PGD Variational vademecum for robot motion planning. A dynamic obstacle case.

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ABSTRACT

Abstract:
A fundamental robotics task is to plan collision-free motions for complex bodies from a start to a goal position among a set of static and dynamic obstacles. This problem is well known in the literature as motion planning (or the piano mover's problem). The complexity of the problem has motivated many works in the field of robot path planning. One of the most popular algorithms is the Artificial Potential Field technique (APF). This method defines an artificial potential field in the configuration space (C-space) that produces a robot path from a start to a goal position. This technique is very fast for RT applications. However, the robot could be trapped in a deadlock (local minima of the potential function). The solution of this problem lies in the use of harmonic functions in the generation of the potential field, which satisfy the Laplace equation. Unfortunately, this technique requires a numerical simulation in a discrete mesh, making useless for RT applications. In our previous work, it was presented for the first time, the Proper Generalized Decomposition method to solve the motion planning problem. In that work, the PGD was designed just for static obstacles and computed as a vademecum for all Start and Goal combinations. This work demonstrates that the PGD could be a solution for the motion planning problem. However, in a realistic scenario, it is necessary to take into account more parameters like for instance, dynamic obstacles. The goal of the present paper is to introduce a diffusion term into the Laplace equation in order to take into account dynamic obstacles as an extra parameter. Both cases, isotropic and non-isotropic cases are into account in order to generalize the solution.

INTRODUCTION

- A fundamental robotics task is to plan collision-free motions among a set of static and known obstacles from a start to a goal position.
- The geometric construction of this planning strategy is computationally hard and hence unfeasible for its use in real-time applications.
- This motion planning (or the piano mover's) problem has motivated many works in the field of robotics.
- Lately, a novel approach called the Proper Generalized Decomposition (PGD) has appeared to approximate the solutions of non-linear convex variational problems.
- It is a new paradigm for solving classical problems in high-dimensional spaces. In the PGD framework, the resulting model is solved once in life in order to obtain a set that includes all the solutions for every possible value of the parameters, that is, a sort of computational vademecum.
- The goal of the present work is to develop a PGD-based computational Vademecum to allow the use of the potential field flow theory in real-time path planning applications in a dynamic environment. We will take into account that the robot will avoid the dynamic obstacles.

PREVIOUS WORK

Consider the functions, $g_S : \Omega_{\underline{X}} \times \Omega_{\underline{S}} \to \mathbb{R}$, $g_T : \Omega_{\underline{X}} \times \Omega_{\underline{T}} \to \mathbb{R}$ as 2D Gaussian density distributions centered in the start $\underline{S} = (s_1, s_2) \in \Omega_{\underline{S}}$ and the target configurations $\underline{T} = (t_1, t_2) \in \Omega_{\underline{T}}$ respectively.

Hence $\Omega_{\underline{X}} = \Omega_x \times \Omega_y$, $\Omega_{\underline{S}} = \Omega_s \times \Omega_r$ and $\Omega_{\underline{T}} = \Omega_t \times \Omega_r$.

Let's assume that the source term f in the equation is non-uniform, that is, $f = g_S - g_T$.

Then, the Poisson equation is now, $-\Delta u(\underline{X}, \underline{S}, \underline{T}) = f(\underline{X}, \underline{S}, \underline{T})$

The PGD-Vademecum is constructed considering that the solution of the potential field u can be constructed as a finite sum of terms, each one consisting of the product of three functions: a function R of the environment $\underline{X}$, a function W of the start configuration $\underline{S}$ and a function K of the target or goal configuration $\underline{T}$:

$$u^{n-1}(\underline{X}, \underline{S}, \underline{T}) = \sum_{i=1}^{n-1} R_i(\underline{X}) \cdot W_i(\underline{S}) \cdot K_i(\underline{T})$$

and where the enrichment step is given by:

$$u^n = u^{n-1} + R(\underline{X}) \cdot W(\underline{S}) \cdot K(\underline{T})$$

PGD-VADEMECUM SOLUTION WITH OBSTACLES

Let's consider a dynamic environment with dynamic obstacles. The mobile robot needs to avoid each obstacle in real time. This situation implies that the domain changes because the path of the mobile robot should change to avoid the dynamic obstacle. For that reason we introduce $K(x,p)$ to change the domain and the equation to solve is:

$$\nabla \left(K(\underline{X}, p) \nabla u \right) = f$$

where,

$$K(\underline{X}, p) = \sum_{i=1}^{M} K^x(\underline{X}) K^p(p)$$

p is the parameter associated to the obstacle

The PGD-solution is expressed as follows,

$$u(\underline{X}, p) = \sum_{i=1}^{N} X_i(\underline{X}) P_i(p)$$

PGD AT A GLANCE

Consider the two dimensional Poisson equation as follows,

$$\Delta u(x, y) = f(x, y)$$

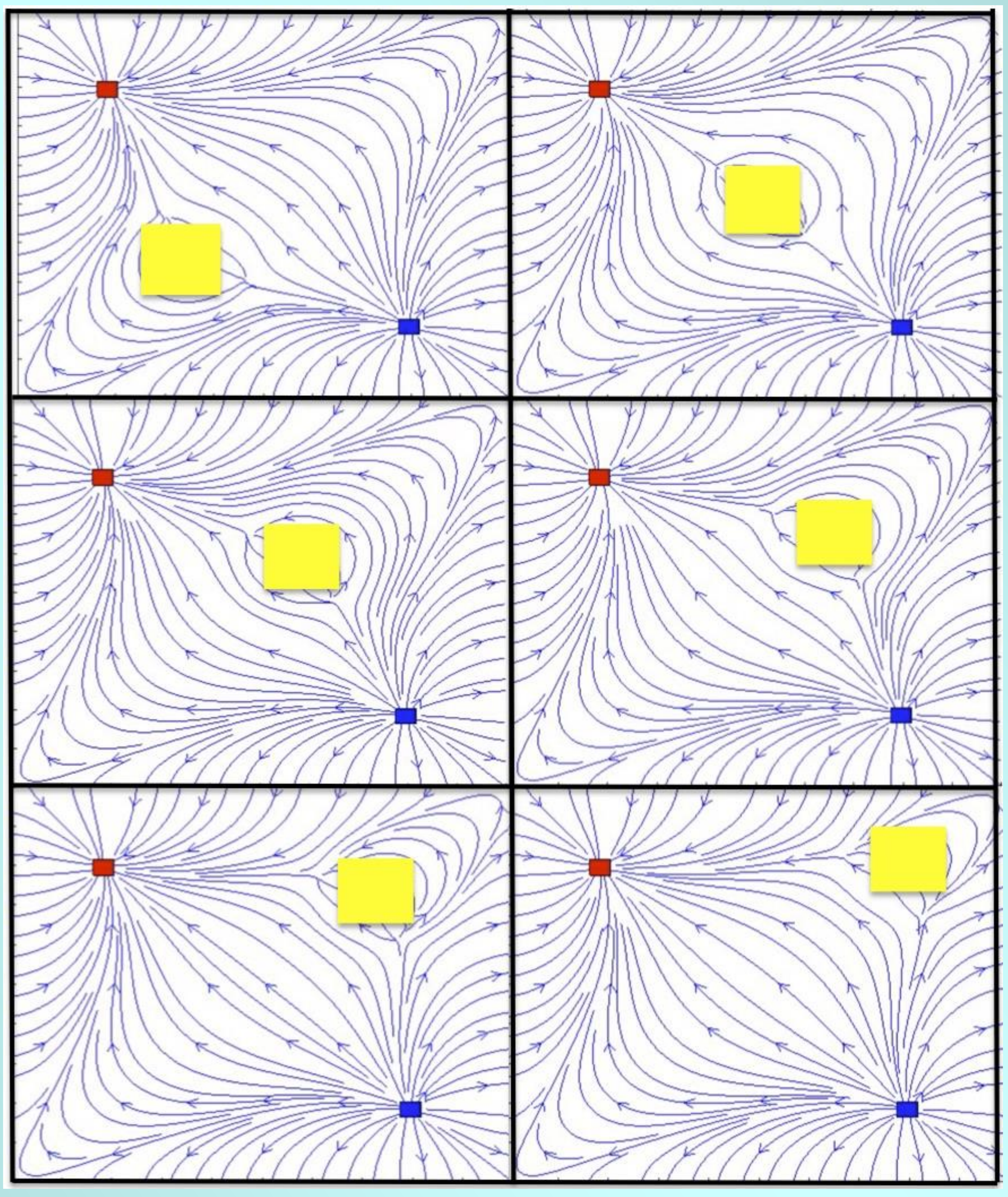
over a two-dimensional rectangular domain: $\Omega_{\underline{X}} = \Omega_x \times \Omega_y \subset \mathbb{R}^2$ with Neumann boundary condition $\left.\frac{\partial u}{\partial n}\right|_{\Gamma} = q$ where q is a real number.

The basic idea is to assume that the exact solution $u(x,y)$ of the Poisson equation in the square domain can be written as an infinite sum of terms,

$$u(x, y) = \sum_{i=1}^{n} X_i(x) \cdot Y_i(y)$$

The computation of the approximation solution in this form can be done by computing each term of the expansion one at a time.

SIMULATION RESULTS



The robot follows the path from the Start (blue point) to the Goal (red point) by means of the PGD-Vademecum solution avoiding the dynamic obstacle (yellow square).

CONCLUSIONS

The present study shows**, for the first time,** the application of the technique known as PGD-Vademecum to global path planning in mobile robots introducing dynamic obstacles. This method produces, for a predefined map, a Vademecum containing all the possible robot paths for any combination of Start and Goal configurations in the map avoiding the dynamic obstacles in the environment.