EMPLOYING ELECTROMAGNETIC ACTUATOR TO HARVEST ENERGY THROUGH A VARIABLE-LENGTH PENDULUM

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Abstract
This article explores the application of electromagnets in energy harvesting via a variable-length pendulum system. Harnessing the principles of electromagnetism, the study investigates the efficiency and feasibility of utilizing electromagnetic forces to extract energy from the motion of a pendulum with variable length, employing sinusoidal excitation as a means for vibrating machinery. This approach enables a more significant number of oscillations, consequently leading to a higher power output. The research investigates various aspects, including the design, implementation, and performance analysis of the electromagnet-based energy harvesting system. Through theoretical modeling, the article provides insights into the potential of electromagnets to generate sustainable energy from oscillatory motion. The findings show that up to 0.25W can be generated, providing power for small devices such as phone chargers and sensor units. The findings contribute to advancing renewable energy technologies and offer promising avenues for developing efficient and environmentally friendly energy harvesting mechanisms.

Keywords: Electromagnets; Energy harvesting; Variable-length pendulum; Renewable energy.

1.0 Introduction
Exploration of innovative technologies is paramount in the pursuit of sustainable energy sources. Electromagnets’ utilization for energy harvesting presents a promising avenue in this endeavor. This article delves into the novel concept of employing electromagnets to harness energy through a variable-length pendulum system. The concept of energy harvesting from mechanical oscillations is not new, yet recent advancements in electromagnetism have opened new possibilities for its implementation [1, 2]. By integrating electromagnets with a variable-length pendulum, we aim to explore the efficiency and viability of this approach in converting mechanical energy into electrical power. The variable-length pendulum serves as a dynamic platform, allowing adjustments to its length to optimize energy extraction under various conditions [3]. Coupled with electromagnets, which can generate electric currents through induced magnetic fields, this system presents a synergistic blend of mechanical and electromagnetic principles for energy conversion. This article elucidates the theoretical framework behind electromagnet-assisted energy harvesting from a variable-length pendulum. Through physical and mathematical models and numerical analysis, we seek to quantify the effectiveness of this approach in generating sustainable energy. Furthermore, we aim to address this technology’s practical considerations, challenges, and potential applications in diverse fields ranging from renewable power generation to autonomous systems. Investigating electromagnets’ integration with a variable-length pendulum for energy harvesting contributes to the growing body of research aimed at
advancing renewable energy technologies [4, 5, 6, 7]. The insights gained from this exploration hold the potential to drive innovation and pave the way for the development of efficient and environmentally sustainable energy solutions. The presented research holds practical significance and value as it aligns with the contemporary trend of energy harvesting through the utilization of variable-length pendulums. The work unfolds in four distinct sections, each contributing to a comprehensive understanding of the subject matter. In Section 1, the groundwork is laid with an introduction that sets the stage for the subsequent discussions. Moving forward, Section 2 delves into the intricacies of the electromagnetic actuator, elucidating its working principles. This section not only includes a detailed model but also provides simulation results for both a single solenoid and a pair of identical solenoids. The analysis encompasses the impact of variables such as current, magnetic plunger diameter, magnetic coil length, and the spacing between the pair of identical solenoids. It also takes a holistic approach by presenting the complete model of the variable-length pendulum. Here, the focus shifts to the incorporation of the electromagnetic device, featuring a thorough exploration of the mathematical model and numerical analysis. Concluding the work, Section 3 encapsulates the key findings and insights derived from the study. Additionally, it offers recommendations for further research and development within this evolving research domain. The comprehensive structure of the presented work ensures that readers gain a nuanced understanding of the energy harvesting potential of variable-length pendulums, paving the way for future advancements in this field.

2.0 Model of the Electromagnetic Actuator
An electromagnetic actuator is a device consisting of essential components, namely a magnetic core, a coil of wire wrapped around the core, and a plunger or armature, as reported by Jebelli et al [8]. When an electric current flows through the coil, it produces a magnetic field that interacts with the magnetic core and the plunger or armature. As a result, a magnetic force is generated, causing the plunger or armature to move and producing the desired mechanical motion. Various electromagnetic actuators exist, including solenoids, relays, and even motors [9]. Solenoids are the most widely used type of electromagnetic actuator and are made up of a coil of wire that generates a magnetic field [10]. The magnetic field produced by the coil is used to move a plunger or armature connected to the coil. Relays are electromagnetic switches that utilize a small current to control a larger current. They consist of a coil of wire that, when energized, creates a magnetic field that attracts a metal switch or contact. This movement can either open or close a circuit, allowing or preventing the flow of electricity [8]. Motors are another type of electromagnetic actuator that transforms electrical energy into rotational mechanical energy. They comprise a stator, which contains a coil of wire that generates a magnetic field, and a rotor, a rotating component that interacts with the magnetic field to generate torque. Electromagnetic actuators are utilized in various applications, including robotics, automation, aerospace, and automotive industries, where precise and rapid movement is necessary [10, 1].

2.1 Single solenoid actuator
The single solenoid actuator is a linear actuator that utilizes a solenoid to generate linear motion through the interaction of the magnetic field produced by the solenoid...
with a ferromagnetic plunger or rod [11]. Typically, the solenoid is positioned within a cylinder or housing, with the plunger or rod located inside the solenoid [12]. The solenoid is induced upon application of an electrical current, resulting in the magnetic field attracting the plunger towards the solenoid and causing linear motion [13]. Subsequent deactivation of the electrical current causes a spring or other mechanism to return the plunger to its original position [13]. Single solenoid actuators are commonly employed in applications such as valves, locks, and latches, where a simple on/off control of linear motion is required [14, 1, 15]. Due to their reliability, longevity, and relatively low cost, they are widely used in various industrial and commercial settings [14, 1]. In the context of a single solenoid actuator, a ferromagnetic plunger interacts with the solenoid’s magnetic field to create linear motion [10, 12]. The plunger is typically located inside the solenoid, and when an electrical current induces the solenoid, the magnetic field attracts the plunger towards the solenoid, causing linear motion. This section details the operation of a magnetomechanical oscillator driven by a cylindrical permanent magnet plunger, which acts on an external magnetic field produced by a solenoid, as illustrated in Fig. 1. The magnetic field along the axial direction of the coils is modeled using the Biot-Savart law and the differential element approach. Additionally, the charge model described in [9, 16, 10, 17] is utilized to determine the force acting on the cylindrical magnet bar within the external magnetic field. It should be noted that, due to symmetry, no lateral force is exerted on the cylindrical magnet if it moves along the axis of the coil. However, in our experimental setup, a significant deflection results in a slight vertical motion of the frame structure.

The black dotted lines represent the coil wire, and the dot-circles and circle-circles depict the current entering and leaving the wire’s cross-section. To compute the magnetic flux density $B$ at each position $P$ along the solenoid axis, the contribution from an infinitesimal element $dl$ of the coil loop, as depicted in Fig. 1, is considered. Using the Biot-Savart law [10], the magnetic flux density at the specified position $P$ can be expressed as a vector. If we consider the magnetic field along only the $x$-axis, the following expression is obtained:

$$\vec{B}_x = \int_{0}^{2\pi} \mu_0 I R^2 \frac{d\theta}{(x^2 + R^2)^{3/2}} dl_x = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} \hat{i},$$

where $I$ represents the flow of current through the coils, $\mu_0$ is the vacuum permeability that is a constant $4\pi \times 10^{-7} \text{H/m}$, $R$ is the radius of the coils, $\hat{i}$ is the unit vector in $x$-axis direction. When calculating the number of coil spins per unit area, the solenoid’s cross-section along its radius plane is considered.

$$n = \frac{N}{(L - \frac{D}{2}) \frac{L}{2}} = \frac{2N}{(D_2 - D_1) \frac{L}{2}}, \quad \text{as the present differential element passes through the unit area, and is provided by:}$$

$$dl = I (ndxdR) = nIdxdR \quad \text{Eq. (2)}$$

Substituting Eq. (2) in Eq. (3), a variable of the magnetic field at an arbitrary point $P$ along the $x$-axis is described as follows:

![Figure 1. A sectional view of the single solenoid actuator consisting of a coil, a magnet cylinder, and a steel rod connecting them.](image)
To obtain the magnetic flux at point $P$, we integrate Eq. (4) over the cross-section mentioned above:

$$
B_s = \frac{\mu_0 N I}{(D_2 - D_1) L} \int \int (x^2 + R^2)^{3/2} \, dx \, dR.
$$

As stated by [9, 10], the charge model states that the force exerted by a permanent magnet in an external magnetic field can be expressed as follows:

$$
\mathbf{F} = \int \rho \mathbf{B}_{ext} \, dv + \int \sigma \mathbf{B}_{ext} \, ds,
$$

where $\rho = -\nabla \cdot \mathbf{M}$ and $\sigma = \mathbf{M} \cdot \mathbf{N}$ are equivalent volume and surface charge density, respectively; $\mathbf{B}_{ext}$ is the external magnetic field, and $\mathbf{M}$ is the magnetic moment of the permanent magnet per unit volume. Additionally, the formula $\mathbf{M} = \frac{\mathbf{B}}{\mu}$ can be used to determine the magnetic moment, where the residual flux density of the permanent magnet is $B_r$. The cylindrical magnet plunger is polarized with fixed and uniform magnetization along its axis, denoted as $\mathbf{M} = M_r \mathbf{r}$ resulting in $\rho = -\nabla \cdot \mathbf{M} = 0$, and the magnetic force along the $x$-axis holds:

$$
\mathbf{F}_x = \int_0 \sigma \mathbf{B}_{ext} \, ds.
$$

If the magnetic plunger is thought of as a hollow cylinder, then, to evaluate $\sigma$, the unit normal is determined as follows:

$$
\begin{align*}
-\mathbf{e}_r, & \quad x_l = x_0 - \frac{l}{2}, \\
\mathbf{e}_r, & \quad x_r = x_0 + \frac{l}{2},
\end{align*}
$$

with the unit vector $\mathbf{e}_r$ in the direction of $\sim r$. If the permanent magnet’s north and south poles are on its left and right sides, respectively, and its end surface’s charge density is described as:

$$
\begin{align*}
M_l, & \quad x_l = x_0 - \frac{l}{2}, \\
-M_l, & \quad x_r = x_0 + \frac{l}{2},
\end{align*}
$$

where $l$, $d_1$, and $d_2$ are the lengths of the inner and outer radii of the magnet cylinder; $x_l$ and $x_r$ are the coordinates of the permanent magnet’s left and right region on the $x$-axis. Eqs. (6) and (9) are used to determine the force acting on the permanent magnet plunger in the manner described below:

$$
F_x = \oint_\gamma \mathbf{B}_r \, dl = M \left[ B_x \left( x - \frac{l}{2} \right) - B_x \left( x + \frac{l}{2} \right) \right] \int \int_{\theta} r \, dr \, d\theta
$$

$$
= B_r \pi \left( \frac{d_2^2 - d_1^2}{4} \right) \left[ B_x \left( x - \frac{l}{2} \right) - B_x \left( x + \frac{l}{2} \right) \right],
$$

Table 1: The parameters that were utilized to generate the figures and display the characteristics of the solenoid actuation forces

<table>
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<th>Line numbers</th>
<th>$D_1$(mm)</th>
<th>$D_2$(mm)</th>
<th>$L$(mm)</th>
<th>$d_1$(mm)</th>
<th>$d_2$(mm)</th>
<th>$l$(mm)</th>
<th>$I$(A)</th>
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</tbody>
</table>

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The force characteristic of the single solenoid actuator

Figure 2. The single solenoid actuator’s force feature for the values shown in Table 1. \( d \) is modified to 20, 22, and 26 mm.

The force characteristic of the single solenoid actuator

Figure 3. The single solenoid actuator’s force feature for the values shown in Table 1. \( L \) is modified to 150, 180, and 210 mm.

Electromagnetic forces of a single solenoid actuator

Figure 4. The single solenoid actuator’s force feature for the values shown in Table 1; with other values unchanged, \( I \) is modified from 1A to 10A.

Electromagnetic forces of a single solenoid actuator

Figure 5. The single solenoid actuator’s force feature for the values shown in Table 1; with other values unchanged, \( I \) is modified from 0.1 to 5A.

Electromagnetic forces of a single solenoid actuator

Figure 6. The single solenoid actuator’s force feature for the values shown in Table 1; with other values unchanged, \( I \) is modified from 0.1 to 5A with the displacement between range between –50-50 mm.

Using the parameter values provided in Table 1 and \( B_r = 0.0057 \), the actuator’s magnetic force is depicted in Figure 8 and 3 as a plot. The images also demonstrate that a solenoid actuator’s highest actuation force is generated when the magnet’s center moves relatively close to the coil’s end. Figure 4-6 depict how a single solenoid actuator’s electromagnetic forces are affected by current. Figure 4 illustrates a rise in current from 1 to 10 A, while Figure 5 shows an increase in current from 0.5 to 5A, and Figure 6 shows that the generated force and the solenoid’s magnet placement have a nearly linear relationship.

2.2 An enhanced electromagnetic actuator design

The amplification of the driving force depends on the displacement variance \( \alpha \), as indicated by the calculations presented earlier for a single solenoid actuator. For mechanical and mechatronic applications, a more effective electromagnetic actuator can be achieved by utilizing two similar and coaxial solenoids with their coils wound in opposite directions and supplied by the same currents, resulting in a constant force that is independent of displacement [10, 13]. Figure 7 shows the details of a double-identical solenoid actuator. The double identical solenoid actuator operates by employing two solenoids mounted in parallel with each other [9, 16]. The use of two identical solenoids in the actuator

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provides several benefits. First, it ensures that the actuator produces a balanced force, as the two solenoids are identical in size and strength. Second, it provides redundancy in case one of the solenoids fails, as the other solenoid can continue to provide motion \[18, 9, 16, 19\]. Finally, it allows for enhanced motion control since the two solenoids can be independently controlled \[9\].

Figure 7. The sectional view of the twin-solenoids actuator shows the position of the magnet in the center of two identical solenoids. The solenoid pair consists of similarly wound coils that carry the same current but in opposite directions.

It is assumed that the distance between the two solenoid centers \((O^0 \text{ and } O^{00})\) is \(d\), and select the center of the axial line \(O^0 \text{ and } O^{00}\) as the coordinate origin \(O\) of the new system \[10, 20\]. It should be stated as well that the magnetic plunger’s center is located at the origin \(O\) along the virtual line. From Equation (10), the electromagnetic force is derived as follows:

\[
F = F_x \left( x - \frac{d}{2} \right) + F_x \left( x + \frac{d}{2} \right), \quad \ldots(11)
\]

where \(F_x(\cdot)\) is a function of the variable \(x\). To provide further detail and explanation, we can present the following expanded form:

\[
F = F_x(B_2 - B_1) - F_x(B_4 - B_3), \quad \ldots(12)
\]

Where: \(F_x = \frac{B_0 \pi}{\mu_0} \left( d_2^2 - d_1^2 \right)\).

To analyze the nature of the magnetic force for the new design, the values corresponding to line 1 of Table 1 are used to construct the pertinent force curves shown in Figure 8.

The characteristics of the electromagnetic force generated by a constant direct current, such as those in Figure 8, can be used to determine one of the four types of the electromagnetic force quasi-constant force (QCF), oblique linear force (OLF), single peak force (SPF), and double peak force (DPF) is being produced.

The force characteristic of the Identical solenoid actuator

Figure 8. Four different sorts of force regions are exhibited in the actuator’s force characteristic, including the QCF, SPF, DPF, and OLF.

The various forces reflect the connections between displacement and force that can be altered by changing the electromagnetic actuator’s properties, such as the solenoid’s length, thickness, and radius, the coil’s number of turns, and the distance between a pair of solenoids. When \(d\) drops and increases, the force curves transform into SPFs and DPFs, for instance, at \(d = 75\) mm and \(92\) mm, respectively. For example, according to Figure 8, the QCF zone is present at \(x = 0\) at \(d = 83\) mm. The OLF region is visible in the plot shown in Figure 8, where the force is between \(-10\) and \(15\) N. The segments of OLF and SPF for the single solenoid actuator can be seen in Figure 2 and 3. One should note that a single solenoid actuator can only produce SPFs and OLFs. On the other hand, an actuator powered by a pair of identical solenoids can produce all four types of forces.
2.3 Practical application of the electromagnetic actuator in the variable-length pendulum

The electromagnetic actuator design that utilizes a pair of identical solenoids and a concept called quasi-constant force (QCF) can be used in various practical applications. The primary objective of the design is to generate a force with nearly constant amplitude despite a continuous input current. The study demonstrates that this electromagnetic actuator offers a wide range of excitation patterns and can be effectively used for experimental investigations requiring precision and energy harvesting. The variable-length pendulum presents an interesting perspective when approached as a second-order nonlinear differential equation, characterized by step function-dependent coefficients. This formulation lends itself to transformation into equivalent discrete dynamical systems, as discussed by Hatvani and Chicone [21, 22]. Furthermore, it can be effectively regarded as a control system, owing to the influence of time-varying control laws that alter its length, as explored by Xin [23]. Beyond its mathematical intricacies, the variable-length pendulum finds application in diverse mechatronic systems. Examples include its integration into robotics, electromechanical setups such as induction motors, purely electrical networks like dc-dc power converters, and lifting devices like mine elevators or cranes. Moreover, its relevance extends to unconventional domains, contributing to seismic activity detection through inverted pendulum concepts and even playing a role in the development of wave energy converters (WEC), as elaborated by Yakubu, Yurchenko, and others [24, 25, 26]. This versatility underscores the variable-length pendulum’s significance in both theoretical and practical contexts within the realm of dynamic systems and mechatronics. In the process of developing the variable-length pendulum equation of motion, the mass \( m \) (see Fig. 9) is viewed as a cylindrical magnet plunger with a fixed and uniform magnetization along its axis where an electromotive force is generated according to Faraday’s law.

2.4 Mathematical model

The complete description of the model is presented in Figure 9. The system is of two degrees of freedom. The 2DOF dynamical model consists of a nonlinear spring pendulum with a stiffness and damping coefficient \( k \) and \( c \), respectively. It is considered that the point of suspension \( O \) oscillates due to the excitation \( Y_0(t) \) of the vibrating plate on which the whole system is mounted. The mass \( m \), which is viewed as a cylindrical magnet whose oscillation passes through the solenoid where \( l_c \) – the inductance of the coil, \( I \) – current of the load resistance \( R_m \), \( \beta I \) – coupling term of the model and the electromagnetic circuit.

Based on the configuration presented in Figure 9, the potential and kinetic energy of the system without the electromagnetic component is expressed in Equation 13 and 14.

Potential Energy:
\[ T = \frac{1}{2} m \left( (\dot{l}g \omega \sin(\omega t) - (l_0 + l(t))\dot{\phi}(t)\sin(\phi(t)) \right. \]
\[ + (l(t)\cos(\phi(t)))^2 + (l_0 + l(t))\dot{\phi}(t)\cos(\phi(t)) \]
\[ + l(t)\sin(\phi(t)))^2 \right) \] (13)

Kinetic energy:
\[ \frac{1}{2} \dot{l}(t)^2 - m f_0 \cos(\omega t) + (l_0 + l(t))\cos(\phi(t))) \] (14)

With the Lagrange equation, \( L = T - U \), we find two degrees of freedom in the directions \( l(t) \) and \( \phi(t) \). The Euler-Lagrange equation yields:
\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} + \frac{\partial R_d}{\partial \dot{q}_i} = 0, \] (15)
where: \( i = 1, 2 \), and \( R_d \) = Rayleigh dissipation function represented as \( \frac{1}{2} c \dot{l}(t)^2 \).

The following equations of motion for the model incorporating the energy harvester are finally obtained as presented in Equation 16–18:
\[ \ddot{l} = f_0 \omega^2 \cos(\omega t) \cos(\phi(t)) + g \cos(\phi(t)) - \frac{c}{m} \dot{l}(t) \]
\[ + \beta \dot{l} - \frac{k}{m} \dot{l} + l_0 \dot{\phi}(t)^2 + l(t) \dot{\phi}(t)^2, \] (16)
\[ \ddot{\phi} = \left( \frac{1}{2l_0^2} \frac{1}{l(t)^2 + l_0^2} \right) \left( \frac{c}{m} \dot{\phi}(t) + (l_0 + l(t)) \sin(\phi(t)) - (f_0 \omega^2 \cos(\omega t) + g) - 2l_0 \dot{l}(t) \dot{\phi}(t) - 2l(t) \dot{l}(t) \dot{\phi}(t) \right), \] (17)
\[ l_i \dot{l} + R_m l_i \dot{l} - \beta \dot{l} = 0. \] (18)

3.0 Results and Discussions

The simulation results detailing the described model concept are depicted in Figure 10. These computations are executed in Python, utilizing Numpy, Scipy, and Matplotlib libraries within the Spider environment of Anaconda Python. The simulation duration spans up to 1000 seconds, allowing for a comprehensive analysis. Figure 10 (a) portrays the periodic motion of the variable-length pendulum sans the electromagnetic device, considering parameters such as \( m = 1.5, \ l_0 = 2.5, \ l(t) = 0.65, \)

\( \varphi(t) = \frac{\pi}{25}, \) with an initial velocity of zero. The radial direction measures \( l(t) \), while the angular direction measures \( \phi(t) \). The illustrated results in Figure 10 (a) reveal nonsingular orbits post-swinging. Intriguingly, certain regimes exhibit compact regions of attraction within the system, enhancing its value for exploration in diverse engineering applications. The findings capture the motion’s periodicity and highlight the system’s potential for specialized engineering endeavors. Figure 10 (b) and (c) depict the time histories of the timedependent variables \( l(t) \) and \( \phi(t) \), respectively, employing the parameter values: \( f_0 = 3.5, \ \omega = 0.325, \ k = 87, \) and \( c = 0.005 \). These parameters are pivotal in shaping the simulation results, and their analysis sheds light on the system’s behavior. Factors such as the pendulum’s mass, swing angle, length of the pendulum, and stiffness (k) are carefully scrutinized to discern their influence.
Fig. 10 An orbit of the variable-length pendulum (a), time histories of the pendulum length $l(t)$ (b), pendulum angle $\phi(t)$ (c), output current (d) and the power output (e).

Of significant importance is the coil’s inductance, which profoundly impacts the output current. Figure 10 (d) illustrates the time history of the output current, utilizing the parameters $R_m = 5$, $\beta = 0.00128$, $l_m = 0.05$, while Figure 10 (e) depicts the power output from the electromagnetic device. The presented results indicate that the output power up to 0.25W, as can be seen in Figure 10 (e), is sufficiently robust to power various devices, including but not limited to sensor power supplies, electronic device chargers, and medical applications. These findings underscore the system’s potential utility across diverse practical domains, offering a promising avenue for further exploration and application.

3.1 Conclusion

In conclusion, the utilization of electromagnetic actuators in harvesting energy through a variable-length pendulum presents a promising avenue for sustainable power generation. The dynamics response of the system undergoes thorough analysis, revealing compelling insights through a range of parameter values, the results presented underscore the system’s effectiveness. It becomes evident that the system exhibits promising characteristics, demonstrating its potential for practical application. However, it is important to note that the results presented thus far represent only the initial evaluation stages, leaving room for further exploration and refinement in subsequent studies. Through our exploration, we have demonstrated the effectiveness of this innovative approach in efficiently capturing kinetic energy and converting it into usable electrical power of up to 0.25W, which could potentially power some devices such as LED night lights or small LED indicators, some low-power sensors or detectors, small electronic gadgets like calculators or small digital clocks, low-power microcontrollers or IoT devices in sleep mode, some types of remote controls or wireless transmitters, small electronic toys or novelty items. By harnessing the inherent dynamics of the pendulum system and leveraging electromagnetic principles, significant strides have been made toward achieving renewable energy solutions and beyond. This research underscores the potential of electromechanical systems in energy harvesting and highlights avenues for further optimization and application in diverse contexts. As we strive towards a greener future, this study’s findings contribute valuable insights into harnessing energy from unconventional sources.
paving the way for more efficient and environmentally friendly energy solutions. Further research and development in this direction could unlock even greater opportunities for practical implementation and widespread adoption of this technology in real-world scenarios, paving the way for more sustainable and self-sufficient energy ecosystems.

**Nomenclature**

ω  
resonance frequency

I  
current flow

μ₀  
vacuum permeability

R  
coils radius

−i  
unit vector in x-axis direction

Bᵣ  
residual flux density of the permanent magnet

d₁, d₂  
lengths of the inner and outer radii of the magnet cylinder

x₁, xᵣ  
coordinates of the permanent magnet’s left and right region on the x-axis

ρ, σ  
equivalent volume and surface charge density

$\vec{B}_{ext}$  
external magnetic field

$\vec{M}$  
magnetic moment of the permanent magnet per unit volume

Bᵣ  
residual flux density of the permanent magnet

m  
pendulum’s mass

k, c  
stiffness and damping coefficient

lᵦ  
inductance of the coil

Rᵢ  
load resistance

βI  
coupling term of the model and the electromagnetic circuit

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**Conflict of interest**

The authors declare no conflict of interest.

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