

How to calculate the McNemar mid- p test

(Supplementary materials)

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Introduction

To calculate the McNemar mid- p test, we start by calculating the McNemar exact conditional test. We condition on the number of discordant pairs ($n_{12} + n_{21}$), such that n_{12} is binomially distributed with parameters $n = n_{12} + n_{21}$ and $p = 0.5$. As shown in the main document, the one-sided exact conditional p -value equals the probability of at least n_{12} successes out of n binomial trials, and the two-sided p -value is twice the one-sided p -value:

$$\text{exact } p\text{-value} = 2 \cdot \text{binomcdf}(n_{12}, n, 0.5), \quad (1)$$

where $\text{binomcdf}()$ is the binomial cumulative distribution function. Here, we have assumed—without loss of generality—that $n_{12} < n_{21}$. If $n_{12} = n_{21}$, the exact p -value equals 1.0.

We calculate the mid- p test by subtracting half the probability of the observed n_{12} from the exact one-sided p -value, then double it. For that we need the binomial probability density function, denoted by $\text{binompdf}(n_{12}, n, 0.5)$. The mid- p -value is thus

$$\begin{aligned} \text{mid-}p\text{-value} &= 2 \cdot \left[\text{binomcdf}(n_{12}, n, 0.5) - \frac{1}{2} \text{binompdf}(n_{12}, n, 0.5) \right] \\ &= \text{exact } p\text{-value} - \text{binompdf}(n_{12}, n, 0.5). \end{aligned} \quad (2)$$

If $n_{12} = n_{21}$, the mid- p -value equals $1 - 0.5 \cdot \text{binompdf}(n_{12}, n, 0.5)$.

The functions $\text{binomcdf}()$ and $\text{binompdf}()$ are readily obtained for most standard software packages. We illustrate the calculations of the mid- p test using data from the study by Bentur et al. [1] given in Table 1 in the main document, which we repeat here.

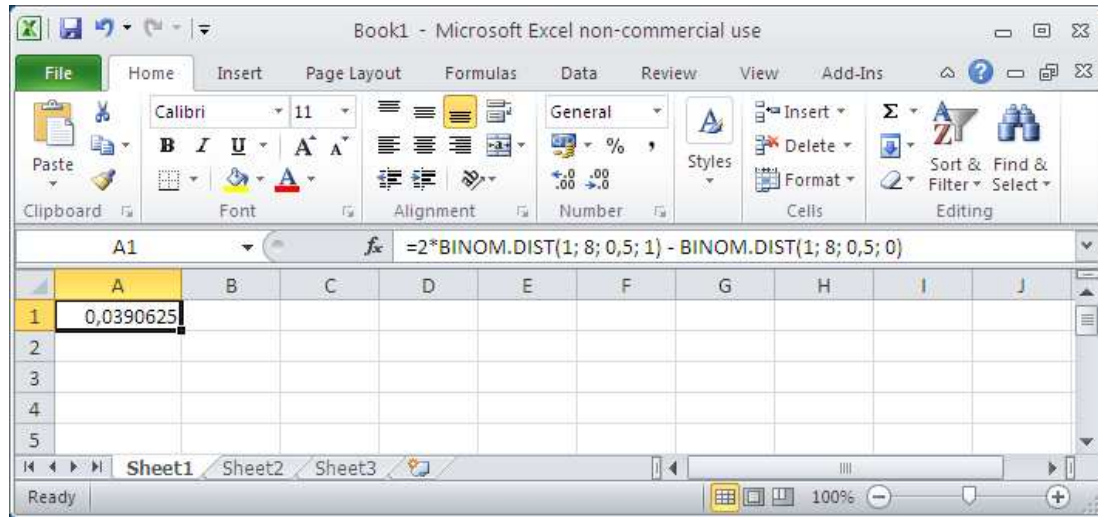
Table 1. Airway hyper-responsiveness (AHR) status before and after stem cell transplantation (SCT) in 21 children [1].

		After SCT		Sum
		AHR	No AHR	
Before SCT	AHR	1	1	2
	No AHR	7	12	19
	Sum	8	13	21

We have only checked the below calculations for the noted versions of the software packages and make no claim that the same approach is applicable to other versions.

1. Microsoft Excel (version 14)

The function `BINOM.DIST(x, n, p, cumulative)` returns both the point probability (`cumulative=0`) and the cumulative probability (`cumulative=1`) of the binomial distribution. Using the data in Table 6, the exact conditional p -value is obtained as `=2*BINOM.DIST(1; 8; 0,5; 1)`, and the mid- p -value is obtained as shown in the figure below. If $n_{12} = n_{21}$, the exact conditional p -value equals 1.0, and we obtain the mid- p -value using the expression `=1-BINOM.DIST(n_{12} ; n; 0,5; 0)`.



2. Matlab (version R2011b)

The two functions needed to calculate the mid- p test are available with the Statistics Toolbox add-on package. They are `binocdf(x, n, p)` and `binopdf(x, n, p)`. For the data in Table 6, we obtain the exact conditional and mid- p tests as shown below. When $n_{12} = n_{21}$, the exact conditional p -value equals 1.0, and the mid- p -value is calculated as `midp = 1 - 0.5*binopdf(n_{12} , n, .5)`.

```
Command Window
>> p = 2*binocdf(1, 8, .5)

p =

    0.0703

>> midp = p - binopdf(1, 8, .5)

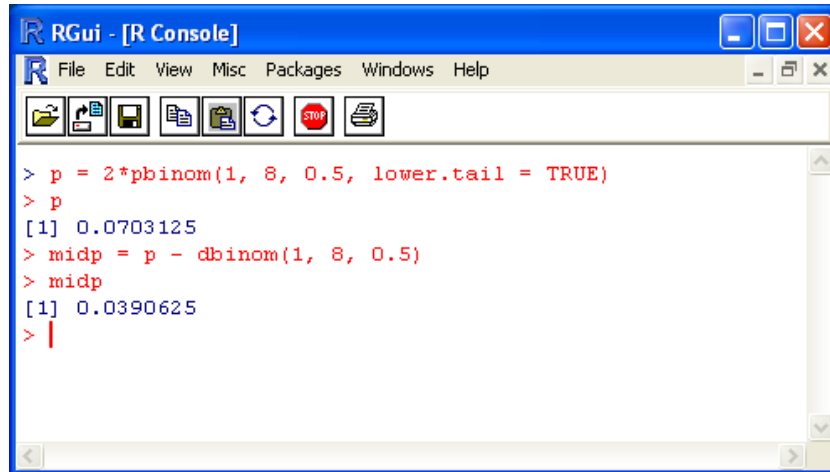
midp =

    0.0391

fx >>
```

3. R (version 2.11.1)

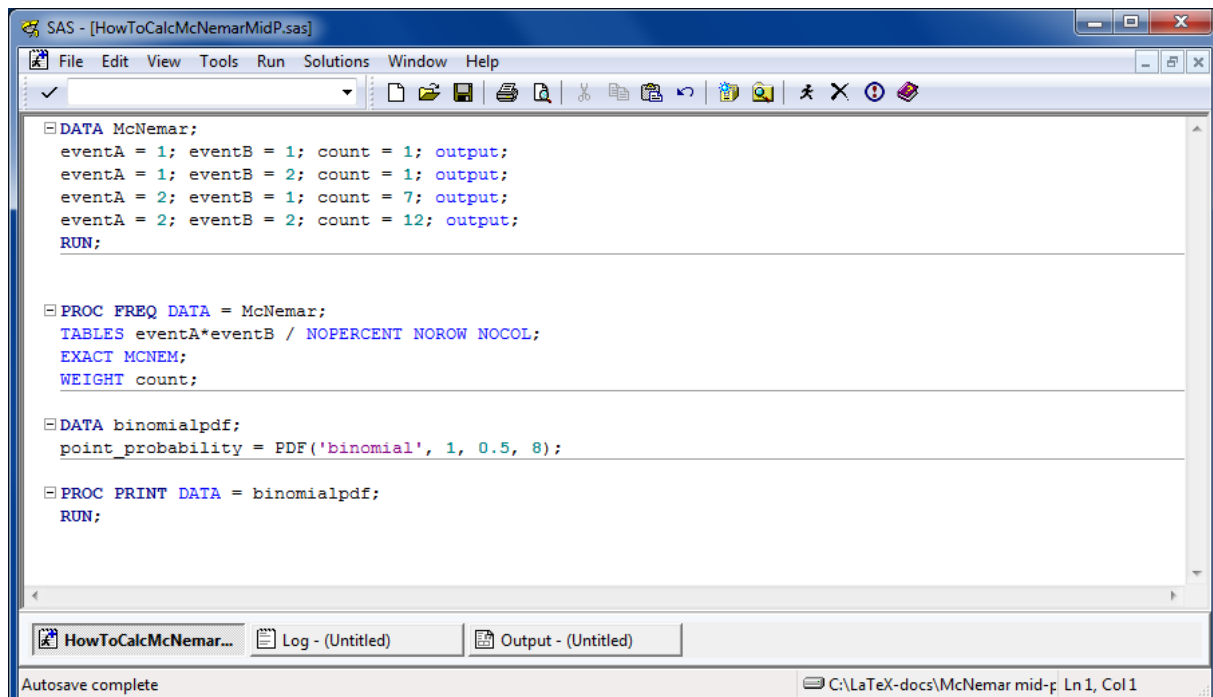
`pbinom(x, n, p, lower.tail = TRUE)` and `dbinom(x, n, p)` compute the binomial cumulative distribution function and the binomial probability density function, respectively. The exact conditional and mid- p tests for the data in Table 6 can be calculated as shown below. If $n_{12} = n_{21}$, let the exact conditional p -value be 1.0 and compute the mid- p -value as $\text{midp} = 1 - 0.5 \cdot \text{dbinom}(n_{12}, n, 0.5)$.



```
RGui - [R Console]
File Edit View Misc Packages Windows Help
[Icons]
> p = 2*pbinom(1, 8, 0.5, lower.tail = TRUE)
> p
[1] 0.0703125
> midp = p - dbinom(1, 8, 0.5)
> midp
[1] 0.0390625
> |
```

4. SAS (version 9.2)

The three figures below show how to calculate the exact conditional and asymptotic tests and the point probability of the observed table. The mid- p test is calculated as $\text{mid-}p\text{-value} = 0.0703 - 0.03125 = 0.0391$. If $n_{12} = n_{21}$, we obtain the mid- p -value as $1 - 0.5 \cdot \text{point_probability}$.



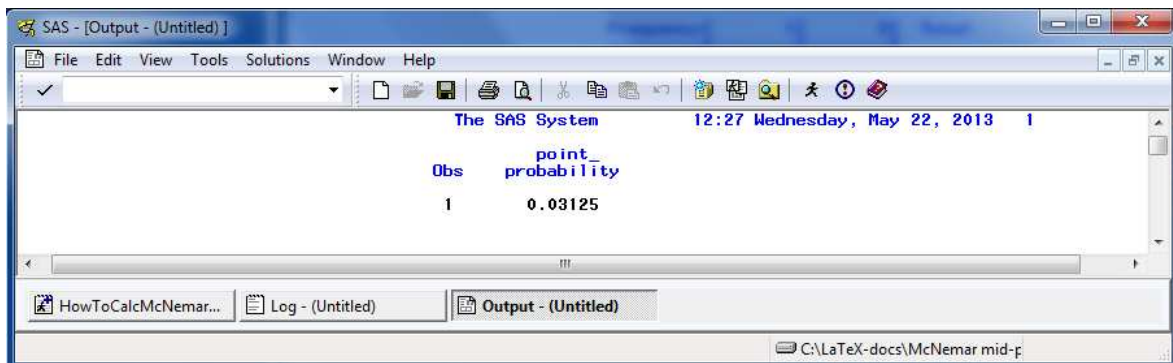
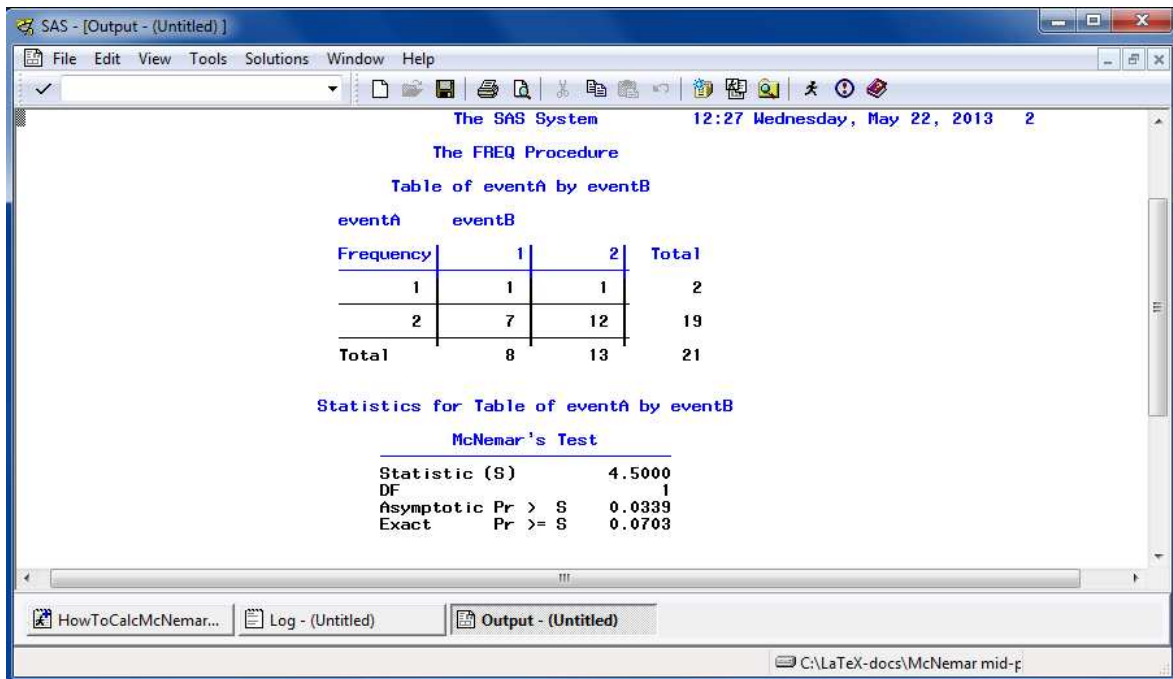
```
SAS - [HowToCalcMcNemarMidP.sas]
File Edit View Tools Run Solutions Window Help
[Icons]
DATA McNemar;
  eventA = 1; eventB = 1; count = 1; output;
  eventA = 1; eventB = 2; count = 1; output;
  eventA = 2; eventB = 1; count = 7; output;
  eventA = 2; eventB = 2; count = 12; output;
RUN;

PROC FREQ DATA = McNemar;
  TABLES eventA*eventB / NOPERCENT NOROW NOCOL;
  EXACT MCNEM;
  WEIGHT count;

DATA binomialpdf;
  point_probability = PDF('binomial', 1, 0.5, 8);

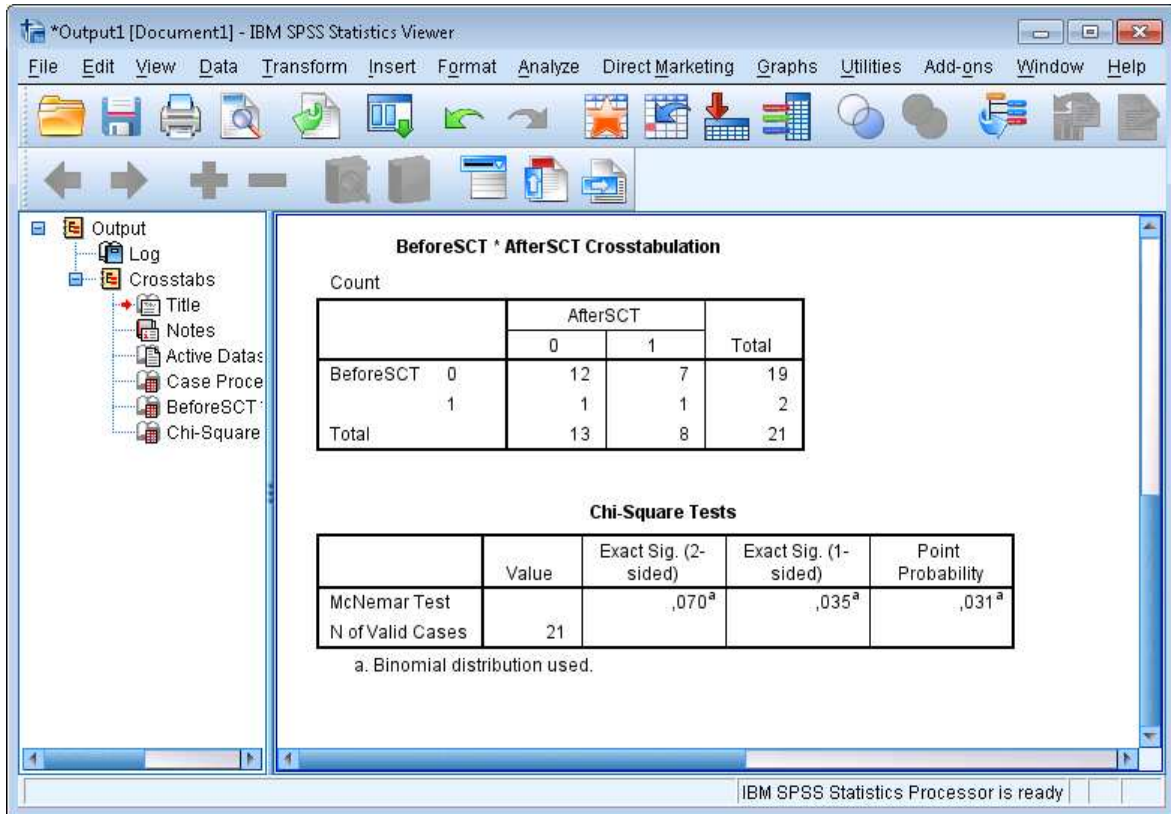
PROC PRINT DATA = binomialpdf;
  RUN;

HowToCalcMcNemar... Log - (Untitled) Output - (Untitled)
Autosave complete C:\LaTeX-docs\McNemar mid-p Ln 1, Col 1
```



5. IBM SPSS Statistics (version 20)

We use the function **Analyze -> Descriptive Statistics -> Crosstabs**. Put the appropriate variables in the boxes named “Row(s)” and “Column(s)” and press the “Exact” button. Select the “Exact” option and press “Continue”. Press the “Statistics” button and place a checkmark next to “McNemar”. Press “Continue”, then “OK”. The McNemar mid-*p*-test is obtained by subtracting the value of “Point Probability” from “Exact Sig. (2-sided)”: mid-*p*-value = 0.070 – 0.031 = 0.039. When $n_{12} = n_{21}$, calculate the mid-*p*-value as $1 - 0.5 \cdot \text{Point Probability}$.



The screenshot displays the IBM SPSS Statistics Viewer interface. The main window shows the output of a Crosstabs analysis. The left sidebar contains a tree view with the following items: Output, Log, Crosstabs, Title, Notes, Active Data, Case Proce, BeforeSCT, and Chi-Square. The main content area is titled "BeforeSCT * AfterSCT Crosstabulation" and contains two tables.

BeforeSCT * AfterSCT Crosstabulation

Count		AfterSCT		Total
		0	1	
BeforeSCT	0	12	7	19
	1	1	1	2
Total		13	8	21

Chi-Square Tests

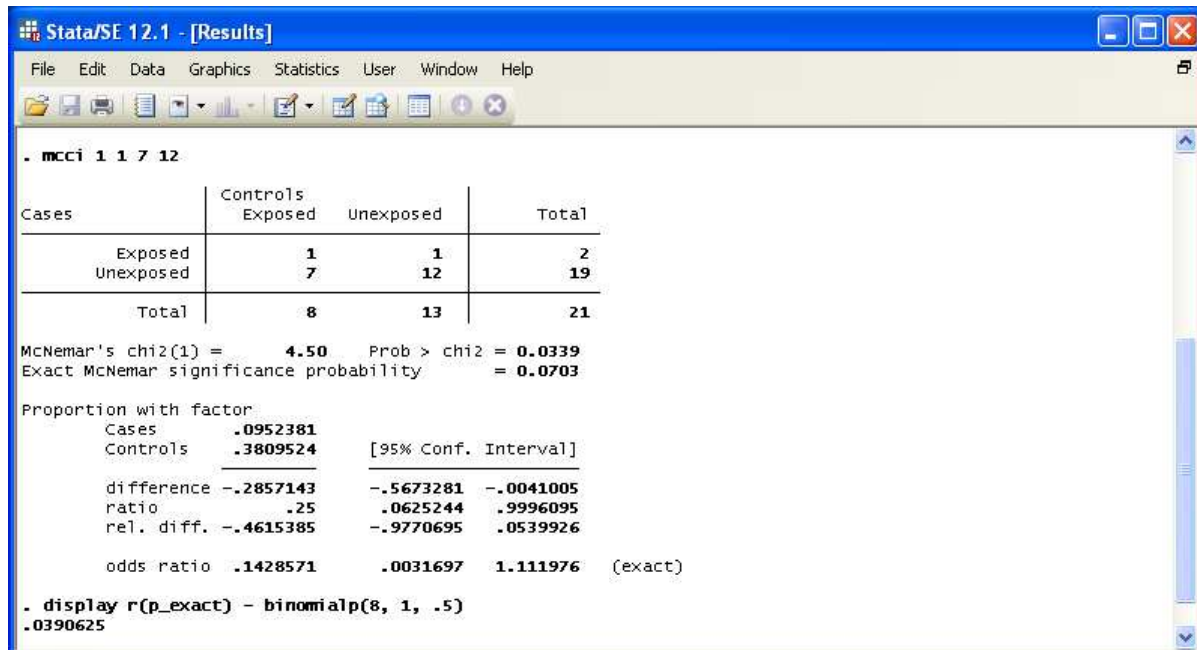
	Value	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
McNemar Test		,070 ^a	,035 ^a	,031 ^a
N of Valid Cases	21			

a. Binomial distribution used.

IBM SPSS Statistics Processor is ready

6. Stata (version 12.1)

The exact conditional and asymptotic McNemar tests are obtained with the commands `mcc` and `mcci`, where the first command uses variables from the current dataset and the second command is a calculator version. Here, we show how to use the calculator version. The probability of the observed table is obtained using the function `binomialp(n, x, p)`. We calculate the mid- p test for the data in Table 6 as shown below. If $n_{12} = n_{21}$, we obtain the mid- p -value as `display 1 - 0.5*binomialp(n, n12, .5)`.



```
. mcci 1 1 7 12
```

Cases	Controls		Total
	Exposed	Unexposed	
Exposed	1	1	2
Unexposed	7	12	19
Total	8	13	21

```
McNemar's chi2(1) = 4.50 Prob > chi2 = 0.0339  
Exact McNemar significance probability = 0.0703
```

```
Proportion with factor  
Cases .0952381  
Controls .3809524 [95% Conf. Interval]
```

difference	-.2857143	-.5673281	-.0041005
ratio	.25	.0625244	.9996095
rel. diff.	-.4615385	-.9770695	.0539926

```
odds ratio .1428571 .0031697 1.111976 (exact)
```

```
. display r(p_exact) - binomialp(8, 1, .5)  
.0390625
```

7. StatsDirect (version 2.5.6)

In StatsDirect, both the exact conditional test and the mid- p test are given when using **Analysis** -> **Proportions** -> **Paired**. The p -value of the exact test is given as "Exact two sided P", and the mid- p -value is given as "Exact two sided mid P". When $n_{12} = n_{21}$, StatsDirect gives **Exact two sided P** > 0,9999 and **Exact two sided mid P** > 0,9999.

Tests on proportions

Proportion Tests

- Single
- Paired
- Two independent

TOTAL number in study

Number responding in BOTH categories

Number responding in FIRST category only

Number responding in SECOND category only

Confidence (%)

Paired proportions

Total = 21, both = 1, first only = 1, second only = 7
Proportion 1 = 0,095238
Proportion 2 = 0,380952
Proportion difference = -0,285714

Exact two sided P = 0,0703
Exact one sided P = 0,0352

Exact two sided mid P = 0,0391
Exact one sided mid P = 0,0195

Score based (Newcombe) 95% confidence interval for the proportion difference:
-0,50692 to -0,025559

8. StatXact (version 9)

The p -value of the exact conditional test and the point probability of the observed table are obtained from Nonparametrics \rightarrow Paired Samples \rightarrow McNemar. In the “Inference” table, the p -value of the exact conditional test is given in the cell with row name “Exact (Conditional)” and column name “2*1-Sided”. The point probability of the observed table is given in the cell with row name “Exact (Conditional)” and column name “Point Prob.”, as shown below. The mid- p test can be calculated as mid- p -value = $0.07031 - 0.03125 = 0.0391$. If $n_{12} = n_{21}$, calculate the mid- p -value as $1 - 0.5 \cdot \text{Point Prob.}$

TableData McNemar's Test [EX]

McNemar's Test

mcnemars (method = exact, time_limit = none);

Data File: TableData

Number of Rows: 2
Number of Columns: 2

Summary of the Test Statistic:

Minimum	Maximum	Mean	Std. Dev	Observed	Standardized
-8	8	0	2.828	-6	-2.121

Inference:

Type	Statistic	Tail	P-Value		
			1-Sided	2*1-Sided	Point Prob.
Asymptotic	-2.121	.LE.	0.01695	0.03389	
Exact (Conditional)	-2.121	.LE.	0.03516	0.07031	0.03125
Exact (Unconditional)	-2.121	.LE.	0.01766	0.03532	

For Help, press F1

References

- [1] Bentur L, Lapidot M, Livnat G, Hakim F, Lidroneta-Katz C, Porat I, Vilozni D, Elhasid R: **Airway reactivity in children before and after stem cell transplantation.** *Pediatr Pulm* 2009; **44**:845–850.