Geometrically nonlinear autonomous reduced order model for rotating structures

1. Context of the study

High fidelity models:

- Very important computational cost
- Difficulties for parametrization
- Good quality of results
- Implemented into industrial FE codes

Moteur GE90-115B [6]

Schéma d'un moteur à réaction [7]

Actual reduced order models:

- Acceptable loss of accuracy
- Proven efficiency for linear elasticity
- Parametrization for simple cases
- Great displacements nonlinearities
- The nonlinear forces are computed by inflation in the FOM

Modélisation d'un rotor [8]

Necessary improvements

- Autonomous ROM (polynomial app.)
- Implement the frictional contact
- Adapt the existing solution methods (HHT- α , HBM)
- Compute for industrial cases (turbomachinery)

3. Numerical application

With /without Φ_f POD correction :

- 3D Hexahedral quadratic FE
- 16 loading cases
- 0,4 m x 0,03 m x 0,01 m
- 1179 dof, 10 LNM

Dynamic response, $\Omega_{rot} = 0 \text{ rpm}$ and 2000 rpm :

Computed relative error:

Linear normal modes:

2. Methodology

Reduced order model:

Equation of motion for a rotating structure:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}(\Omega)\dot{\mathbf{u}} + \mathbf{K}(\Omega)\mathbf{u} + \mathbf{g}_{NL}(\mathbf{u}) = \mathbf{f}_e(t) + \mathbf{f}_c(\mathbf{u}, \dot{\mathbf{u}})$$

Equation for the reduced order model:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{C}}(\Omega)\dot{\mathbf{q}} + \tilde{\mathbf{K}}(\Omega)\mathbf{q} + \tilde{\mathbf{g}}_{NL}(\mathbf{q}) = \tilde{\mathbf{f}}_e(t) + \tilde{\mathbf{f}}_c(\mathbf{q}, \dot{\mathbf{q}})$$

High fidelity model

Projection:
 $\mathbf{u} = \Phi\mathbf{q}$
 $\dot{\mathbf{X}} = \Psi^T\mathbf{X}\Phi$

Reduced order model

Computation of nonlinear forces:

Inflation method:

$$\tilde{\mathbf{g}}_{nl}(\mathbf{q}) = \Phi^T \mathbf{g}_{nl}(\Phi\mathbf{q}) = \Phi^T \mathbf{g}(\mathbf{u}_s + \Phi\mathbf{q}) - \Phi^T \mathbf{g}(\mathbf{u}_s) - \tilde{\mathbf{K}}_s \mathbf{q}$$

STEP polynomial method [10]:

$$\tilde{\mathbf{g}}_{nl}^p(q_1, \dots, q_r) = \tilde{\mathbf{g}}_{nl_{Quad}}^p + \tilde{\mathbf{g}}_{nl_{Cub}}^p = \sum_{i=1}^r \sum_{j=i}^r A_{ij}^p q_i q_j + \sum_{i=1}^r \sum_{j=i}^r \sum_{m=j}^r B_{ijm}^p q_i q_j q_m$$

Φ_f POD based correction :

$$\mathbf{g}_{nl}(\mathbf{u}) \approx \Phi_f \mathbf{q}_{nl}^f = \mathbf{g}_{nl}^f(\mathbf{u})$$

$$\tilde{\mathbf{g}}_{nl}(\mathbf{q}) = \Phi^T \mathbf{g}_{nl}^f(\mathbf{u}) = \Phi^T \Phi_f (\Phi_f^T \Phi_f)^{-1} \Phi_f^T \mathbf{g}_{nl}(\mathbf{u}) = \mathbf{B}^T \mathbf{g}_{nl}(\mathbf{u})$$

4. Concluding remarks

Conclusion:

- Development of a POD based correction for nonlinear forces in the ROM
- The solutions are in good accordance with the FOM solution
- Accurate results for different rotating velocities and loading intensity

Table 1: Computational cost in seconds for a single simulation with 3000 time steps, $\Omega = 0 \text{ rpm}$ and $\alpha_f = 0.7$.

	FOM	Linear	t_{Lin}/t_{FOM}	STEP	t_{STEP}/t_{FOM}
HHT- α	1147.18	34.76	3.03 %	53.76	4.68 %
HBM	-	24.48	2.13 %	32.09	2.79 %

Perspectives:

Evaluate the nonlinear effects for an academic case study

Construct of the autonomous ROM

Perform aeroelastic coupling analysis

Test the proposed ROM for an industrial structure

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