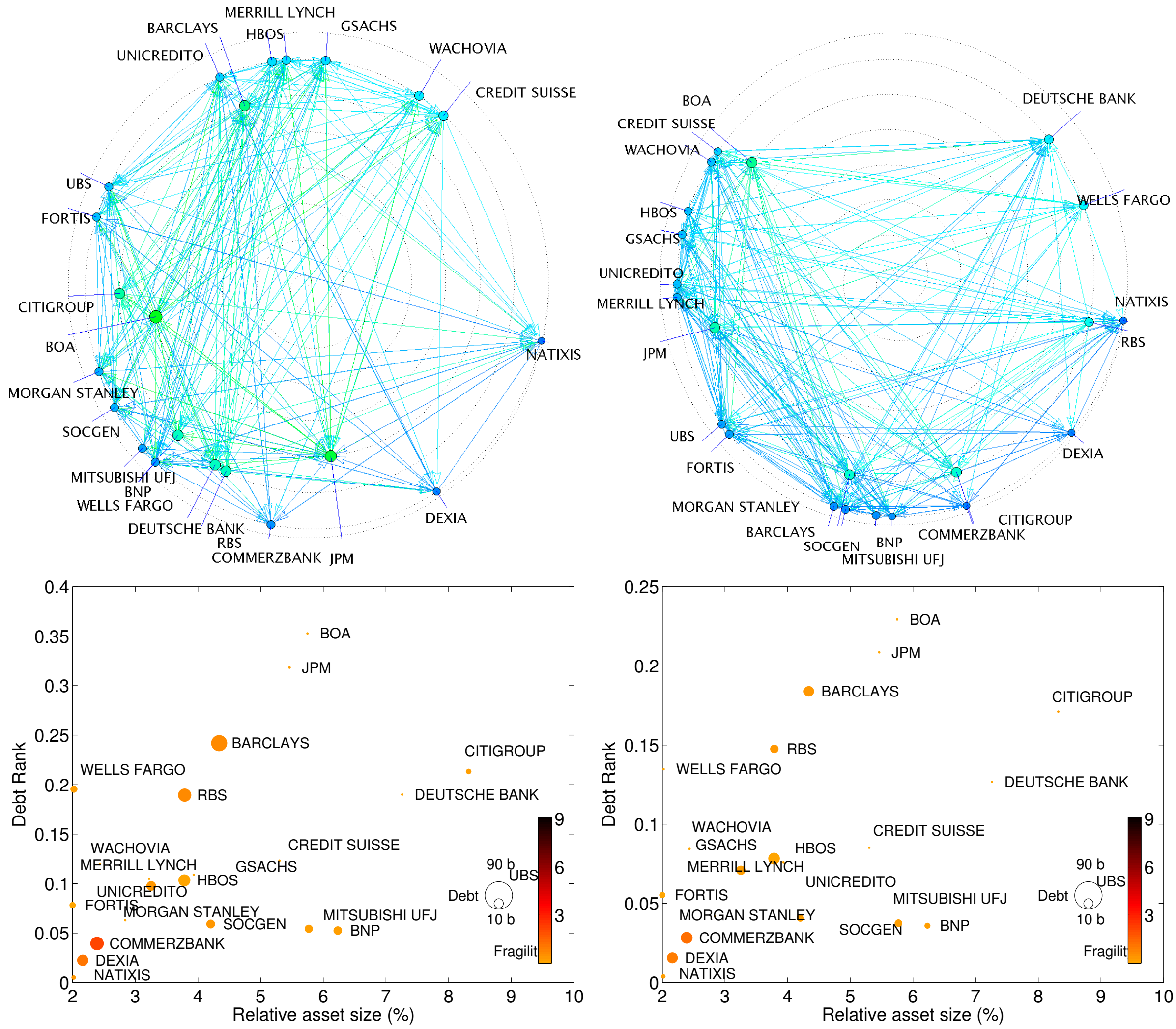

Figure 11: Same as Fig. 2 for period 9 and 10

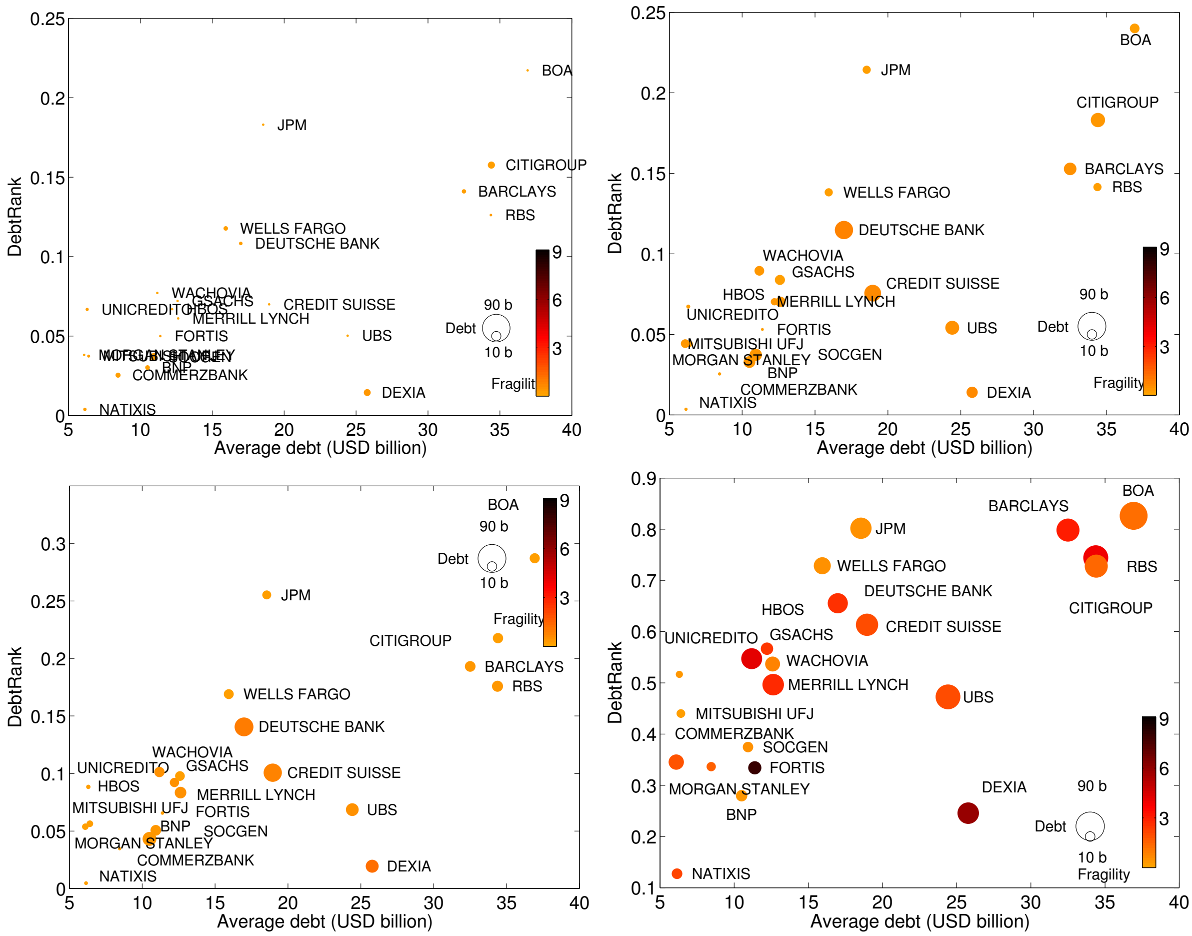

Figure 12: **DebtRank versus average debt per institution**. Periods 1-4 from top left to bottom right. Colors are proportional to the fragility (debt over market capitalization) while the size is proportional to the debt in the given period.

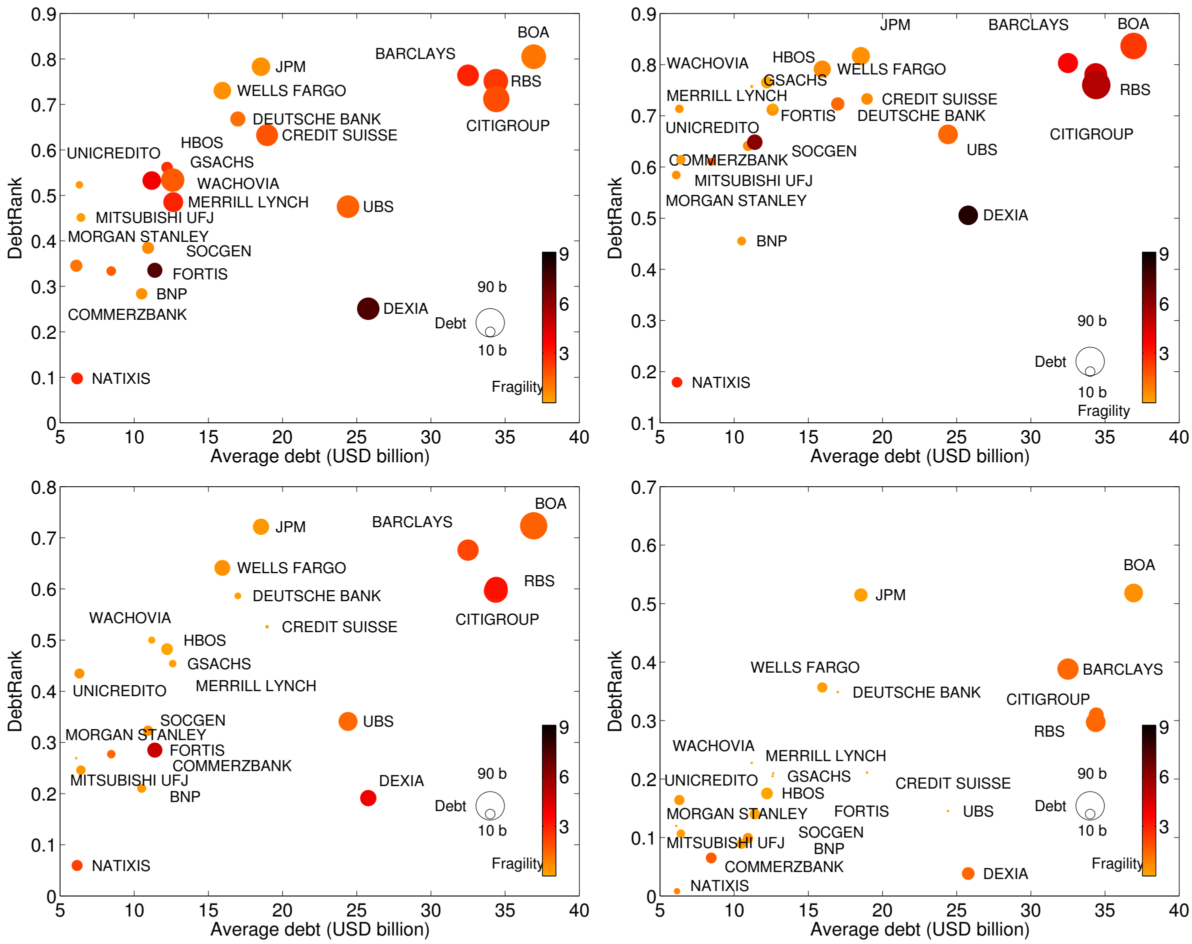

Figure 13: Same as 12 for periods 5-8

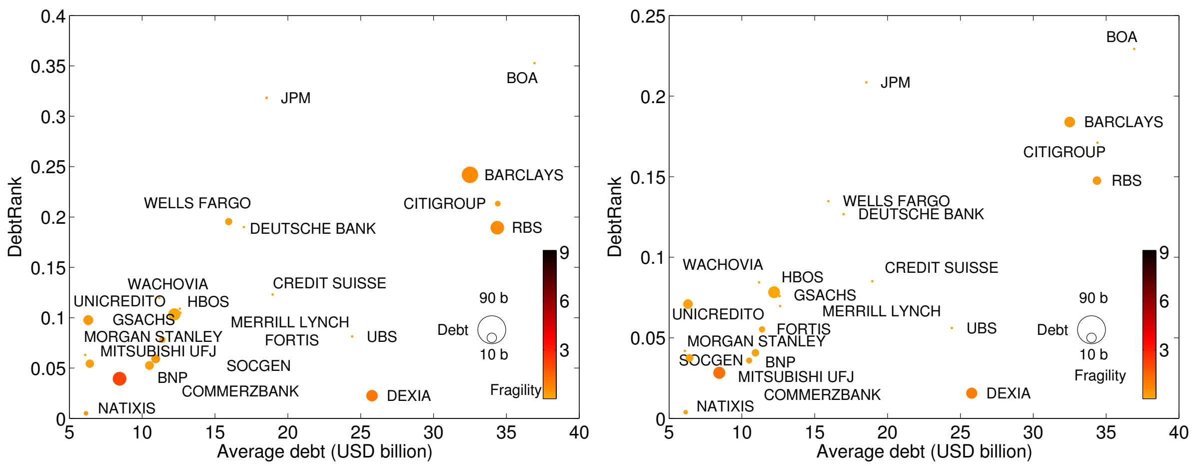

Figure 14: Same as 12 for periods 9 and 10

3.5 Comparing DebtRank with Feedback Centrality Measures and with Standard Default Cascade Measures

In this section, we carry out a systematic comparison over time and across institutions of DebtRank to two measures of feedback centrality and one standard measure of default cascade impact. We first present the definitions of the measures. We then report the results for 3 different values of the *impact scaling factor* introduced in the Section Methods of the main text.

Feedback centrality refers to all those measures in which the centrality of a node depends recursively on the centrality of the neighbours.

Eigenvector Centrality An example is eigenvector centrality, a classic measure introduced initially in the field of social networks to assess the importance or influence of nodes in networks in various contexts. In a strongly connected graph, the eigenvector centrality of node i is the value of the i-th component of the eigenvector associated to the largest eigenvalue of the, possibly weighted, adjacency matrix. In vector notation, the eigenvector centrality v is the vector that solves the equation $Wv = \lambda_1 v$. In terms of recursive expression we can define the centrality of node i as $c_i = (1/\lambda) \sum_j W_{ij} c_j$.

Impact Centrality Other examples of feedback centrality include an additional constant term in the equation that represents the intrinsic importance of each node, in absence of the network (Brandes and Erlebach, 2005).

As illustrated in the main text, if want to take into account the impact of i on its indirect successors, we can define the impact in terms of the following recursive equation $I_i = \sum_j W_{ij} v_j + \beta \sum_j W_{ij} I_j$, where the second term accounts for the indirect impact via the neighbours. The parameter $\beta < 1$ is a dampening factor. In vector notation we have $I = Wv + \beta W v$, which yields $I = (\mathbb{I} - \beta W)^{-1} W v = \sum_{k=0}^{\infty} (\beta^k W^k) W v$. One natural way of ensuring that the expression is well defined is to normalize the matrix W along the columns and to make it column stochastic, that is, $\sum_i Wij = 1$ for all i. In this way its largest eigenvalue is one and the matrix $(\mathbb{I} - \beta W)$ is invertible. The meaning of the normalization is that the impact that a node receive from the others always sums up to one. This measure is the natural extension of feedback centrality to the financial context. However, the normalization of the impact matrix implies that the measure is not sensitive to situations in which the impact on a node overall increases, as it may occur during a crisis.

Default Cascade Impact A standard approach to estimate the impact of a default in network of banks connected by interlocked balance-sheets consists in the following algorithm. One bank is assumed to default and its liabilities to be devalued. In the worst scenario, the new value is zero. The equity of the banks exposed to the defaults bank is recalculated as difference between the new value of the assets minus their liabilities. Because the assets included the liability issued by the defaulted bank, equity decreases. If equity of any these banks becomes negative, then they are also assumed to default. The process continues until no new default occur. The impact of the default is calculated as the sum of the total assets of the banks defaulted as a consequence of the initial default. The impact is then normalized with respect to the sum of the asset values across institutions. The algorithm is illustrated in (Battiston et al., 2012b; Mistrulli, 2011).

3.5.1 Results

To make the comparison consistent, we have computed each measure using the same impact matrices that we used to compute DebtRank over time. In the case of Impact Centrality we had to make the matrix column-stochastic in order for the measure to be well-defined. The results across institutions for different values of the scaling factor α are illustrated in Fig. 3.5.1-3.5.1-3.5.1. Because in our exercise the impact matrices (one every day) have been constructed by rescaling the initial impact by the inverse of the market capitalization (see Methods), we obtain by construction that all measures reflect the behaviour of market capitalization. The purpose of this exercise is to illustrate what could be done if we had the actual values of the exposures. The first thing to notice is that the order of the colors is similar in each panel, i.e., the ranking provided by the various measures are consistent among each other across time. However, the measures differ in the timing and in the magnitude of their response. Recall that days 500-700 correspond to the period around March 2009 where the market capitalization dropped on average to its minimum values. DebtRank (top left panel in each figure) is the only one that delivers a very clear response at the peak of the crisis but, at the same time, starts raising much before the peak.

The strength of the default cascade impact is to provide, as also DebtRank does, the monetary value of total loss (as a fraction) induced by a single default. However, as we can observe in the top right panel of the figures, it is non-zero only in the days around the minimum of the market capitalization. The reason is that because this is a threshold process, the default cascade is not triggered unless the values of equity across banks are sufficiently low. This is problematic because it provides a signal only when the situation is already very deteriorated. Eigenvector centrality, is not poorly sensitive to the increase in impact that institutions had on each other. Even normalizing the values of each day by the maximum value of eigenvector centrality the picture does not improve. Impact centrality performs a little better as we see some increase corresponding to the peak of the crisis but it does not deliver the build-up of the systemic importance of each institution as the peak of the crisis approaches. When we look at the average values of these indicators across institutions the results are even stronger. Fig. 18 shows the comparison between the average values of the various measures for two different values of α.

In conclusion, DebtRank delivers some advantages w.r.t the other indicators, especially as a candidate early-warning indicator. Let us emphasize again that DebtRank has a precise meaning is absolute terms, i.e. it is the fraction of economic loss, measured in dollars, caused by the distress or default of a node. In contrast, eigenvector centrality does not have any economic interpretation in absolute terms. For impact centrality there is some interpretation in terms of the flow of the distress (Kaushik and Battiston, 2012), but not as immediate as the monetary loss.

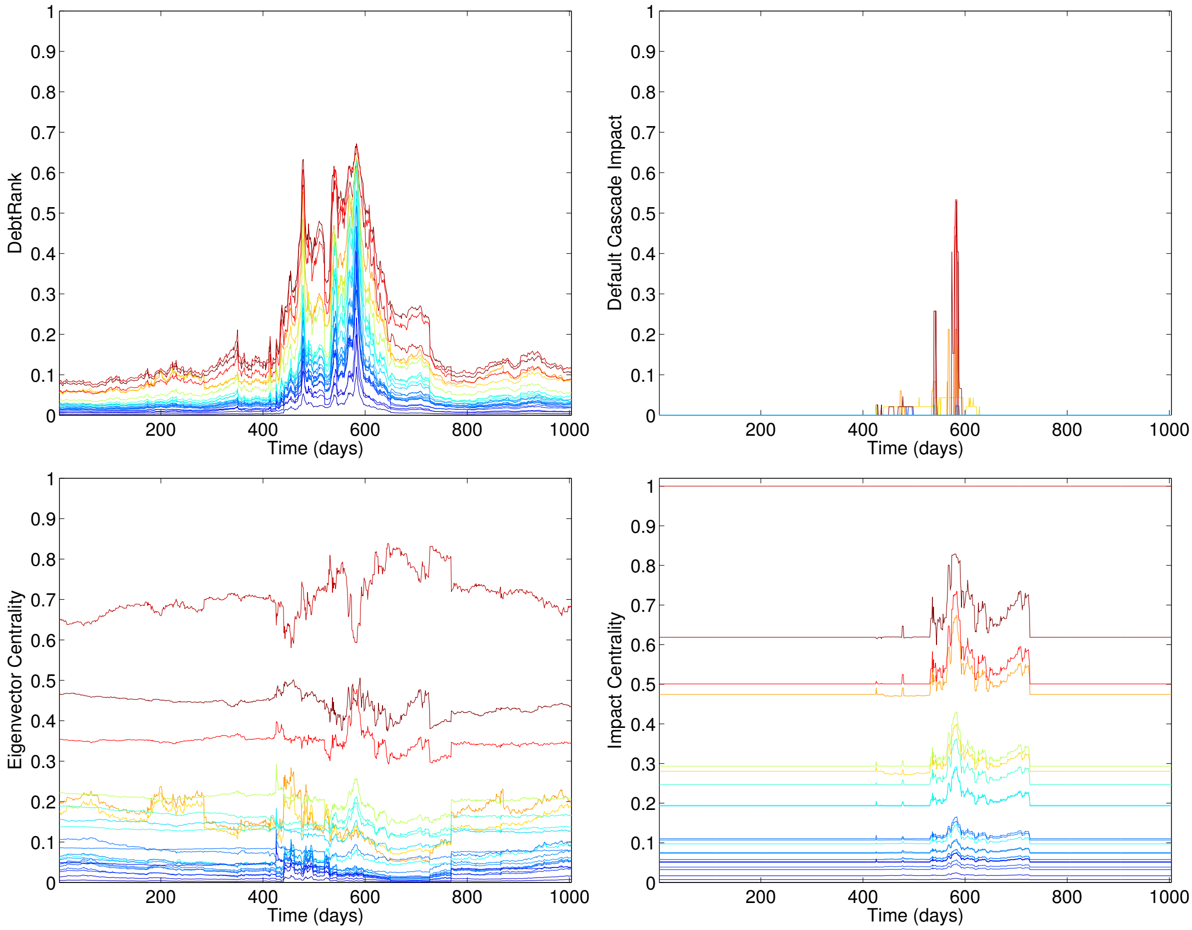

Figure 15: **Comparison of DebtRank with feedback centrality and default cascade impact**. Each curve refers to one institution over time. Case with scaling factor in the impact matrix $\alpha = 0.1$. Colors are assigned according to the ranking obtained with DebtRank, dark red (blue) are the most (least) systemically important. Top left: DebtRank. Top right: default cascade impact. Bottom left: eigenvector centrality. Bottom right: impact centrality.

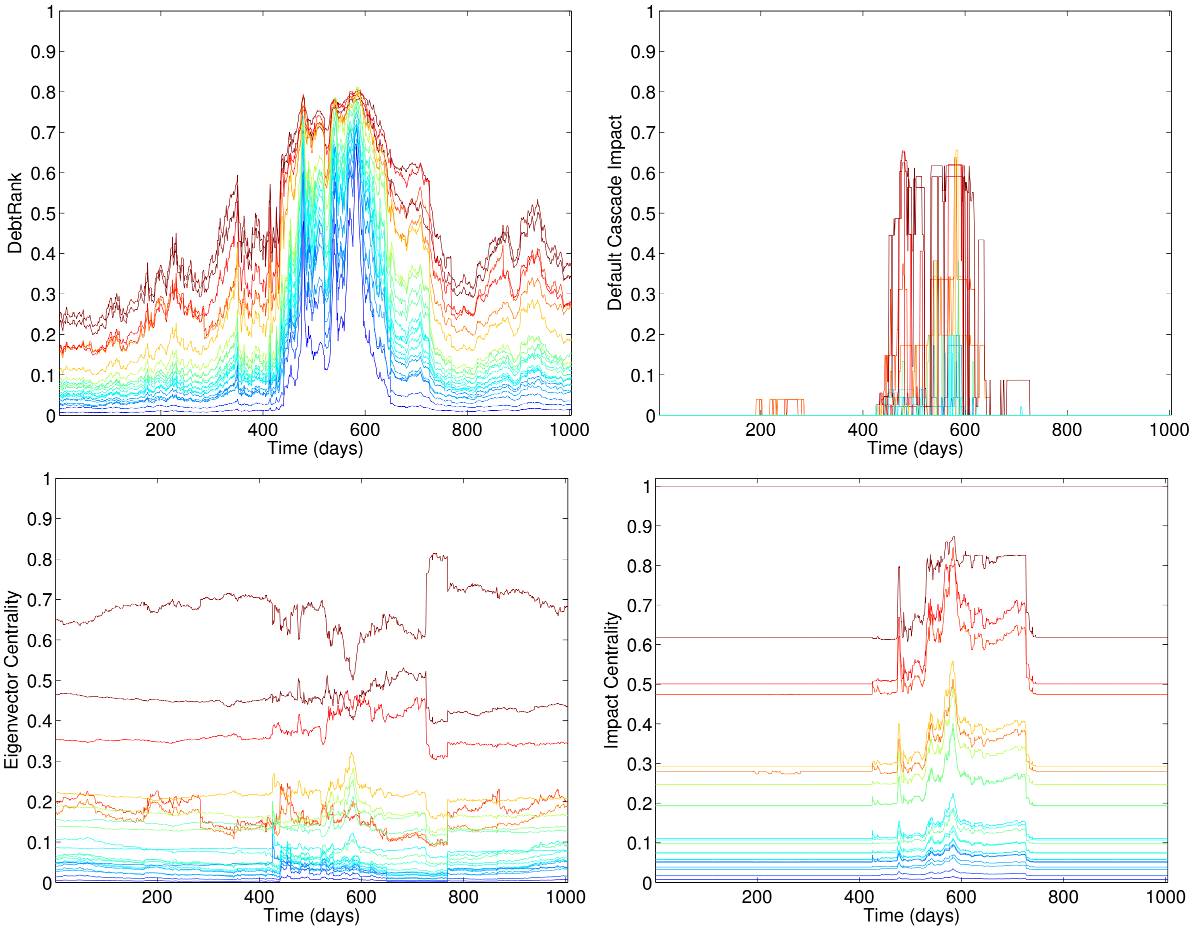

Figure 16: **Comparison of DebtRank with feedback centrality and default cascade impact**. Each curve refers to one institution over time. Case with scaling factor in the impact matrix $\alpha = 0.2$. Colors are assigned according to the ranking obtained with DebtRank, dark red (blue) are the most (least) systemically important. Top left: DebtRank. Top right: default cascade impact. Bottom left: eigenvector centrality. Bottom right: impact centrality.

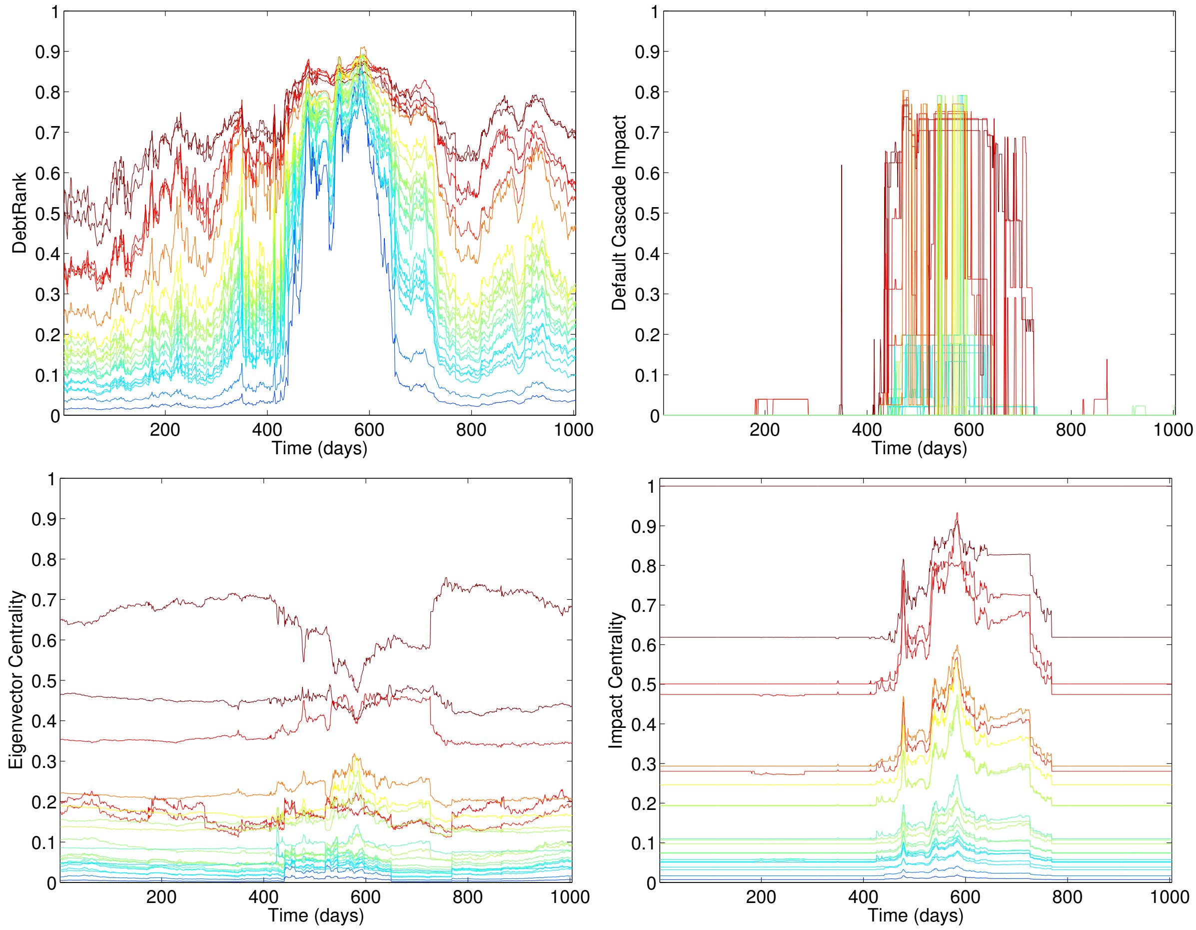

Figure 17: **Comparison of DebtRank with feedback centrality and default cascade impact**. Each curve refers to one institution over time. Case with scaling factor in the impact matrix $\alpha = 0.3$. Colors are assigned according to the ranking obtained with DebtRank, dark red (blue) are the most (least) systemically important. Top left: DebtRank. Top right: default cascade impact. Bottom left: eigenvector centrality. Bottom right: impact centrality.

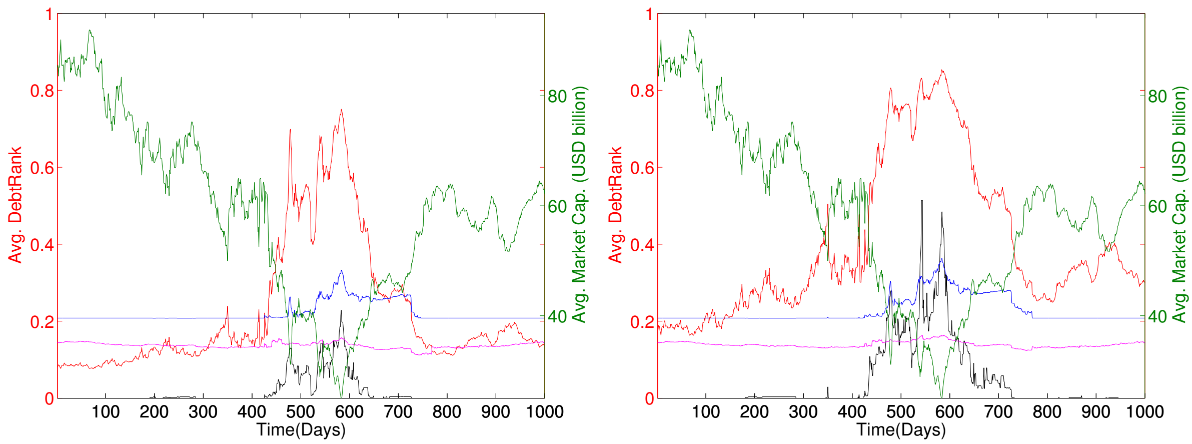

Figure 18: **Comparison of average values across institutions**. Each curve refers to the average value across institutions of one measure: average market capitalization (green); average default cascade impact (black); average eigenvector centrality (magenta); average impact centrality (blue). **Left**: case with scaling factor in the impact matrix $\alpha = 0.2$. **Right**: $\alpha = 0.3$.

3.5.2 Impact-Matrix Weight Evolution

Figure shows the evolution in time of the distribution of weights in the impact matrices utilized in the previous section. For the sake of readability, only one every ten days is considered for constructing the histogram of relative frequencies of the weights.

We recall that, as explained in the main text, the initial values of the impact matrix are the investments made by banks in each other's equity shares, normalized. This is used as a proxy of the real exposure for the purpose to show what could be done with the real data. Therefore, the weights evolve in time because they are rescaled according the variations in the market capitalization of banks. The smaller the market capitalization of a bank, the larger the impact that the distress or default of a counterparty would have. Accordingly, the weights of the counterparty increase as the banks market capitalization decreases and viceversa.

As we can observe in the figure, the distributions are peaked over small values of the weights until around day 300 (from blue to cyan). During days 500-700 (green to yellow), corresponding to the period around March 2009, the distribution display a significan shift in probability mass at values close to 1. This is because the impact of many institutions over the others become much stronger (the maximum impact is 1 by construction). Finally, in the following months the weights distribution becomes again peaked at small values.

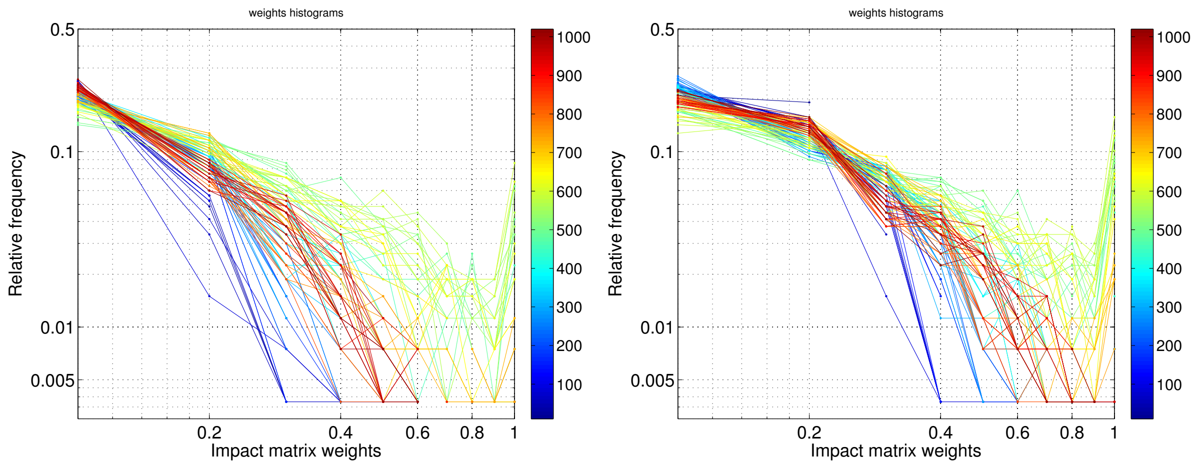

Figure 19: Evolution in time of the distribution of weights in the impact matrices. Only one every ten days is considered for constructing the histogram of relative frequencies of the weights. The network is not fully connected and only the non-zero weights in the adjacency matrix are considered, which are 0.58% of the total. Left: case with scaling factor in the impact matrix $\alpha = 0.2$. Right: $\alpha = 0.3$.

3.6 GroupDebtRank: Systemic Importance of a Group of Institutions

A very interesting aspect of DebtRank is that it can also be used to measure the impact of the simultaneous default or distress of several nodes in the network (see Methods in the main text). We have conducted a series of experiments in which in each day we assign an initial level of distress, equal to $\psi < 1$, to all nodes in the network and we measure the total loss that would derive in the system. For instance, a distress $\psi = 0.1$ represents here a devaluation of 10 % of the core capital of the institution. This exercise aims to capture the situation of a dispersed shock to all the players. The same procedure could be applied to any subset of nodes of interest. We refer to this procedure as **GroupDebtRank**, where the set of nodes initially in distress and the level of distress have to be specified. Figure 20 shows the results obtained with $\psi = \{0.1, 0.2, 0.3\}$ (corresponding to a devaluation of 10, 20, 30 % of the core capital) and with the scaling factor $\alpha = \{0.2, 0.3\}$. As explained in the section Methods of the main text, $\alpha = \{0.2\}$ can be considered a conservative scenario because it implies that each bank can withstand the default of at least five counterparties. As we can observe in the figure, with $\psi = \{0.1$, GroupDebtRank is already at levels above 0.2 even when the market capitalization has not yet declined significantly. This means that a shock of 10 % gets amplified by the network into a global devaluation of 20 %. At the peak of the crisis, the same shock of 10 % gets amplified into a 70 % loss. These numbers would change if DebtRank is computed using the actual data, but they give a concrete idea of how significant can be the network effect.

Finally, notice that GroupDebtRank differs not only conceptually but also numerically from the average of DebtRank across institutions (compare Fig. 20).

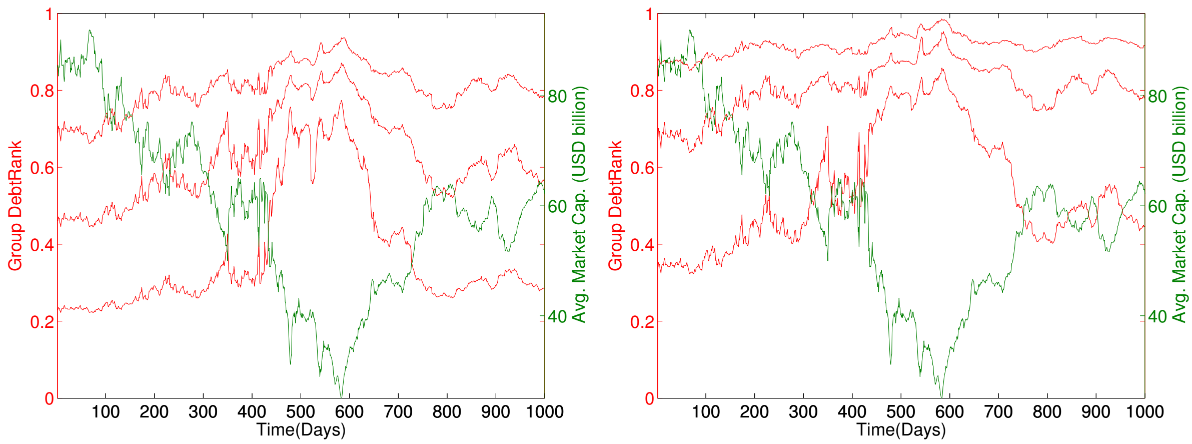

Figure 20: **Group DebtRank evolution in the case of dispersed shocks**. Plot over time of the value of Group DebtRank (red curves). Each value represents the total loss due to an initial level ψ of distress on all nodes. The three red curves correspond to $\psi = \{0.1, 0.2, 0.3\}$ (the higher ψ, the higher the curve). For sake of a comparison, the average value of the market capitalization across the institutions is plotted in green. **Left**: case with scaling factor in the impact matrix $\alpha = 0.2$. **Right**: $\alpha = 0.3$.

3.7 DebtRank vs Asset Size

One could argue that a higher asset size implies a higher chance to attract investments from others and therefore implies, as a tendency, a higher impact on others and a higher DebtRank. However, it is also easy to construct configurations in which the institutions with the highest DebtRank are not those with the highest asset size. The contribution of this paper to the discussion on too-big too fail goes beyond the fact of providing a ranking. Indeed, by accounting explicitly for the network effects, DebtRank provides an estimate of the total loss, regardless of whether the institutions with the highest DebtRank happen also to be those with the highest asset size.

To better clarify this issue, we have compute the correlation coefficient between DebtRank and asset size both in the empirical data and in a sample of daily realizations in which we reshuffle the values of DebtRank across nodes. We find a weak but significant level of correlation that decreases towards the peak of the crisis. The coefficient ranges from $\rho = 0.4$ in the beginning of the crisis to $\rho = 0.2$ in the middle of the crisis (corresponding to period 4). In Fig. 21 we plot the daily random correlation between assetsize and debtrank (blue), the daily estimated correlation between assetsize and debtrank (red) and we compare it with the debtrank computed daily. The fact that the correlation between asset size and debtrank decreases towards the peak of the crisis suggests that asset size alone is not a good proxy of systemic importance.

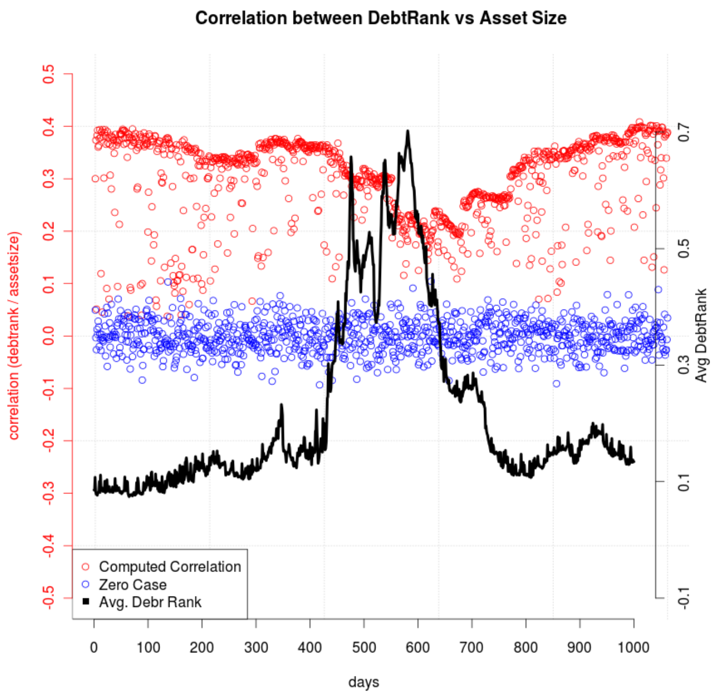

Figure 21: Correlation between asset size and DebtRank for the 22 nodes of the network. Values are computed daily. The correlation in the case of randomly reshuffled series (in blue) serves as "zero" reference level. It represents the correlation level existing by chance between the two variables. The daily value of the average debt rank across institutions is plotted in black. We observe that the correlation between the DebtRank and the AssetSize is always below 0.4 and it decreases during the peak of the crisis.

4 Top players

The list of major institutions granted loans by the FED is reported in Table 2. Notice that not all the institutions having a high peak in their outstanding debt had also a high average debt. Many institutions are not US-based, such as Bank of Scotland or Deutsche Bank, but all of them had branches in the US and were thus eligible for funding by the FED.

Institution FED peak debt		**Institution FED daily average debt**	
Institution	**USD billions**	**Institution**	**USD billions**
Morgan Stanley	107.29	Royal Bank of Scotland	21.34
Citigroup	99.45	Bank of America	20.71
Bank of America	91.4	Citigroup	19.61
Royal Bank of Scotland	84.5	Barclays	19.10
State Street	77.80	Dexia	15.37
UBS	77.15	UBS	13.89
Goldman Sachs	68.96	Credit Suisse	13.29
JPMorgan Chase	68.35	Deutsche Bank	12.48
Deutsche Bank	66.01	JPMorgan Chase	11.78
Barclays	64.89	Hypo Real Estate	10.71
Merrill Lynch	62.11	Wells Fargo	8.45
Credit Suisse	60.8	Merrill Lynch	8.33
Dexia	58.45	HBOS	8.26
Wachovia	50.09	Norinchukin Bank	7.5
Lehman Brothers	46.023	Goldman Sachs	7.53
Wells Fargo	45.00	State Street	7.07
Bear Stearns	30.00	BNP Paribas	7.05
BNP Paribas	29.25	Societé Generale	6.90
Hypo Real Estate	28.7	Wachovia	6.87
Fortis Bank	26.27	Morgan Stanley	6.73

Table 2: **List of the largest borrowers of the FED in 2008-2010**. (Left) Top 20 borrowers by *peak* value of their debt. (Right) Top 20 borrowers by *averaged* debt

5 Further Background Information

5.1 Emerging Loans Programs

The following list quoted from (GAO, 2011) and from Investopedia (2011) dictionary describes the FED funding emergency programs (or funding facilities) put in place during the credit crisis and used by Bloomberg to classify the loans.

- **CPFF** The CPFF is an institution created by the Federal Reserve Bank of New York on Oct. 27, 2008, as a result of the credit crunch faced by financial intermediaries in the commercial paper market. The Commercial Paper Funding Facility (CPFF) provides liquidity to U.S. issuers of commercial paper registered with the CPFF through a special purpose vehicle (SPV) that is funded by the Federal Reserve Bank of New York. The peak amount of outstanding debt reached by this program was of USD 348 billions.

- **PDCF**. The Primary Dealer Credit Facility (PDCF) was an institution created by the FED to provide overnight loans to primary dealers through their clearing banks in exchange for eligible collateral. The PDCF operated from Mar. 2008 to Feb. 2010. The largest five borrowers accounted for approximately 82.5% of the total amount of the PDCF loans (see GAO (2011)). The peak outstanding amount reached USD 130 billions.
- **TSLF** A lending facility through the Federal Reserve that allows primary dealers to borrow Treasury securities on a 28-day term by pledging eligible collateral. The eligible securities under the term securities lending facility include 'AAA' to 'Aaa' rated mortgage-backed securities (MBS) not under review for downgrade, and all securities eligible for tri-party repurchase agreements. In exchange for this collateral, the primary dealers receive a basket of Treasury general collateral, which includes Treasury bills, notes, bonds and inflation-indexed securities form the Fed's system open market account.TSL loans outstanding, included TSLF Option Program loans, peaked at USD 236 billion.
- **DW** The discount window (DW) is an instrument of monetary policy that allows eligible institutions to borrow money from the FED, usually on a short-term basis to meet temporary shortages of liquidity. During the period 2007-2009, DW has been altered. On August of 2007, the discount rate was cut by 50 bp (basis points) and the term of loans was extended from overnight to thirty days. Then, on March 16, 2008, the rate was further cut and the term was further extended to ninety days. From January of 2007 to January of 2010 the discount rate was cut by 620 bp (from $6,25\%$ to 0.5%).
- **ST-OMO**. The lending program single-tranche open-market operations (ST-OMO) was supervised by the New York Fed and consisted in lending 28-day loans from March of 2008 to December of 2008. The FED used an auction process in which the banks bid an interest rate they were willing to pay for the credit. At the program's peak, financial companies asked the Fed for a total of USD 80 billions. The last auction was on 30th of December, 2008, when Goldman Sachs, got USD 200 millions at an interest rate of 0.01% when the FED's main lending facility was at the same time charging an interest of 0.5%.

5.2 Glossary

For the reader convenience we report here several definitions of financial tools from the Investopedia (Investopedia, 2011) dictionary .

- **ABCP**. Asset backed commercial papers are a short-term investment vehicle with a maturity that is typically between 90 and 180 days. The security itself is typically issued by a bank or other financial institution. The notes are backed by physical assets such as trade receivables, and are generally used for short-term financing needs.
- **MBS**. A type of asset-backed security that is secured by a mortgage or collection of mortgages. These securities must also be grouped in one of the top two ratings as determined by a accredited credit rating agency, and usually pay periodic payments that are similar to coupon payments. Furthermore, the mortgage must have originated from a regulated and authorized financial institution.
- **SPV**. A special purpose vehicle, also referred to as a "bankruptcy-remote entity" whose operations are limited to the acquisition and financing of specific assets. The SPV is usually a subsidiary company with an asset/liability structure and legal status that makes its obligations secure even if the parent company goes bankrupt.

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