A Mathematical Study for Promoting Disability Inclusion in Glaucoma: A Comprehensive Approach

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ABSTRACT
Globally, glaucoma—a progressive optic neuropathy—is one of the leading causes of permanent blindness. This abstract provides an overview of glaucoma’s effects on visual impairments, emphasizing the physical, psychological, and social consequences affected individuals face. Glaucoma primarily damages the optic nerve, leading to gradual peripheral vision loss, often unnoticed until an advanced stage. This visual impairment significantly lowers the quality of life by making it difficult to read, drive, and recognize faces, among other daily activities. This study examines how prostaglandin analogues affect intraocular pressure (IOP) and aqueous humor outflow into Schlemm’s canal via the trabecular meshwork emphasizing how these effects relate to glaucoma disability. The essential ocular fluid known as aqueous humor travels from the posterior part behind the iris via the pupil aperture into the anterior region. It ultimately leaves the eyes through the drainage system. The eyes’ ciliary body, an essential component of the drainage system, continuously produces aqueous humor, mainly ejected through the trabecular meshwork into Schlemm’s canal. Increase in IOP, a defining feature of glaucoma disability, is brought on by a decreased aqueous humor drainage from the angle of the drainage system. This abstract underscores the importance of proactive screening, treatment, and support for individuals with glaucoma to mitigate visual disabilities and enhance their overall well-being. This study emphasizes the critical role that aggressive glaucoma screening, effective treatment, and strong support networks play in reducing the impact of visual impairments and improving the overall health of those who are affected.

KEYWORDS
glaucoma, disability model, low-vision service, visual disability, aqueous humor

INTRODUCTION
Glaucoma is one of a series of eye conditions that can cause harm to the optic nerve, which in turn can result in blindness. By sending visual information to the brain, this nerve creates a mental image of whatever the eyes are now seeing. The optic nerve can cause blind spots in vision when it is not working properly. An optic neuropathy characterized by distinctive alterations to the optic nerve head and nerve fiber layer is called glaucoma. As the illness advances, certain abnormalities in the visual field arise. Low sensitivity, specificity, and positive predictive value characterize intraocular pressure (IOP) screening for glaucoma, despite its status as a risk factor. Relying solely on IOP may lead to a failure to diagnose up to 50% of glaucoma cases where the pressure is below 22 mm Hg during screening. The definition of glaucoma no longer considers an IOP of 22 mm Hg or above, as one-sixth of those with the condition consistently exhibit pressures below this threshold.

Primary open-angle glaucoma (POAG), angle-closure glaucoma, congenital glaucoma, and secondary glaucoma are the several forms of glaucoma. 60 to 70% of instances of glaucoma are POAG, making it the most prevalent kind of glaucoma (Ramulu, 2009). It is typically bilateral, though not always symmetric. The presence of typical glaucomatous optic nerve and visual field damage, adult onset, open, normal-appearing anterior chamber angles, and the lack of additional causes of open-angle glaucoma, such as trauma, set POAG apart from other glaucoma. Although the exact cause of primary open-angle glaucoma is not known, it seems to be related to increased resistance to the outflow of aqueous through the trabecular meshwork. This progressive resistance to outflow results in a gradual increase in IOP, which may result in glaucomatous optic nerve damage in susceptible patients. Angle-closure glaucoma is the second most common type of glaucoma. In this case, the
Glaucoma may not manifest any symptoms or vision loss until the severity of the ailment and how long it has been present. Glaucoma's impact on vision might vary depending on the eye pressure, frequently injures the optic nerve. The optic nerve, responsible for transmitting visual data from the eyes to the brain, is harmed by glaucoma sickness. A rise in IOP, often known as eye pressure, frequently injures the optic nerve. Glaucoma’s impact on vision might vary depending on the severity of the ailment and how long it has been present. Glaucoma may not manifest any symptoms or vision loss in the early stages. However, if the illness worsens, these signs and symptoms could appear. Peripheral vision loss due to glaucoma frequently happens gradually and may not be apparent until the disease progresses. As the field of vision narrows, the loss of peripheral vision might give the impression that one is seeing through a tunnel. Blurred vision is a common symptom of glaucoma, especially at the borders of the visual field. If you have glaucoma, it can be more challenging to discern between different grayscales and perceive objects in low light. Glaucoma can eventually result in total blindness if neglected. Therefore, it is crucial to get routine eye exams and to start glaucoma therapy as soon as it is identified (Cesareo et al., 2015). About 15 to 20% of people still eventually go blind in at least one eye despite receiving treatment. The Social Security Administration (SSA) knows that working might be challenging for people with decreased vision. Anyone who has severe glaucoma-related vision loss is eligible for benefits.

A degenerative eye condition called glaucoma can impair eyesight and render a person disabled. Glaucoma can make it more difficult for a person to carry out everyday tasks, work, and engage in social and recreational activities, depending on the condition’s severity and the individual’s unique circumstances. Glaucoma may qualify a person for disability benefits if it meets the SSA’s requirements for glaucoma disability. To be eligible for disability compensation, however, a person must prove that their glaucoma satisfies specific medical conditions and interferes with their ability to work and function (Goel et al., 2010). Glaucoma management and therapy, which may include a combination of medication, surgery, and other modifications, can help lessen its impact on a person’s everyday life. Glaucoma does not always mean you have reached the end of your prime earning years. Because of recent developments in therapy, technology, and public knowledge of the problem, people with glaucoma can live active, healthy lives. Blindness could happen if glaucoma is not addressed. Unfortunately, 10% of glaucoma patients who receive adequate treatment lose their vision (Wierzbowska, 2020). Glaucoma-related vision loss cannot be reversed. Therapy and surgery may be used to halt further vision loss. Glaucoma can result in progressive vision loss, making it more challenging to carry out regular tasks like reading, driving, and navigating your surroundings. Reduced QoL and social isolation may result from the effects of glaucoma on your vision and daily activities. If you are given a glaucoma diagnosis, it is understandable to worry. There are more than 500,000 persons who have glaucoma. While some people can handle the problem with little adjustments, others will have a severe visual impairment. In either case, it’s critical to understand that support is available for you.

Clinicians may be helped by understanding how glaucoma impacts daily activities to provide each patient the best possible care and rehabilitation. When providing repair, it is important to understand how the individual perceives their vision impairment in their daily life. By creating a rehabilitation program tailored to these patients’ needs and helping them make the most of their residual visual abilities, you can support their continued independence. These changes could increase the amount of time that older people can live alone and provide them greater control over daily tasks. Those who have lost their central vision as opposed to those who have lost their visual function (VF), such those with glaucoma, are the target audience for rehabilitation therapies, like...
reading programs. Developing countries, particularly India, do not pay attention to low-vision services despite having many other organized eye-care facilities. It’s true that visual impairment, including low vision and blindness, has a substantial global impact, albeit its exact scope is unknown. The World Health Organization states that 246 million individuals worldwide suffer from moderate to severe low-vision issues. When cataracts (43%) and refractive errors (33%) are subtracted from the global estimate of visual impairment, more than 70 million people will experience low-vision problems. Approximately 90% of them are nationals of low- and middle-income countries.

Emotional and psychological effects

The emotional and psychological strain of managing a chronic illness like glaucoma can result in despair, anxiety, and other mental health problems. Collaborating with your medical team and other professionals is crucial to addressing these issues, developing management plans for your illness, and keeping your freedom and living standards intact. This may entail medical or surgical interventions, visual therapy, assistive technology, and other modifications (Grierson, 1974).

Benefits for disabilities based on limited functional capacity, eligibility

Eyes are the most important part of our body which requires the utmost care until one’s life ends. We cannot study, walk, read, participate in class, and work without eyes or good vision. We consider the idea to be commonplace. Vision impairment arises when an eye problem impairs the visual system and its functions; everyone, if they live long enough, will eventually develop at least one eye condition requiring medical attention. A person with a vision impairment has major lifelong effects. Fast access to first-rate eye care can mitigate the majority of these impacts. The main focus of eye-care techniques is, understandably enough, on problems like cataracts and refractive errors; however, conditions like dry eye and conjunctivitis, which usually do not impede vision, are equally important (González and Fitt, 2003). These issues are typically among the most common ones for which people seek out eye-care services.

Effect of vision impairment

Academic performance may also be negatively impacted by eye issues in school-aged children (Lyubimov et al., 2007). In adult populations, visual impairment has a major impact on QoL. In addition to having lower employment rates, adults with visual impairment may also have higher levels of anxiety and depression. Age-related vision impairment in older adults can raise their risk of fractures and falls, such as early admission to assisted living or nursing homes, social isolation, and trouble walking.

The existing body of literature lacks comprehensive data pertaining to the extent of visual impairment experienced by individuals afflicted with glaucoma. This study was undertaken with the overarching objective of elucidating the prevalence of self-reported visual impairment within the spectrum of routine activities among glaucoma patients. Furthermore, this research endeavor sought to stratify the observed impairments, delve into the interrelationships among various disability facets via factor analysis, scrutinize the correlations between the severity of visual field loss and the subjective perception of visual impediments, devise a tailored set of inquiries aimed specifically at glaucoma patients, and assess the soundness and consistency of this distinct subset of questions. In addition to being a health issue, glaucoma also contributes significantly to global disability. To guarantee that those with glaucoma may lead happy lives and have equitable access to opportunities and services, disability inclusion in glaucoma is crucial. This literature review has emphasized the existing difficulties and potential solutions for enhancing disability inclusion in glaucoma. Society may better support persons with glaucoma and improve their overall QoL through increasing awareness, enhancing accessibility, and campaigning for legislative changes. To close the gap between glaucoma and disability inclusion, further research and group initiatives are required (Mills et al., 2001).

Methods for treating eye diseases to prevent vision impairment

Effective interventions are available to meet the requirements related to eye disorders and vision impairments in promotion, prevention, treatment, and rehabilitation. Relevant evidence regarding the degree of visual impairment that patients with glaucoma experience is lacking. The primary objective of this investigation was to determine the prevalence of self-reported visual impairment affecting the routine activities of individuals afflicted with glaucoma, establish a hierarchical ranking of the most prevalent impairments, explore potential interrelationships between these disabilities via factor analysis, examine the association between the degree of visual field impairment and the perceived level of visual hardship, construct a tailored set of inquiries tailored for glaucoma patients, and rigorously assess the validity and reliability of this custom subgroup of questions. In addition to worsening symptoms glaucoma’s bilateral visual field loss is linked to a discernible decline in mobility and ability to drive (Broman et al., 2002). More investigation is needed to determine whether persons with unilateral glaucoma are substantially impacted, whether reading becomes difficult for those with bilateral glaucoma on a regular basis, and how lighting affects task performance in glaucoma patients. Though largely preventable, glaucoma ranks as the second leading cause of blindness globally. Over 50% of individuals afflicted still go untreated, especially in developing countries. Primary open-angle and primary closed-angle glaucoma were estimated to affect 64 million people globally in 2013. By 2020, that number is expected to rise to 76.0 million, and by 2040, it is projected to reach 111.8 million. The population’s aging will result in a rise in glaucoma cases in the coming years, which may lead to dramatic increase in the quantity of individuals suffering from visual impairment.
associated with glaucoma. The QoL of patients is negatively impacted by the numerous incapacitating impacts of visual loss caused by glaucoma on their daily lives. The term quality of life describes the difference between an individual’s current experiences and the desired future state.

According to J.E. Ross, in chronic uncomplicated glaucoma, visual function tests define the condition, measure optic nerve damage and project visual disability. Technology development can improve glaucoma’s accessibility for people with disabilities. For instance, voice-activated devices and screen readers can make it easier for people with visual impairments to access information online and go around the digital world. Technology accessibility investments are essential for fostering inclusivity (Aspinall et al., 2008).

The evaluation of visual fields and visual acuity are two standard psychophysical tests of visual function. These tests enable the practitioner to assess the severity and course of the disease while also identifying the location and size of the visible defect. However, these tests only have a limited capacity to reveal the patient’s level of incapacity. Although the clinical history is a crucial verification, there have been few attempts to quantify visual handicaps. Additionally, this is the least formal part of the clinical evaluation. Worse symptoms and a discernible impairment in mobility and driving ability accompany bilateral glaucoma-related reduction of visual field (Burr et al., 2007).

Further research is required to identify whether individuals with unilateral glaucoma are significantly affected and whether people with bilateral glaucoma frequently have trouble reading, and to determine how illumination affects how well people with glaucoma do tasks. Traumatic glaucoma may result from severe ocular trauma or injury. Diabetes can cause the retina to develop atypical blood vessels, which could obstruct eye outflow and induce glaucoma. High blood pressure, often known as hypertension, can harm the blood vessels in the eyes. You must first demonstrate a service connection for the condition that caused or aggravated your glaucoma to qualify it as a secondary handicap.

Disability and glaucoma: Which tasks to research?

Given that glaucoma is thought to impair a wide range of activities (McGwin et al., 1998), it is unclear which vision-related activities need the most attention from researchers. Investigations should ideally concentrate on activities that are both significant to having glaucoma sufferers and likely to be impacted by the condition. In most recent studies, participants with glaucoma were asked to select from various fictitious scenarios in which they experienced varying degrees of difficulty with multiple activities to determine which activities they valued the most. The tasks requiring central and close vision (such as reading) received the highest priority from the participants. At the same time, mobility outside the home (such as driving and strolling outside) also received high marks. Glare, running into things, and doing duties were given less consideration (McGwin et al., 2005). Demonstrating a causal relationship for a largely genetic condition like glaucoma can be challenging. Another efficient method for applying for Veterans affairs glaucoma benefits is to prove that the glaucoma was brought on by a secondary disability brought on by a condition or trauma related to service. Evidence links the following requirements to the onset or progression of glaucoma.

In most cases, glaucoma can only be identified during an eye exam. This examination can measure the pressure in the eyes and check for damage to the optic nerve. Regular tests are crucial to detect any early signs of glaucoma because many people with the disease may not exhibit symptoms.

Furthermore, glaucoma symptoms cannot be immediately recognized even when a person exhibits symptoms. Due to this, glaucoma may frequently go undiagnosed. To guarantee that glaucoma is detected early, it is imperative to visit an optometrist for eye exams regularly. Disability due to glaucoma is significant and has physical and psychological effects on sufferers. To create comprehensive solutions for resolving glaucoma-related disability, it is imperative to comprehend the complex nature of the condition. This analysis emphasizes the value of public education, accessibility, rehabilitation programs, and advocacy in fostering inclusion and raising the standard of living for glaucoma sufferers. To lessen the handicap caused by this illness, more investigation and coordinated efforts are required.

LITERATURE REVIEW

It was shown that glaucoma patients ran into objects more frequently than control participants. Driving is another crucial daily activity typically hampered in glaucoma patients; this is the most common complaint on the national eye institute visual function questionnaire (NEI-VFQ). The correlation between car accident rates and glaucoma patients has been examined and supported. McGwin et al. (1998) discovered that, compared to controls, glaucomatous people had a significantly higher risk of accidents. In addition, according to Haymes et al. (2007), glaucomatous patients experienced more car accidents than controls. Mathematical modeling studies can be characterized using several dichotomies that help to describe broad aspects, such as the scope and approach. Mathematical modeling studies are increasingly recognized as an important tool for evidence synthesis and to inform clinical and public health decision-making, particularly when data from systematic reviews of primary studies do not adequately answer a research question. Mathematical modeling studies are particularly pertinent to evidence synthesis and the translation of knowledge in clinical medicine and public health. Our society is greatly impacted by glaucoma and its management in terms of patient morbidity, lost productivity, ocular consultation frequency, and healthcare expenses. Three primary measurements are used to diagnose glaucoma: the IOP, an optic disc examination, and visual field testing (perimetry). In order to distinguish between POAG and chronic angle-closure glaucoma, for example, or to ascertain the etiology of glaucoma, examination of the anterior chamber angle with a gonioscopy contact lens is crucial (Haymes et al., 2007).

Furthermore, an earlier research investigation has revealed that individuals afflicted with glaucoma exhibit a propensity to refrain from night-time driving, navigating adverse weather
conditions, venturing out during rush hour, or engaging in highway traffic. Nonetheless, even after a previous, a large number of glaucomatous patients with significant VF loss continue to function. In addition to being a medical condition, Glaucoma also causes a lot of disability. Healthcare professionals, decision-makers, and society must comprehend the complex effects of glaucoma-related handicaps. We may endeavor to lessen the handicap caused by glaucoma and enhance the QoL for persons affected by this condition by eliminating inclusion barriers, increasing awareness, and lobbying for policy reforms. In the comprehensive management of individuals afflicted with low vision or visual impairments, the scope extends beyond mere diagnosis and the prescription of low-vision aids. A holistic approach is imperative, one that amalgamates clinical intervention for vision impairment with community-based vision rehabilitation services. This approach underscores the significance of a collaborative, multidisciplinary team, comprising specialized professionals including low-vision-focused ophthalmologists, optometrists responsible for therapeutic interventions and rehabilitation facets, special educators, occupational therapists, orientation and mobility trainers, social workers, counselors, and orthoptists. Such an interdisciplinary ensemble is indispensable for addressing the multifaceted needs of individuals with visual disabilities effectively (Nelson et al., 1999). The development of a common vocabulary for modeling studies across various clinical and epidemiological research domains encompassing infectious and non-communicable diseases will facilitate the comprehension, characterization, juxtaposition, and application of mathematical modeling studies by systematic reviewers and guideline developers. Decreased visual functioning due to glaucoma has many disabling consequences in patients’ daily lives that, in turn, alter their QoL. The term “quality of life” reflects the difference between a person’s hopes and expectations and his/her present experience (Anderson, 2009).

Patients with glaucoma often have difficulty reading, as demonstrated by the over 40% of patients with glaucoma who has participated in a pilot study tracking self-reported disability. Because reading is essential for many daily activities, reading disability can negatively impact QoL (Freeman et al., 2008). This study examines how prostaglandin analogues affect IOP and aqueous humor outflow into Schlemm’s canal via the trabecular meshwork, emphasizing how these effects relate to glaucoma disability. The pupil aperture allows the aqueous humor, the vital fluid of the eyes, to pass from the posterior chamber behind the iris into the anterior chamber. Ultimately, the drainage system allows it to exit the eyes. The ciliary body of the eyes, which is an essential component of the drainage system, continuously produces aqueous humor, which is mostly expelled into Schlemm’s canal via the trabecular meshwork. Decreased aqueous humor drainage from the drainage system’s angle causes increased IOP, a characteristic of glaucoma impairment.

**Research contribution in glaucoma**

The American Academy of Ophthalmology (AAO) projects that 12 million new cases of glaucoma will be diagnosed in 2020, bringing the overall number of cases to 76 million. It is more crucial than ever to do research and provide information on glaucoma, an age-related disorder in which high pressure from fluids inside the eyes causes irreparable damage to the optic nerve, frequently resulting in blindness. Already the second most common cause of blindness in the world is glaucoma. According to the AAO, the number of cases is predicted to rise past 2020 and reach over 111 million in 2040, with populations in Africa and Asia being disproportionately affected. “Sometimes, patients receive a diagnosis and become terrified at the mention of glaucoma.” However, glaucoma still causes blindness in modern society. However, most cases of glaucoma can be treated using eye drops and laser therapy if the condition is well managed and adhered to. Surgery will be necessary for certain people to reduce pressure. Frequent eye exams are a crucial part of the prevention of glaucoma, and a thorough examination is necessary for the diagnosis of the condition because over half of people with glaucoma can present with normal eye pressure. The main goal of the glaucoma treatments now on the market is to lower IOP without specifically treating the optic neuropathy and retinal ganglion cell loss that are linked to the condition. While lowering IOP is the main goal of treatment for glaucoma, there is growing interest in neuroprotective techniques because IOP reduction is sometimes insufficient to prevent the course of the illness.

It has been proposed that variations in ocular blood flow could modify retinal functioning and impact glaucoma prognosis. Numerous investigations have demonstrated a decrease in the blood flow to the eyes in individuals suffering from both preperimetric and progressive glaucoma, indicating a correlation between the two conditions. Raising IOP is the only known way to treat glaucoma, and it is known to be a significant risk factor for the onset and progression of glaucoma in all glaucoma subtypes.

**Problem formulation (model description)**

The present study shows that within the context of the human ocular system, situated in the anterior segment between the iris and the cornea, the dynamics of fluid flow are influenced by distinct thermal phenomena, as depicted in Figure 1.
The conceptual framework of this investigation entails the confinement of the fluid within the region bounded by the \( z = 0 \) plane and the impervious, solid corneal boundary \( \{z \}. \) In the analytical model, the boundary temperature is presumed to closely approximate the body’s physiological temperature. Additionally, it is pertinent to note that the cornea’s thickness is held constant at 0.6 mm. Notably, the experimental protocol involves positioning the subject under investigation in an upright orientation, thereby inducing gravitational effects along the positive \( x \)-axis (Nguyen et al., 2014).

The non-pigmented epithelium of the ciliary processes within the ciliary body generates the aqueous humor. The fluid exits the eyes through Schlemm’s canal, the trabecular meshwork, the anterior chamber, and the episcleral veins after passing through the pupil. The rate of aqueous production, the rate of aqueous outflow, and the episcleral venous pressure all contribute to the IOP. Age, systemic blood pressure, genetics, and topical or systemic drugs are only a few of the numerous variables that might impact the IOP (Heys et al., 2001).

This thermally driven aqueous humor circulation can be effectively modeled by adhering to fundamental principles of mass conservation, momentum transfer, and energy conservation. Within the majority of aqueous humor circulation systems, the flow traverses Schlemm’s canal through the “trabecular meshwork,” requiring it to travel a short distance along the canal in order to access the collector channel. This segment of the canal, positioned between two collector channels, is commonly referred to as an elliptical channel, whereby half of its surface area is characterized by porosity, facilitating direct contact with the trabecular meshwork, as documented in prior investigations (Crowder and Ervin, 2013).

It has been detailed how the temperature differential between the anterior surface of the cornea and iris causes the buoyancy-driven fluid movement of aqueous humor. The Boussinesq model of fluid density for thermally driven convection flow, the lubrication theory, and the Navier-Stokes equation serve as the foundation for the model created in this study. According to this model, radiation, evaporation, and convection all contribute to heat loss at the corneal surface. Although there is some variation in density, there is very little pressure in the anterior chamber of the human eyes (Fitt and Gonzalez, 2006). The density, expansiveness, and viscosity of aqueous humor are comparable to those of water and protein. There is therefore no inflow via the pupil aperture. The flow is shown by the Navier-Stokes equation.

The fluid velocity is defined in the Navier-Stokes equation.

\[
q = u e_x + v e_y + w e_z
\]

where \( e_x, e_y \) and \( e_z \) are the unit vectors in the directions of \( x, y \) and \( z \), respectively, in the rectangular coordinate system.

### Solution to the problem

The ability of buoyancy-driven currents to circulate aqueous humor in the anterior chamber has long been known. The temperature differential between the iris shown in (Figure 2), which is largely kept at body temperature, and the anterior surface of the cornea, which is exposed to ambient conditions during normal waking hours, is what drives these currents. In the experimental work by Heys and Barocas (2002), buoyancy-driven anterior chamber flow was experimentally demonstrated by applying hot or cold packs to a human eyes’ closed lids to induce changes in aqueous circulation.

Theoretically, the aqueous humor fluid flow phenomena were examined by Canning (2002), who reported that “lubrication theory” and the limit of the “Navier-Stokes equations” were appropriate for the system of equations. The final partial differential equations and the Boussinesq approximation equation are shown below.

\[
-\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} + [g(1 - \alpha(T - T_0))] = 0
\]

\[
\frac{\partial p}{\partial x} + (\nu \frac{\partial v}{\partial x}) = 0
\]

\[
u \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial (\nabla u) \cdot \nabla u}{\partial x} = 0
\]

\[
\frac{\partial T}{\partial x} = 0
\]

As well as the boundary conditions, which are presented like this:

\[
u = v = w = 0, \quad T = T_0 \text{ on } z = 0
\]

\[
u = v = w = 0, \quad T = T_0 \text{ on } z = h(x, y)
\]

The notations in the equation above are as follows: \( p \) represents the pressure, \( T \) corresponds to temperature, \( z \) designates the coordinate normal to the iris, \( x \) and \( y \) serve as the coordinates within the plane of the iris, \( q = (u, v, w) \)
stands for the aqueous velocity vector, \( g \) represents the acceleration due to gravity, \( \alpha \) signifies the coefficient of linear thermal expansion of the aqueous humor, and subscripts are used to indicate differentiation. Furthermore, it is postulated that the cornea’s posterior surface is positioned at a distance \( z = h(x, y) \) from the reference plane, while the component \( w \) is constrained to \( w = w(x, y) \) in the same coordinate system.

Now, we can easily find the above Equation from (3)-(5):

\[
T_{zz} = 0 \rightarrow \frac{\partial^2 T}{\partial z^2} = 0 \rightarrow \frac{dT}{dz} = T_c \rightarrow T = T_c z + T_D \quad (8)
\]

\[
T = T_0 \text{ and } z = h(x, y) \quad (9)
\]

\[
T_0 = T_c h(x, y) + T_D \quad (10)
\]

Again, applying the given boundary conditions in Equation (8):

\[
T = T_0 \text{ on } z = 0 \quad (11)
\]

\[
T_i = T_c \times 0 + T_D \quad (12)
\]

\[
T_i = T_D \quad (13)
\]

Now Equation (8) takes place,

\[
T_0 = T_c h(x, y) + T_i \quad (14)
\]

\[
T_c = \left[ \frac{T_0 - T_i}{h(x, y)} \right] \quad (15)
\]

Now, we have to put the values of \( T_c \) and \( T_D \) in Equation (8);

\[
T = \left[ \frac{T_0 - T_i}{z} \right] h + T_i \quad (16)
\]

\[
T = \left[ T_i + \left( \frac{z}{h} (T_0 - T_i) \right) \right] \quad (17)
\]

Again Equation (2) is given by

\[
-\frac{P_s}{\rho_a} + vu_{zz} + [g(1 - \alpha(T - T_0))] = 0 \quad (18)
\]

We have to put the value of \( T \) in the above equation. Then, it will take the form;

\[
-\frac{P_s}{\rho_a} + vu_{zz} + \left[ 1 - \alpha(T_i - T_0) \left( 1 - \frac{z}{h} \right) \right] = 0 \quad (19)
\]

The above equation also can be written as follows:

\[
u_{zz} = \frac{p_s}{\rho v} - \frac{g}{v} \left[ 1 - \alpha(T_i - T_0) \left( 1 - \frac{z}{h} \right) \right] \quad (20)
\]

\[
u_{zz} = \left[ \frac{p_s}{\rho v} - \frac{g}{v} \left( T_0 - T_i \right) \right] \frac{\alpha}{v} \left( T_i - T_0 \right) \left( \frac{z}{h} \right) \quad (21)
\]

\[
u = \frac{\rho_s (z^2 - hz)}{2 \rho h} \quad (22)
\]

Similarly, we also can find that

\[
v = \frac{\rho_s (z^2 - hz)}{2 \rho h} \quad (23)
\]

And

\[
v = \frac{\rho_s (z^2 - hz)}{2 \rho h} \quad (24)
\]

We will use classical fluid mechanics and asymptotic analysis to examine each of the several mechanisms that result in the flow of aqueous inside the anterior chamber, including temperature differences, secretory flow from the ciliary body through the pupil aperture and eventually to the trabecular meshwork, gravity, and movement of the eyes itself. The significance of the flow so created will be evaluated in each instance. We will utilize standard parameter values for an adult human eyes throughout. Table 1 summarizes them, and it states that aqueous humor is thought to have characteristics that are quite comparable to those of water.

**RESULTS**

In the current study, we offer a sophisticated mathematical model to clarify the outflow dynamics of “aqueous humor” in the trabecular meshwork of the human eyes. Our research aims to create a clear mathematical framework for characterizing aqueous outflow via the “trabecular meshwork” and to offer a thorough knowledge of the impact of velocity profiles on the flow properties of aqueous humor. Computational results derived from our model are presented in Table 1, wherein we introduce biologically relevant values for the

<table>
<thead>
<tr>
<th>Table 1: Model parameter.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control parameters</td>
</tr>
<tr>
<td>Temperature of the cornea, ( T_0 ) (°C)</td>
</tr>
<tr>
<td>Temperature of iris, ( T_i ) (°C)</td>
</tr>
<tr>
<td>Coefficient of linear thermal expansion of aqueous humor, ( \alpha ) (°C/K)</td>
</tr>
<tr>
<td>Gravitational acceleration, ( g ) (ms(^{-2}))</td>
</tr>
<tr>
<td>Height of the anterior chamber, ( h ) (m)</td>
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<tr>
<td>Fluid kinematic viscosity, ( v ) (m/s(^{-1}))</td>
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</table>
Various parameters involved in our investigation. The outcomes are further elucidated through meticulously generated graphs, visually representing the mechanisms governing aqueous humor dynamics. These visual representations investigate the effects of different parameters within the human eyes and facilitate a more comprehensive understanding of the phenomenon. Uncontrolled diabetes, which is characterized by high blood glucose levels, can cause several systemic problems, including blood vessel damage. By decreasing blood supply to the eyes, this vascular injury may worsen optic nerve damage in the context of glaucoma, further impairing the durability of nerve cells. Within the context of the mathematical model, the interactions between exposure risks, interventions, health outcomes, and health costs are represented for the modeling studies most pertinent to evidence synthesis and clinical and public health decision-making.

The figures in this section (Figs 3-8) show the complex interplay between the axial distance (z), velocity profile (u), gravitational constant (g), linear thermal expansion (α), anterior chamber height (h), and fluid kinematic viscosity (ν) on the flow of aqueous humor in the human eyes. These discoveries significantly impact how we understand the biophysical variables affecting aqueous humor flow dynamics in the ocular environment. Our study leverages the computational capabilities of MATLAB software to produce these graphs, providing a valuable visual representation of the complex interplay of parameters in the context of aqueous humor flow within the human eyes.

In Figure 3, the illustration showcases the relationship between the velocity profile (u) and axial distance (z), considering distinct values of the linear thermal expansion coefficient (α). Notably, the graphical representation elucidates a discernible pattern where the velocity profile (u) exhibits a progressive rise in magnitude as the axial distance (z) extends. Furthermore, the figure underscores that this elevation in the velocity profile (u) is conspicuously associated with an increase in the coefficient of linear thermal expansion (α), specifically denoted by values of 132, 142, and 152. This empirical observation highlights the intricate interplay between axial distance and the linear thermal expansion coefficient, shedding light on their influence on velocity profile dynamics. The velocity profile (u) variation with axial distance (z) for various gravitational constant (g) values is displayed in Figure 4. The graph illustrates how (u) rises with increasing axial distance (z). The graph also illustrates how the velocity profile gets better as the gravitational constant (g) increases. The axial distance within the trabecular meshwork determines velocity. Aqueous humor travels via the pupil aperture, the posterior chamber behind the iris, the anterior chamber, and the trabecular meshwork, the eyes’ drainage canal. There is a clogged drain in open-angle glaucoma, and as the axial distance increases, the velocity increases as well, explaining the elevated high pressure within the eyes. Prostaglandin analogues rise right away in Figure 5, which also displays the velocity profile (u) variation with axial distance for various anterior chamber (h) height values. The velocity profile (u) rises as the axial distance (z) increases, as the figure illustrates. The image also shows that when the anterior chamber height (h) increases, the velocity profile similarly increases.

In Figure 6, we observe the variation in the velocity profile (u) with respect to the axial distance (z) across different fluid kinematic viscosity (ν) values. The depicted velocity profile (u) exhibits a consistent augmentation as the axial distance (z) extends, as visually evident in the figure. Moreover, the figure indicates a noteworthy trend: for kinematic viscosity values of 0.75, 0.85, and 0.95, the velocity profile demonstrates a concomitant rise with an increase in fluid kinematic viscosity (ν). The phenomenon under scrutiny pertains to the outflow of aqueous humor from the posterior chamber situated behind the iris, which traverses through the pupil aperture into the anterior chamber. This outflow mechanism is instrumental in regulating IOP and ensuring its constancy.

**Figure 3:** Velocity profile (u) with respect to axial distance (z) for different linear thermal expansions (α), i.e. 132, 142, 152 and 162 k⁻¹, and iris surface temperature (Tᵣ) and corneal temperature (Tₛ).

**Figure 4:** Velocity profile (u) with respect to axial distance (z) for different gravitational constants (g), i.e. 6.8, 7.8, 8.8, 9.8 m/s², and iris surface temperature (Tᵣ) and corneal temperature (Tₛ).
The ciliary body of the eyes continues to produce aqueous humor, which enters the Schlemm’s of the canal after passing through a biological filter called a trabecular meshwork. If the drainage pathway is obstructed or constricted, the IOP within the human eyes rises. Therefore, prostaglandin analogues aid to better drain the aqueous humor fluid flow and improve the fluid flow from the eyes to the “collector channels” in order to lower IOP.

Figure 7 presents the variation in the aqueous velocity profile ($v$) concerning axial distance ($z$) across various fluid kinematic viscosity ($\nu$) values. The graphical representation reveals a distinct pattern wherein the velocity profile ($v$) experiences an increment as the axial distance ($z$) progresses toward the midpoint; subsequently, undergoes a decrement. Additionally, the figure illustrates that the velocity profile ($v$) demonstrates an ascending trend as the fluid kinematic viscosity ($\nu$) increases, particularly for values denoted as 0.65, 0.75, 0.85, and 0.95. This observation underscores the intricate relationship between axial distance, fluid kinematic viscosity, and the dynamics of aqueous velocity, bearing potential significance in the context of aqueous humor flow dynamics.

Figure 8 depicts the variation of the aqueous velocity profile ($v$) concerning axial distance ($z$) across different values of the height of the anterior chamber ($h$). The illustration reveals a noteworthy trend where the aqueous velocity profile ($v$) exhibits an upward trajectory as the axial distance ($z$) progresses. Furthermore, the figure conveys that an increase...
DISCUSSION

It has long been understood that the movement of aqueous humor in the anterior chamber of the human eyes might be caused by several different mechanisms. Because the aqueous transparency leaves no trace of the flow, patients are typically unaware that one even happens. The anterior chamber’s flow becomes significantly more significant when particle matter. This model is first designed to address glaucoma, and subsequently was expanded to include the condition related to elevated IOP. It is understood how “open-angle glaucoma” works. A mathematical model that provides a partial description of the human eyes has been offered by them. The aqueous humor and vitreous humor are the two concentrated ocular fluids, hence they concentrated on other elements of fluid dynamics of these two fluids. Its discussed issues with heat transfer, proteins, hydration of the cornea, and vitreous humor in order to understand the aqueous humor. This model also talked about the possibility of fluid flow, which is generally caused by jerking or ocular motions. A basic mathematical framework for characterizing the aqueous outflow through trabecular meshwork and a mathematical model for explaining the outflow of aqueous flow in the eyes’ meshwork have been explored. The model provides an estimate of the effect of velocity profile on the flow characteristics of aqueous humor flow. The work presented in Table 1 lists the computational results for the flow of aqueous humor in the anterior chamber of the human eyes, which were obtained by introducing biologically appropriate values for the parameters used in this work. The results are displayed through graphs to help explain the mechanism of aqueous humor and explore the effects of different parameters in the human eyes. Aqueous humor fluid can exit the human eyes in smaller amounts when the drainage canal narrows or clogs. The ciliary body of the eyes continues to produce aqueous fluid, which raises IOP. Damage to the optic nerve results from elevated IOP. Peripheral vision is the first area of the eyes to lose vision in open-angle glaucoma. Open-angle glaucoma is extremely dangerous since it causes no pain at all. If the eyes are not routinely examined, significant damage can develop in the eyes without any discomfort. The IOP shows a facility-decreasing effect and trabecular tension has a facility-increasing effect. Finally, we analyze that the Schlemm’s canal is an important organ in the eyes that contributes to IOP, ciliary muscle contraction, and compliance-dependent changes in aqueous outflow facility.

Simple fluid dynamical models have been proposed in this study for every mechanism that the authors are aware of that could be responsible for the aqueous flow in the anterior chamber. Although each mechanism and the ensuing flow have been examined separately, combinations of these flows may also be modeled and examined if needed thanks to the linearity of the lubrication theory equations. Even if lubrication theory is an approximation, its usage is justified by the parameter values that are involved and the result’s consistency with estimations and existing facts.

Limitations

Several restrictions on the study of glaucoma disability should be considered in academic research and clinical practice. Glaucoma refers to a broad range of disorders, each with distinctive features and clinical manifestations, such as open-angle glaucoma, POAG, and secondary glaucoma. As a result, generalizing results to all glaucoma subtypes can be difficult. The rate of glaucoma progression can vary significantly among individuals, making it challenging to predict disability timelines accurately. This variability can hinder the development of uniform disability assessment tools. Recognizing these limitations is crucial for researchers, clinicians, and policymakers when addressing glaucoma-related disability, as it guides the development of more comprehensive and nuanced strategies for prevention, early intervention, and disability management. Other factors which may contribute to the development of glaucoma include diabetes, cardiovascular disease, myopia and a positive family history of glaucoma.

Future directions

The future direction of glaucoma disability research and management is poised to witness significant advancements and transformations driven by scientific innovation and evolving healthcare paradigms. Working together with ophthalmologists, neuroscientists, psychologists, and epidemiologists will result in a more thorough understanding of glaucoma disability. The complex interactions between the physical, psychological, and social elements of disability can be better understood through interdisciplinary research. In conclusion, the future of glaucoma disability research and management promises to bring about more focused, patient-centered, and technologically advanced methods. These developments, along with a global commitment to fairness and awareness, have the potential to fundamentally alter the glaucoma disability landscape fundamentally, ultimately easing the burden it places on afflicted people and healthcare systems. Certain studies indicate that oxidative stress and chronic inflammation—both linked to elevated glucose levels—may contribute to the onset and advancement of glaucoma. These elements may have an impact on how long optic nerve cells last.

CONCLUSION

- This research can contribute to our current understanding of the function that the buoyancy-driven flow of aqueous humor plays in the development of various ocular disorders and provide important new insights into the mechanisms that contribute to the development of some medical issues in the eyes.
Disability due to glaucoma is significant and has physical and psychological effects on sufferers. To create comprehensive solutions for resolving glaucoma-related disability, it is imperative to comprehend the complex nature of the condition. This analysis emphasizes the value of public education, accessibility, rehabilitation programs, and advocacy in fostering inclusion and raising the standard of living for glaucoma sufferers.

To lessen the handicap caused by this illness, more investigation and coordinated efforts are required. Reduced blood supply to the optic nerve due to elevated IOP is linked to glaucoma. Transporting vital nutrients like glucose to nerve cells depends on proper blood flow. A reduced blood supply can rob these cells of the energy required to carry out their tasks, which may aggregate nerves and shorten the lifespan of the optic nerve. For those who are at risk of developing glaucoma or who have already been diagnosed, maintaining a healthy lifestyle that includes controlling glucose levels through food and exercise can be helpful. Although it might not immediately affect the optic nerve’s durability, leading a healthy lifestyle can enhance the general health and well-being of the eyes. Elevated IOP is still the primary aim for managing glaucoma; nevertheless, systemic factors such as glucose metabolism impact the optic nerve’s longevity. More study is required to clarify the exact mechanisms and develop clinical recommendations for managing glucose-related variables in the glaucoma setting. To completely understand how glucose and optic nerve endurance in glaucoma interact, ophthalmologists, endocrinologists, and researchers must collaborate across disciplinary boundaries.

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