Parametric model order reduction for large-scale systems using Krylov subspace

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Problem Setup

Applications Design optimization, parameter estimation, model-based control design, simulation...

Example Parameter calibration

Thm: Given a system Σ of state dimension n . Let $(s_0, p_0) \in \mathbb{C} \times \mathbb{P}$ and $r < n$. The reduced order system Σ_r of the state dimension r satisfies

$$\eta_m(s_0, p_0) = \eta_{r,m}(s_0, p_0), \quad \text{for } |m| = 0, \dots, M.$$

m th moment of Σ

m th moment of Σ_r

Error bound

- Mean value theorem

$$f(b) = f(a) + R_n(b)$$

- Remainder in Lagrange form

$$R_n(b) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (b-a)^{n+1}$$

- Error bound

$$\|f(x) - f_n(x)\| \leq \mathcal{M} \left| \underbrace{[x-a] \otimes \dots \otimes [x-a]}_{n+1 \text{ copies}} \right|; \forall x \in I_a$$

$$\mathcal{M} := \sup_{\xi \in I_a} \left\| \frac{D^{(n+1)} f(\xi)}{(n+1)!} \right\|$$

Prior computed value with the given reduced order and expansion points

Contribution II

- Prior error bound is proposed and it enables finding optimal expansion points and optimal reduced order, iteratively.

Project statement

Model order reduction techniques are needed for designs in ultra-high precision systems. Of particular interest is the preservation of the physical relevance of the reduced model. We present a parametric model order reduction method with explicit error bound.

Multi-frequency and multi-parameter moment-matching

Assumptions

- Parameters are not varying with time
- System matrices are affine functions of parameters.

Laplace variables and physical parameters both expand

Contribution I

- Generalize moment-matching method and extend to multi-frequency and multi-parameter matching

A heat transfer case study

Goal:

Reduce the model while preserving the heat capacitance parameter

Input:

heat generated by the coil

Output:

Temperature at POIs

$$c_p \rho \cdot \dot{T} = \Delta \cdot (\kappa \Delta T) + q$$

$$T(0) = T_0$$

$$\begin{cases} E(p) \dot{T} = AT + Bu \\ y = CT \end{cases}$$

With $\dim T(t) \in \mathbb{R}^{6539}$ and $C_p = [200, 500, 900]$

Reduced model: $T_r(t) \in \mathbb{R}^{32}$ and $E_r(p) = E_{r,0} + E_{r,1} C_p$

I. D. Lou and S. Weiland. Parametric model order reduction for large-scale and complex thermal systems. Control Conference (ECC), 2018 European, 2018