Investigation of the Mathematical Relationship between the Aortic Valve and Aortic Root: Implications for Precise Guidance in Aortic Valve Repair

Luyao Ma, MD1,a, Kangting Tang, MD2,a, Guanyu Yang, PhD3,a, Hong Liu, MD1,a, Chengxiao Xu, MD4, Yinsu Zhu, MD4 and Yongfeng Shao, MD1

1Department of Cardiovascular Surgery, The First Affiliated Hospital with Nanjing Medical University, Nanjing, Jiangsu Province, China
2Department of Cardiology, The First Affiliated Hospital with Nanjing Medical University, Nanjing, Jiangsu Province, China
3School of Computer Science and Engineering, Southeast University, Nanjing, Jiangsu Province, China
4Department of Radiology, The First Affiliated Hospital with Nanjing Medical University, Nanjing, Jiangsu Province, China

Received: 19 February 2023; Revised: 15 May 2023; Accepted: 31 May 2023

Abstract

Background: The study was aimed at investigating the mathematical relationship between the aortic valve and aortic root through CTA imaging-based reconstruction.

Methods: We selected 121 healthy participants and analyzed the measurements of aortic root dimensions, including the sinotubular junction (STJ), ventriculo-arterial junction (VAJ), maximum sinus diameter (SD), sinus height (SH), effective height (eH) and coaptation height (cH). We also reconstructed 3-D aortic valve cusps using CTA imaging to calculate the aortic cusp surface areas. Data were collected to analyze the ratios and the correlation between aortic valve and aortic root dimensions.

Results: Among healthy participants, the STJ was approximately 10% larger than the VAJ, and the SD was 1.375 times larger than the VAJ. The average eH and cH were 8.94 mm and 3.62 mm, respectively. The aortic cusp surface areas were larger in men than women. Regardless of sex, the non-coronary cusp was found to be largest, and was followed by the right coronary cusp and the left coronary cusp. Although the aortic root dimensions were also significantly larger in men than women, the STJ to VAJ, SD to VAJ, and SH to VAJ ratios did not significantly differ by sex. The mathematical relationship between the aortic cusp surface areas and VAJ orifice area was calculated as aortic cusp surface areas \( \left( \text{mm}^2 \right) = 1.512 \times \left( \frac{\text{VAJ}(\text{mm})}{2} \right)^2 \times \pi + 166.866. \)

Conclusions: The aortic root has specific geometric ratios. The mathematical relationship between the aortic valve and aortic root might be used to guide aortic valve repair.

Keywords: aortic cusp surface area; aortic root; aortic valve; aortic valve repair
Abbreviations and acronyms: AI, aortic insufficiency; AVR, aortic valve replacement; BSA, body surface area; cH, coaptation height; eH, effective height; SD, maximal sinus diameter; SH, sinus height; STJ, sinotubular junction; V AJ: ventriculoarterial junction.

Introduction

Aortic valve repair, including isolated aortic valve repair and valve-sparing aortic root replacement (the David procedure), are not performed as commonly as mitral valve repair. However, enthusiasm for aortic valve repair has increased among cardiac surgeons [1]. The purpose of aortic valve repair is to avoid any prosthetic valve-associated complications, including bleeding, thromboembolism, endocarditis and tissue valve deterioration [2–4]. The value of aortic valve repair lies in its long-term maintenance of aortic valve function and favorable quality of life, particularly for young patients [5]. Factors contributing to aortic insufficiency (AI) include deformed aortic root configuration, diseased aortic valve or a combination of both. Normal aortic valve function has been demonstrated to depend on the specific relationship between the aortic valve and aortic root dimensions [6–8]. For patients with AI, prioritizing aortic valve repair is believed to restore a normalized relationship between the aortic valve and aortic root. However, no objective methods are available for choosing graft size in the David procedure, and determining STJ and V AJ size in performing isolated aortic valve repair. To provide a reference for aortic valve repair, we hypothesized that the aortic valve cusp areas should be considered the core anatomic geometry, which should be referenced and matched to the aortic root dimensions. Therefore, in this study, we retrospectively analyzed 121 healthy participants with tricuspid aortic valve and used MeVislab software to reconstruct the aortic valve cusps in three dimensions. This process enabled us to measure the individual aortic cusp surface areas and aortic root dimensions, and determine the mathematical relationship between them, to provide a surgical reference.

Materials and Methods

Study Population

The study protocol conformed to the ethical guidelines of the 1975 Declaration of Helsinki and was approved by the Institutional Review Board of the First Affiliated Hospital of Nanjing Medical University (July 28th, 2021; No. 2021-SR-381). All participants provided informed consent. We retrospectively analyzed dual-source cardiac CT scan images of the participants between January 2020 and October 2021. All participants underwent dual-source cardiac CT scanning and echocardiography because of chest pain. The cardiac CT and echocardiography scans were ordered to verify the presence of any coronary artery abnormalities and aortic diseases. Finally, 121 participants (61 men and 60 women) among a total of 306 patients, who had a normal aortic root, as confirmed by cardiac CT, and normal tricuspid aortic valve anatomy and function, as confirmed by echocardiography, were considered healthy participants and selected for inclusion in this study. All participants denied any cardiovascular disease history, such as hypertension. The exclusion criteria were bicuspid aortic valve; any evidence of aortic valve diseases, such as stenosis, regurgitation or calcification; and any evidence of aortic root anomalies.

Cardiac CTA Imaging Data

Cardiac CTA images were obtained with dual-source CT (Somatom Definition; Siemens Medical Systems, Erlangen, Germany) with a 64 row detector and retrospective ECG gating. Four ECG leads were attached to the patient’s chest in standard positions, and the ECG was continuously recorded throughout the scan. Bolus tracking was used for timing, and scanning started automatically 6 seconds after the contrast enhancement reached 100 HU in the region of interest placed in the descending aorta. The scanner settings were as follows: tube voltage of 120 or 100 kV, and effective tube current of 380 mAs for both tubes. The effective radiation dose ranged from 6 to 10 mSv. The scan direction was craniocaudal, starting above the carina of the trachea and ending at the diaphragm below
all cardiac structures. High concentration contrast material (Ultravist iopromide, 370 mg I/mL; Bayer HealthCare Pharmaceuticals, Leverkusen, Germany) was administered at 4–6 mL/s with a mechanical power injector (Dual Shot, MedRad Inc., Indianola, PA, USA) via a 20-gauge cannula inserted into an antecubital vein. The image slice thickness was 0.75 mm or 0.625 mm. All CCTA data were transmitted to the workstation (syngo. ViaVB10, Siemens) for post-processing. Any images whose quality rendered image analysis unfeasible were excluded [9, 10].

CT Image Reconstruction and Analysis

The 3-D surface of each cusp of the tricuspid aortic valve was reconstructed from cardiac CTA scans through the following steps:

1. The original CTA images were first interpolated along the z-axis to generate an isotropic volumetric image.
2. The orthogonal reformatted images were manually defined, to generate a series of planar cross-sectional images perpendicular to the root of the aorta. The image size of the reformatted cross-sectional images was fixed to 256 × 256 × 80 mm³.
3. In the planar cross-sectional images, several points located on a single aortic cusp were annotated manually by the cardiac surgeon (Figure 1). After the aortic cusp was delineated, the triangulated surface of each leaflet was created from a scattered points cloud by using Delaunay 3-D triangulation (Figure 2) (https://www.mathworks.com/matlabcentral/fileexchange/63730-surface-reconstruction-from-scattered-points-cloud).
4. The surface area of each aortic valve cusp was computed from the reconstructed triangulated 3-D surface. The other parameters, comprising the diameters of the sinotubular junction (SJT) and ventriculo-arterial junction (VAJ), and the maximum sinus diameter (SD), sinus height (SH), effective height (eH) and coaptation height (cH), were measured in the reformatted images (Figure 3).

The above steps were performed in Mevislab (Ver 2.1) (https://www.mevislab.de/) and Matlab (Ver 2018).

Statistics

Data are presented as frequency (percentage) for categorical variables, and mean (standard deviation) or median (Interquartile range) for continuous variables according to the nature of the data. Differences between groups at baseline were assessed with χ² test or Fisher’s exact test for categorical variables, and t test or Mann-Whitney U test for continuous variables. Linear regression was performed to investigate the associations among the VAJ, STJ, SD, SH and body surface area (BSA), and to calculate the aortic cusp surface areas with a regression coefficient (β) and 95% confidence interval (CI). We considered a two-sided P value < 0.05 to indicate statistical significance. Statistical analysis was performed in Stata version 14 (Stata Corp, College Station, TX, USA) and R version 3.5.0 (R Foundation for Statistical Computing).

---

Figure 1  Aortic Cusps.  
(A) Left aortic cusp, delineated with points. (B) Right aortic cusp. (C) Non-coronary aortic cusp.
Results

All 121 healthy participants with tricuspid aortic valve were divided by sex into two groups comprising 61 male and 60 female participants. The age ranges were 42.1 ± 14.1 years in men and 46.3 ± 15.3 years in women. All baseline characteristics are shown in Table 1.

Aortic Root Dimensions Size and Ratios

Among all participants, the men had larger aortic roots than the women, with larger dimensions across all anatomic structures (Table 1). Our data demonstrated that, across sexes, the STJ was approximately 10% larger than the VAJ, and the SD was approximately 1.375 times larger than the VAJ. Although the aortic root size was larger in men than women, the ratios of the aortic root dimensions did not significantly differ by sex (Table 1). Specifically, the ratio of the STJ to VAJ was 1.09 in men and 1.10 in women; the ratio of the SD to VAJ was 1.39 in men and 1.36 in women; and the ratio of the SH to VAJ was 0.89 in men and 0.86 in women. Among all participants, the three separate SH values were larger in men than women; the non-SH was largest and was followed by the right SH and left SH. The average eH and cH was 8.94 mm and 3.62 mm, respectively. The ratio of cH to eH was 0.42 in men and 0.40 in women; and the ratio of eH to SH was 0.42 in men and 0.43 in women. Therefore, the aortic root dimensions have specific geometric ratios that are essentially independent of sex (Figure 4).
Table 1  Healthy Participants’ Characteristics, and Aortic Root Dimensions, Sizes and Ratios.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Male (n = 61)</th>
<th>Female (n = 60)</th>
<th>Average of all</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Range</td>
<td>Value</td>
<td>Range</td>
</tr>
<tr>
<td>Age (years)</td>
<td>42.15 ± 13.03</td>
<td>19–77</td>
<td>46.30 ± 15.37</td>
<td>22–74</td>
</tr>
<tr>
<td>Weight (kg)</td>
<td>71.23 ± 9.77</td>
<td>47–95</td>
<td>57.69 ± 7.66</td>
<td>45–75</td>
</tr>
<tr>
<td>Height (cm)</td>
<td>171.58 ± 6.08</td>
<td>160–186</td>
<td>160.48 ± 5.87</td>
<td>147–174</td>
</tr>
<tr>
<td>BSA (m²)</td>
<td>1.81 ± 0.15</td>
<td>1.49–2.20</td>
<td>1.56 ± 0.11</td>
<td>1.40–1.78</td>
</tr>
<tr>
<td>STJ (mm)</td>
<td>27.04 ± 2.92</td>
<td>17.29–33.16</td>
<td>24.84 ± 2.57</td>
<td>19.77–31.57</td>
</tr>
<tr>
<td>SD (mm)</td>
<td>34.62 ± 3.03</td>
<td>29.37–43.37</td>
<td>30.76 ± 3.00</td>
<td>24.31–39.17</td>
</tr>
<tr>
<td>VAJ (mm)</td>
<td>24.98 ± 1.91</td>
<td>20.17–28.49</td>
<td>22.64 ± 1.43</td>
<td>19.34–25.17</td>
</tr>
<tr>
<td>Average SH (mm)</td>
<td>21.36 ± 2.06</td>
<td>18.20–25.59</td>
<td>18.68 ± 1.63</td>
<td>13.27–26.02</td>
</tr>
<tr>
<td>Left SH</td>
<td>20.71 ± 2.01</td>
<td>16.30–24.93</td>
<td>18.17 ± 1.60</td>
<td>15.17–22.68</td>
</tr>
<tr>
<td>Right SH</td>
<td>21.18 ± 2.04</td>
<td>16.77±32.17</td>
<td>18.51 ± 1.61</td>
<td>15.2–22.38</td>
</tr>
<tr>
<td>Non-SH</td>
<td>22.18 ± 2.12</td>
<td>17.35–28.31</td>
<td>19.35 ± 1.68</td>
<td>16.06–23.67</td>
</tr>
<tr>
<td>eH (mm)</td>
<td>9.27 ± 1.16</td>
<td>6.65–11.61</td>
<td>8.61 ± 1.05</td>
<td>6.50–11.33</td>
</tr>
<tr>
<td>cH (mm)</td>
<td>3.89 ± 0.68</td>
<td>2.76–5.55</td>
<td>3.35 ± 0.53</td>
<td>1.95–4.89</td>
</tr>
<tr>
<td>STJ/VAJ</td>
<td>1.09 ± 0.13</td>
<td>0.76–1.35</td>
<td>1.10 ± 0.12</td>
<td>0.88–1.48</td>
</tr>
<tr>
<td>SD/VAJ</td>
<td>1.39 ± 0.12</td>
<td>1.14–1.62</td>
<td>1.36 ± 0.13</td>
<td>1.16–1.83</td>
</tr>
<tr>
<td>SH/VAJ</td>
<td>0.89 ± 0.09</td>
<td>0.70–1.10</td>
<td>0.88 ± 0.10</td>
<td>0.59–1.18</td>
</tr>
<tr>
<td>eH/SH</td>
<td>0.42 ± 0.05</td>
<td>0.32–0.53</td>
<td>0.43 ± 0.05</td>
<td>0.33–0.53</td>
</tr>
<tr>
<td>cH/eH</td>
<td>0.42 ± 0.07</td>
<td>0.31–0.58</td>
<td>0.40 ± 0.06</td>
<td>0.24–0.53</td>
</tr>
</tbody>
</table>

Aortic Valve Cusp Surface Areas

The total aortic cusp surface areas were 947.8 ± 138.9 mm² for men and 714.9 ± 108.2 mm² for women, thus demonstrating that the men had larger aortic cusps than the women. Regardless of sex, a comparison of the three aortic cusps indicated that the non-coronary cusp (339.6 ± 57.9 mm² in men, 269.7 ± 46.1 mm² in women, P < 0.001) was largest, and was followed by the right coronary cusp (320.1 ± 52.5 mm² in men, 246.0 ± 41.8 mm² in women, P < 0.001) and then the left coronary cusp (288.0 ± 47.0 mm² in men, 226.1 ± 35.5 mm² in women, P < 0.001) (Figure 5).

Correlation between Aortic Cusp Surface Areas and Aortic Root Dimensions

Linear regression analysis indicated a good correlation between the aortic cusp surface areas and aortic root dimensions, including the STJ, VAJ, SD, SH and BSA (Table 2). Because all aortic root and aortic valve dimensions were intercorrelated, as shown above, we aimed to provide a reference to normalize the aortic root dimensions on the basis of measurements of the corresponding aortic cusp surface area. The mathematical relationship between the aortic cusp surface areas and VAJ orifice area was calculated with linear regression (Figure 6). The equation was expressed as aortic cusp surface area (mm²) = 1.512 × (VAJ(mm))² × π + 166.866, which can be applied to determine the optimal aortic root dimensions for the purpose of aortic valve repair.

Discussion

The overall prevalence of aortic insufficiency is 13% in men and 8.5% in women, as documented by color Doppler echocardiography in the Framingham...
Offspring Study [11]. The factors contributing to aortic insufficiency include aortic root dilatation causing malcoaptation of the aortic valve; aortic valve leaflet anomalies, such as prolapse; or both [12]. Traditionally, the surgical treatment for AI has been aortic valve replacement (AVR) which is highly straightforward and reproducible. Currently, AVR remains the first choice for most cardiac surgeons worldwide [13]. However, with increased understanding of the mechanism of aortic insufficiency, aortic root anatomy and geometric relationships, interest in aortic valve repair has markedly increased among cardiac surgeons [14–16]. The advantage of aortic root repair is its avoidance of any valve-related complications including bleeding, prosthetic thromboembolism, endocarditis and tissue valve deterioration [2–4]. Previous publications have shown that aortic valve repair provides better quality of life, with a minimal transvalvular gradient and normal life expectancy, than observed in a general matched population [5]. Therefore, aortic valve repair is currently recognized as a better choice than AVR in selected patients with AI, particularly young patients.

The techniques for aortic valve repair, such as modification of annuloplasty and sinus reconstruction, have substantially evolved in the past two decades [17, 18]. Considering isolated aortic valve repair, use of plasty of the STJ and VAJ to reshape the aortic root is considered crucial [19, 20].

![Figure 6](image)

**Table 2** Correlation between Aortic Valve Cusp Areas and Aortic Root Dimensions.

<table>
<thead>
<tr>
<th>Aortic valve cusp areas</th>
<th>r</th>
<th>r²</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0.11</td>
<td>0.013</td>
<td>0.21</td>
</tr>
<tr>
<td>Weight</td>
<td>0.49</td>
<td>0.239</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Height</td>
<td>0.55</td>
<td>0.298</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>BSA</td>
<td>0.48</td>
<td>0.235</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>STJ</td>
<td>0.65</td>
<td>0.418</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>SD</td>
<td>0.84</td>
<td>0.699</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>VAJ</td>
<td>0.71</td>
<td>0.508</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>SH</td>
<td>0.68</td>
<td>0.458</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>STJ orifice area</td>
<td>0.65</td>
<td>0.428</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>VAJ orifice area</td>
<td>0.72</td>
<td>0.524</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>

BSA: body surface area, STJ: sinotubular junction, VAJ: ventriculoarterial junction, SD: maximal sinus diameter, SH: sinus height.

Aortic cusp surface areas (mm²) = 1.512 × \( \left( \frac{VAJ(mm)}{2} \right)^2 \) × π + 166.866

**Figure 6** Linear Regression Correlation between Aortic Cusp Surface Areas and VAJ Orifice Area.
Dr. Schäfers proposed the method of intra-operative measurement of eH (criterion ≥ 9 mm) [21, 22]. Our data on eH (9.26 ± 1.16 mm in men, 8.61 ± 1.04 mm in women) were consistent with those studies. Despite technical progress, aortic valve repair remains technically challenging. The long-term outcomes of aortic valve repair vary widely and are highly dependent on surgeons’ experience [23, 24]. To date, no consensus exists regarding how to choose the graft size and the aortic sinus reconstruction size in valve-sparing aortic root replacement, and how to determine STJ, SD, SH and VAJ size in performing isolated aortic valve repair. In the first report of the David procedure by the pioneer Dr. Tirone David, the graft size was chosen by doubling the average height of all leaflets and multiplying by two-thirds [25]. Dr. Laurent de Kerchove believed that the height of the commissure is equal to the external diameter of the STJ and therefore applied the height of the commissure between non-coronary and left coronary leaflets to determine the graft size [26]. However, our findings revealed that the SH is approximately 20% smaller than the STJ diameter. In addition, Dr. Morishita has chosen a graft 15% larger than the average distance between commissures [27].

Herein, we sought to determine a reference normalized relationship between the aortic valve and aortic root in healthy participants with tricuspid aortic valve, to provide an objective method for guiding aortic valve repair. Our research question was how to accurately determine this normalized relationship. As we observed, the entire aortic root dimensions should match its own corresponding aortic cusp surface areas, to achieve normal aortic valve function. Therefore, we hypothesized that the aortic cusp surface areas could be considered the core geometric parameter for the entire aortic root dimensions during aortic valve repair. In fact, the aortic valve cusp has a specific curved 3-D structure, and the application of the 3-D reconstruction of the aortic valve cusps, as illustrated in our study, was more accurate and representative than 2-D parameters such as free margin length, cusp width and height.

Our data indicated that the STJ was approximately 10% larger than the VAJ, similarly to findings in other publications [22, 28]. However, our findings contrasted with Dr. Kunzelman’s report [29] that the VAJ is larger than the STJ. We consider this discrepancy to be due to differences in measurement circumstances: our measurement was derived from healthy individuals in diastolic phase under physiological blood pressure and cardiac cycle, whereas Dr. Kunzelman used formalin-fixed in vitro specimens under non-pressurized conditions, which might have caused a lack of aortic elasticity.

Our data also demonstrated that the SD was 1.375 times larger than the VAJ, and the SH was 12.5% smaller than the VAJ. The average eH was 8.94 mm in all healthy participants, similarly to Dr. Schäfers’s suggestion that the eH should be ≥9 mm. The eH has been shown to account for 42.5% of the SH; therefore, the coaptation line should be close to the middle of the SH. In addition, our study indicated an average coaptation height of 3.62 mm in all healthy participants, accounting for 41% of the eH.

We reconstructed aortic cusps in 3-D by using specific software based on cardiac CTA imaging to calculate the aortic cusp surface areas, whereas most previous studies have used the excised formalin fixed homograft to measure the aortic cusp surface areas in vitro. Among all participants, the total aortic cusp surface areas were 947.8 ± 138.9 mm² in men and 714.9 ± 108.2 mm² in women. Our data demonstrated that the non-coronary cusp is largest, followed by the right coronary cusp, and the left coronary cusp is smallest. More importantly, we established the mathematical relationship between the aortic cusp surface areas and the VAJ. Our ultimate goal is the application of the equation to objectively calculate the desired aortic root dimensions in patients with AI, to precisely guide aortic valve repair.

Clinical Implications and Innovation

In our study, we assembled measurement data of aortic cusp surface areas and diameters of the entire aortic root dimensions in healthy participants. We established the reference ratios of the aortic root dimensions and determined the mathematical relationship between the aortic cusp surface areas and VAJ orifice area. The predicted normalized aortic root dimensions in patients with aortic insufficiency can be calculated on the basis of pre-operative measured aortic cusp surface areas, and can be used to
precisely guide the individualized selection of graft size and reshaping of the entire aortic root during aortic valve repair.

Conclusion

The aortic root has specific geometric ratios and a mathematical relationship with corresponding aortic cusp in healthy individuals. We propose that restoration of the normalized relationship between the aortic valve and aortic root in patients with aortic insufficiency might serve as an objective method for successful aortic valve repair.

Ethics Statement

The study protocol conformed to the ethical guidelines of the 1975 Declaration of Helsinki and was approved by the Institutional Review Board of the First Affiliated Hospital of Nanjing Medical University (July 28th, 2021; No. 2021-SR-381). All participants provided informed consent.

Limitation

Aortic sizes may vary among racial groups – an aspect requiring further consideration. In addition, the number of specimens in our study was not sufficiently large. In the future, we plan to expand the number of participants to reexamine the relationship between the aortic valve and aortic root. This study focused on theoretical principles, and data from surgical practice are needed to further confirm our findings.

Author Contribution Statement

Luyao Ma designed the study and wrote the manuscript; Guanyu Yang developed the software programming and methods; Kangting Tang and Hong Liu performed the data analysis; Chengxiao Xu supported this study and contributed materials; and Yinsu Zhu and Yongfeng Shao supervised the study and revised the manuscript.

Funding

This work was supported by the Youth Foundation of Natural Science Foundation of Jiangsu Province (grant number: BK20191069).

Conflict of Interest

None declared.

Data Sharing Statement

The data underlying this article will be shared upon reasonable request to the corresponding author.

REFERENCES