Data-driven parameterized modeling of LTI systems with guaranteed stability

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Outline

- Main objective:** automated extraction of **parameterized ROMs** from **sampled frequency responses**, for **Electronic Design Automation**
- Requirements:** models must be **stable in the parameter space**, enabling transient numerical simulations during design flow
- Problem:** state-of-the-art techniques **may not** guarantee model stability
- This work:** we present a new framework based on a **parameterized Sanathanan-Koerner iteration with stability constraints**
- Main result:** a **sufficient condition** and an **algorithm** for extracting **guaranteed stable ROMs** from sampled responses
- Enabling technologies:** **Hamiltonian pencil eigenvalue perturbation** for adaptive construction of stability constraints, enforcing **positive realness** of denominator submodel

Model structure and identification

Input data samples: $\tilde{\mathbf{H}}_{k,m} = \tilde{\mathbf{H}}(s_k; \boldsymbol{\vartheta}_m)$

Model: parameterized Sanathanan-Koerner form

$$\mathbf{H}(s; \boldsymbol{\vartheta}) = \frac{\mathbf{N}(s; \boldsymbol{\vartheta})}{\mathbf{D}(s; \boldsymbol{\vartheta})} = \frac{\sum_n \sum_l \mathbf{R}_{n,l} \xi_l(\boldsymbol{\vartheta}) \varphi_n(s)}{\sum_n \sum_l r_{n,l} \xi_l(\boldsymbol{\vartheta}) \varphi_n(s)}$$

Laplace variable s → Parameters $\boldsymbol{\vartheta}$ → Model coefficients

$\xi_l(\boldsymbol{\vartheta})$ Parameter-space basis functions

- multivariate orthogonal polynomials;
- mixed Fourier + polynomials;
- any other choice...

Frequency-domain basis functions: partial fractions

$\varphi_0(s) = 1$, $\varphi_{n>0}(s) = (s - q_n)^{-1}$ "basis poles", fixed

$$\sum_{n=1}^N \frac{\mathbf{R}_n(\boldsymbol{\vartheta})}{s - \mathbf{p}_n(\boldsymbol{\vartheta})} + \mathbf{H}_\infty(\boldsymbol{\vartheta})$$

Pole-residue form not used! ⚠

Model poles: $\mathbf{p}_n(\boldsymbol{\vartheta}) = \text{zeros of } \mathbf{D}(s; \boldsymbol{\vartheta})$

Parameter-dependent

Uniform stability: $\Re\{\mathbf{p}_n(\boldsymbol{\vartheta})\} < 0, \forall \boldsymbol{\vartheta}$

Extract **denominator submodel** $\mathbf{D}(s; \boldsymbol{\vartheta})$ and construct its state-space realization

$$\mathbf{D}(s; \boldsymbol{\vartheta}): \begin{aligned} \dot{\mathbf{w}} &= \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{u} \\ \mathbf{y} &= \mathbf{C}(\boldsymbol{\vartheta}) \mathbf{w} + \mathbf{D}(\boldsymbol{\vartheta}) \mathbf{u} \end{aligned}$$

$$\text{Fit model to data: } \mathbf{H}(s_k; \boldsymbol{\vartheta}_m) = \frac{\mathbf{N}(s_k; \boldsymbol{\vartheta}_m)}{\mathbf{D}(s_k; \boldsymbol{\vartheta}_m)} \approx \tilde{\mathbf{H}}_{k,m}$$

Parameterized GSK iteration

$$\mathbf{D}_0 = 1$$

for $\mu = 1, 2, \dots$

$$\min \sum_{k,m} \left\| \frac{\mathbf{N}_\mu(s_k; \boldsymbol{\vartheta}_m) - \mathbf{D}_\mu(s_k; \boldsymbol{\vartheta}_m) \tilde{\mathbf{H}}_{k,m}}{\mathbf{D}_{\mu-1}(s_k; \boldsymbol{\vartheta}_m)} \right\|_F^2$$

$$\text{s. t. } \Re\{\mathbf{D}_\mu(s_k; \boldsymbol{\vartheta}_m)\} > 0$$

end

adaptive constraints see below

$$\text{Cast as } \min \|x\|^2 \text{ s. t. } \mathbf{Q}x < b$$

Key result: building constraints for uniform stability

Theorem: if $\mathbf{D}(s; \boldsymbol{\vartheta})$ is a (strictly) **Positive-Real** function, then $\Re\{\mathbf{p}_n(\boldsymbol{\vartheta})\} < 0, \forall \boldsymbol{\vartheta}$

- $\mathbf{D}(s; \boldsymbol{\vartheta})$ regular for $\Re\{s\} > 0$
- $\mathbf{D}^*(s; \boldsymbol{\vartheta}) = \mathbf{D}(s^*; \boldsymbol{\vartheta})$
- $\Re\{\mathbf{D}(s; \boldsymbol{\vartheta})\} \geq \varepsilon > 0$ for $\Re\{s\} > 0$

guaranteed by model structure
check for $s = j\omega$

Hamiltonian pencil $(\mathbf{M}(\boldsymbol{\vartheta}) - \lambda \mathbf{K})$ associated to denominator submodel

$$\mathbf{M}(\boldsymbol{\vartheta}) = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{B} \\ \mathbf{0} & -\mathbf{A}^T & -\mathbf{C}^T(\boldsymbol{\vartheta}) \\ \mathbf{C}(\boldsymbol{\vartheta}) & -\mathbf{B}^T & \mathbf{D}(\boldsymbol{\vartheta}) + \mathbf{D}^T(\boldsymbol{\vartheta}) \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Bottom line: passivity of denominator submodel implies uniform model stability → Let us enforce denominator passivity!

S. Grivet-Talocia, "A Perturbation Scheme for Passivity Verification and Enforcement of Parameterized Macromodels," *IEEE Trans. on CPMT*, Vol. 7, No. 11, Nov. 2017, pp.1869-1881

Examples

A reverberation chamber

An integrated inductor

A complex Printed Circuit Board interconnect